Arbeidsnotater

STATISTISK SENTRALBYRÅ

Dronningensgt. 16, Oslo-Dep., Oslo 1. Tlf. 41 38 20

WORKING PAPERS FROM THE CENTRAL BUREAU OF STATISTICS OF NORWAY

IO 76/11 8 April 1976

AN APPROACH TO CONSISTENT AND EFFICIENT PLANNING

by

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Acknowledgement

I am deeply honored to have been invited by different Norwegian planning agencies to observe how the planning machinery operates, particularly in Norway and to some extent in other countries.

I have spent most of the time of my stay in Norway in the Research Division of the Central Bureau of Statistics. I am very thankful for the time and considerable generosity given to me by several members of the research staff. Mr. Olav Bjerkholt has been responsible for coordinating my study program and I am very thankful to him for preparing a highly intensive introduction to Norwegian planning models. I am deeply grateful for the encouragement Dr. Petter Jakob Bjerve, Dr. Odd Aukrust, Mr. Per Sevaldson and Mr. Arne Amundsen, have given me to pursue the work along the lines of rational planning. On special questions I have also been helped by Mr. Erik Homb, Mr. Svein Longva, Mr. Nils Terje Furunes and Mr. Lorents Lorentsen. Miss Benthe Ringstad has skilfully typewritten my notes to my greatest satisfaction.

During my three weeks work at the Institute of Economics at the University of Oslo I have acquired an excellent knowledge of the planning problems and the possible approaches, and I feel it as a pleasant duty to give due credit to Professor Leif Johansen who has contributed much to my work at the University. I must also acknowledge pertinent suggestions received from Professor Tore Thonstad and Professor Preben Munthe, Mr. Michael Hoel and Mr. Tore Johansen.

I must also acknowledge pertinent discussions with a number of civil servants of the Royal Norwegian Ministry of Finance, in particular with Mr. Amund Utne of the Planning Division, and Mr. Fred Olav Sørensen of the Economic Division.

My program in Norway has included a profitable visit to the Chr. Michelsen Institute at Bergen. I must express my thanks to the Director and the members of the Development Economics Research and the Advisory Project, in particular, to Mr. Just Faaland and Mr. Carl Anonsen.

Finally, during my stay in Norway - January through April, 1976, - I have learned a great deal on how to narrow the gap between theoretical

analysis and its application, and I am deeply impressed by what I have seen. For this I am very thankful. The following is a collection of notes on rational planning, as a first draft presentation.

Oslo, April, 1976

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"What is most often lacking is the statistical base. But the input-output table is now being very properly used in about 120 countries. If I had to say which country makes the best use of it, I should probably say Norway."

An Interview
with
Wassily Leontief
Economic Impact
Washington 1975/1

INTRODUCTION

During the elaboration of a development plan, a number of alternatives are usually prepared. The task of the coordination unit is to appraise their consistency and efficiency and to evaluate their effectiveness in realizing the country's main objectives. In the absence of preconditions for constructing optimization models, techniques have to be developed in order to evaluate alternatives, introduce amendments, and suggest economic policy instruments, so as to approach optimality as far as possible.

Assume that there are only two prepared alternative plans. The task then is to select one of them. One approach is to construct a formalized model which will not be considered as a third alternative but to use it as a test to see which one of these alternatives pass in terms of consistency and efficiensy. This paper is concerned with this approach.

There is, however, another approach which transfers the targets of one alternative to be constraints in the other alternative. This approach will be considered in a separate paper: Criteria for Evaluation of National Plans, which is under way. These two papers along with another study: A Discussion with Norwegian Model-Builders, though each study can be considered as self-contained to some extent, will be revised to form one topic: Plan Appraisal and Plan Evaluation, suggested by the Iraqi Ministry of Planning and sponsored by the United Nation Development Program.

The main theme of all of these studies is based on narrowing the gap between theoretical analysis and its practical application in a developing country under conditions of relative abundance of capital resources, with rapid growth envisaged for the Iraqi economy, but with limited absorptive capacity and other physical and human constraints.

PLANNING: PROBLEMS AND APPROACHES

The Planning Problem

The planning problem can be interpreted as a constrained maximization type |1| or in the form of generating feasible plans by evaluating all constraints in the economy |2|. The planning procedure of the first type of the planning problem can be summarized as follows: the object is to choose the value of a variable, say x, in such a way as to make the value of something dependent upon x, say f(x), as large as possible. If the maximization problem is unconstrained, i.e. there is no restrictions on the values of x that might be chosen, then the solution is to find the value of x at which $\frac{df}{dx} = 0$, and the second order conditions for maximum are satisfied.

If the maximization problem is constrained, i.e. x must be chosen from within a specified set of values, say x must be positive, or x must not exceed some number m in absolute value, then the solution is to choose x so as to maximize f(x) subject to g(x) = 0. The term f(x) is referred to as the objective function which has to be constructed and its nature has to be examined |3|. The term g(x) represents resource constraints and technical constraints governing production. The certain features of these constraints is that they can be varied within limits over long enough time periods and their variations depend on the economic and social plans being adopted and their time-horizon.

The planning procedure of the second type of the planning problem could be summed up as follows. The planner collect and evaluate information about the future and confronting the existing constraints with target values that are either feasible or infeasible. The criterion of the optimal plan then is to choose an efficient target vector t in such a way that there is no other feasible t' that is at least as great as in all components, and strictly greater in some.

Some planners prefer adopting the first type of the planning problem over the second. For instance Heal |1| on page 23, has preferred the first type. His conclusion is that a constrained maximization type is required to be solved in all cases. If feasible targets are chosen, then it is needed to investigate about the efficiency of the plan. If infeasible targets are chosen, then it is required to find the feasible stat of the economy.

However, in this paper, the two types of the planning problem are assumed to be two alternatives for the following reasons: (i) because of considerable diffisulties involved in constructing an objective function needed for the first type, the second type has been used in practice, and (ii) the preference of adopting one over the other depends on certain circumstances related to the stage of social and economic plan-making development and the plan coverage. In fact, the second type of the planning problem is adopted for comprehensive national plans while the first type is related to sectoral, regional, or partial plans, since the aim of this study is to narrow the gap between theoretical analysis and its applications.

Possible Approaches

The possible approaches to the two types of the planning problem may be isolated according to distinct characteristics. For the sake of discussing consistent and efficient plans, it seems desirable to characterise these different approaches separately. They are: prediction, conventional, optimization and compromise approaches.

(i) Prediction Approach

Some planners believe that the future development of a country is determined by its given existing conditions. This approach has different implications. If it is based on merely forecasting or systematizing alternative guesses without attempting to influence the course of affairs, then the attitude of these planners is simply that of "an onlooker" to use a Frisch phrase |4|. If this approach is based on equalizing abstraction with the reality, then it would be a "fatalistic philosophy of planning", Kornai |2|. For example, using a static Leontief-model where no substitution between activities is assumed for the sake of simplicity. This is an abstraction and not an attribute of reality.

However, these planners rely heavily on econometric and statistical techniques for estimating future needs and long-run trends. The results of this approach supplemented by value judgment are to be taken for planning purpose. In discussing the difference between econometric forecast and judgment forecast one has to consider that the former is not a

substitute but a supplement to the latter. Econometric forecast yields consistent forecast components, save labor, and protect objectivity. The econometric forecasters can identify where they went wrong. They can also analyze the impact of particular policy actions that are proposed |5|.

(ii) Conventional Approach

If emphasis is placed on influencing the structure of the economy by using some instruments, then this philosophy may take the "ad-hoc instrument approach", Frisch |4|. The inadequacy of this approach is implied by many preconditions, such as: a predetermined set of objectives is required, a number of feasible plans are to be defined, data on which the plans are based are reasonably accurate and complete, the resulting plans are internally consistent (see below), and the social needs or the objectives of the decision-makers are adequately reflected by the targets |3|. Furthermore, an important question related to the use of this approach is when and what instrument is to be used |4|.

Though there are techniques |3| based on minimizing the loss, if any, in not meeting all the targets, and on sensitivity analysis of various combinations of loss functions to estimate the relative importance of the objectives, there are difficulties for applying them in practice. The main source of these difficulties is based on the definition of the approporiate targets and relative loss functions.

(iii) Optimization Approach

The philosophy of this approach is closely realted to conventional approach in that both require a priori knowledge of the decision maker's preferences. But preferences are assumed here as objectives rather than targets. A set of feasible plans is given. The task of the planners is to seek optimal solutions in light of these objectives, given the available resources. |2|, |3|, |4|. This approach may be taken as a solution of the first type of the planning problem. The solution is in the form of an optimizing problem of a constrained minimum or maximum case: the planner should choose that plan which satisfies the feasibility constrained (i.e. the consistency problem) and which is preferred to any other feasible plan (i.e. the efficiency problem).

The central problem in this philosophy is that the objectives of the political decision-maker are not well-defined before the beginning of the planning process. This is so, because they do not know what they want prior to knowing what they can have. On the other hand, the short-run problems of less developed countries are very specific and do not lend them selves easily to treatment by models based on this approach |6|.

(iv) Compromise Approach

The philosophy of this approach is based on generating feasible plans by evaluating all quantitative and qualitative social, economic, and political factors that will affect the future development of a country in question |2|. This approach is taken as the solution to the second type of the planning problem. The main difference between the two solutions is that preferences are revealed only ex post after evaluating comparable alternatives in the second type where as preferences are revealed ex ante in the first type based on a preconstructed objective function.

In accordance with the aim of this paper and because the preconditions for optimization are absent in the less developed countries, the second type of the planning problem is assumed and thus the compromise approach will be adopted hence forth.

PLANNING: CONSISTENCY AND EFFICIENCY

Plan Consistency

Any particular approach to any type of the planning problem will meet different requirements for consistency in different degrees. Richard Stone |7|, has presented seven classes of consistency. Their relevancy to the present study permits listing them and comment on them. They are:

(1) Consistency with Arithmetic Identities.

(2) Consistency with Accounting Identities.

(3) Consistency with what we know about Past Behaviour and Technology.

(4) Consistency with what we expect about Future Behaviour and Technology.

(5) Consistency with Transitional Possibilities.

(6) Consistency with all Aspect of the Problem.

(7) Consistency with all our long-term Aims.

One added:

(8) Consistency with International Relations.

The philosophy which guides the suggested approach should now be clear. Any other approaches can be passed on the basis of some of the requirements. Requirements (1), (2) and (3) may be more relevant to the prediction approach. If one added requirement (4) and (5), then conventional and optimization approaches may pass. But for the compromise approach, all seem to be relevant and that is the definition of consistent planning we may suggest.

Richard Stone's analysis was independent of the degree of aggregation and made it to be applied as much to high aggregation as to high disaggregation. The aggregation problem may be defined as a process where by a part of the information available for the solution of a problem is sacrificed for the purpose of making the problem more easily and manageable |8|. While consistent aggregation may be defined as reaching the same results of the analysis of the problem at hand from the use of information more detailed than that contained in the aggregation |8|. However John Green left some factors to be studies later.

Leif Johansen |9| presented a study where the relationships between the detailed description and the aggregated description of policy, exogenous factors and resulting states of the economy were analyzed. The analysis included both quantitative variables and qualitative factors, with aggregation problems encountered in planning. These problems were grouped in three categories. The first is related to "operationality": An operational plan is that one which contains specific meaningful messages to the agents to execute the various parts of the plan. The second is concerned with "evaluation": The description has to be as detailed as required by the authority in order to judge the desirability of the state. The third category deals with problems of "inference" or "accuracy" of analysis which is related to the previous two categories, and with respect to the plan elaboration.

Plan Efficiency

If a national development plan was to be evaluated in terms of a single objective, then the task of evaluation would not be so difficult as if there were multiple objectives. But in actual planning there are several objectives and some of them conflict with each other. For example, the Iraqi national preference composes two objectives: accelerating economic growth and equity in distribution of income. Therefore, there is a strong need for techniques for evaluation based on multiple objectives. One common procedure may be used in such a case is that of trade-offs between the different kinds of objectives.

Examining the mentioned welfare function, one realizes by inspection that there are two distinct and separate parts. The first is related to economic efficiency of production based on objective analysis where optimization techniques can be applied. The second part is concerned with social efficiency of distribution based on ethical and subjective judgment where optimization techniques can't be applied. Thus can the two parts be combined?

To some economists, the two parts cannot be combined |10|, while to others, the combination of the two parts is conditioned on appropriate specification of welfare function, and a simple process for the determination of a social welfare function does not exist |11|.

After Arrow's demonstration of the impossibility theorem which is based on ordinary polls to obtain a useful assessment of a social welfare function, ideas such as the search for dominance among subsets of the alternatives were formalized |12|. Dominance exists if one alternative results in an equal or higher value for all objectives than do all other alternatives, and if at least one objective has a strictly higher value. Dominance can be a useful method for reducing the number of alternatives |2|. The results can be obtained in practice, by compromises between members of a group.

One main method for evaluating economic efficiency such as consumers' surplus (as utility in terms of money,) based on willingness to pay |13| can not be taken as the criterion for the allocation of public resources regardless of who receives the benefits. Hence, a distributional criterion such as the compensation principle based on Pareto optimality |14| is needed. But the value of compensation is difficult to be determined, especially on the national level, and particularly in less developed countries.

Other criteria such as cost benefit method assume the existence of a unique objective function, and it also has other deficiencies with respect to monetary valuation and measurements of benefits and costs |15|.

Since the theoretical literature does not provide an operational procedure for plan efficiency evaluation, and since the Iraqi national preference combines both quantitative and qualitative objectives, further search, especially on the practical level, is desirable. The evaluation of any plan requires a careful balance between the economic efficiency and the social efficiency.

The following section presents examples on plan consistency and plan efficiency. The basic principles of such evaluations are outlined but those who require a more detailed exposition of the subject should refer to some of the work listed in the Notes and References.

APPLICATIONS: APPRAISED AND EVALUATED CASES

In order to develop practical criteria suitable to the Iraqi conditions, one has to simulate actual planning procedures so as to learn about their properties with respect to accuracy, evaluation and operationality.

Example 1. Informational Decentralization Procedure

The first example illustrates a procedure for decentralizing plan elaboration based on iteration method in solving information problem with the achievement of consistency as the main objectives. The iteration method is to indicate a series of simple steps which approaches approximations to the solution of a big problem. The main purpose of this example is to show that in the process of convergence towards the correct solution of overall problem will be much consistent and efficient if it can be subjected to intuitive evaluations based on general insight and practical experience. We shall follow the path of Leif Johansen |16| in presenting how a Central Planning Agency (CPA) and a series of lower level units can cooperate in eleborating a plan without using optimization approaches. But let the CPA informs each of the production sectors about the desired final output plus intermediate output from the sector:

$$(1) \quad X_1^{(1)} = \bar{X}_1,\ X_2^{(1)} = \bar{X}_2,\ \ldots\ X_n^{(1)} = \bar{X}_n$$

where $X_1, \ldots, X_n$ are total output from the various sectors, and $Y_1, \ldots, Y_n$ represent final demand, and $\bar{X}_j^{(S)}$ is proposed output plan for sector No. j in stage No. S of the procedure.

Assume that the sectoral input-output coefficent (A) in the matrix form: $X = AX + Y$, i.e. a_{ij} is known to the sector but not to the CPA. Then the needed input deliveries will be calculated by the sector:

$$(2) \quad X_{1j}^{(2)} = a_{1j}X_j^{(1)}, \ \ldots, \ X_{nj}^{(2)} = a_{nj}X_j^{(1)}$$

and to be compared with the proposed output. The sector informs the CPA about the result, (i.e. consistency). If the proposed output is met by the input requirements with no other effects on other sectors, then the plan is said to be efficient.

This type of procedure will be carried on between the CPA and each sector. The CPA will add all the requested input deliveries for intermediate goods. Then the output plans including the final demand for the output produced by sector 1 is

$$(3) \quad X_1^{(2)} = Y_1 + \sum_{j=1}^{n} X_{1j}^{(2)}, \ \ldots, \ X_n^{(2)} = Y_n + \sum_{j=1}^{n} X_{nj}^{(2)}.$$

From the given an arbitrary initial approximation to the solution, this procedure indicates how the solution must be modified in order to produce a better approximation to it. Successive applications of the modification procedure lead to better and better approximation. This is known as an algorithm or routine planning process.

The exchanges between the CPA and the sectors continue until all sectors are covered, then:

$$(4) \quad \begin{aligned} X_1^{(S)} &= Y_1 + \sum_{j=1}^{n} a_{1j}X_j^{(S-1)} \\ &\cdot \quad\quad \cdot \quad\quad \cdot \\ &\cdot \quad\quad \cdot \quad\quad \cdot \\ &\cdot \quad\quad \cdot \quad\quad \cdot \\ X_n^{(S)} &= Y_n + \sum_{j=1}^{n} a_{nj}X_j^{(S-1)} \end{aligned}$$

or in the matrix form:

$$(5)\quad X^{(S)} = Y + AX^{(S-1)}$$

Either of the last two results corresponds to a well known method of iterative solution of linear equations. This procedure is equivalent to a step-wise elaboration of a consistent plan. By a repeated application of (5) the result will be:

$$(6)\quad X^{(S)} = (I + A + A^2 + \ldots A^S)Y$$

where I is the unit matrix with n element. The expansion formula for inverse of the matrix (I-A) is:

$$(7)\quad (I-A)^{-1} = I + A + A^2 + A^3 \ldots$$

and the solution of the system X = AX + Y is:

$$(8)\quad X = (I-A)^{-1}Y$$

The central idea, here, is that the series in (6) after stage S approximates the correct solution when the series in (7) approximates the true inverse matrix $(I-A)^{-1}$.

This paper is concerned with the proporties of this procedure: Monotonicity and feasibility |2|. The strict monotonicity property indicates a process with each step leads to a new plan with a higher value of the target than that in the previous plan (unless the previous plan was a solution to the system). The property of monotonicity refers to when the plan proposed at each step is at least as good as the previous one. If every plan proposed during the iterative process is feasible. Then the second property is achieved. Another property related to this analysis is the convergence. The sufficient conditions for convergence are the non-negativity condition for input-output coefficient: $a_{ij} \geq 0$ for i, j=1,2, ..., n, and the sum of input coefficients for inputs to a sector to be:

$$\sum_{i=1}^{n} a_{ij} \leq 1 \qquad \text{for } j=1, \ldots, n$$

$$\text{or} \quad \sum_{i=1}^{n} a_{ij} < 1 \qquad \text{for at least one sector } j \ .$$

The property of convergence is less important than the two other properties in practice. But in theory the property of convergence is used to indicate at the latest stage of a series of stages, the plan will converge to local and global optimum |2|.

From the applicability point of view, it has been argued that this "administrative iteration" between the CPA and the lower units of the economy is related to the USSR plan elaboration |16|.

This procedure can be shown in terms of non-linear input coefficients |17| and in terms of shadow prices |18|. Also, this procedure can be adopted for information exchange between the CPA and consumers for private consumption plan elaboration |19|. Finally, this procedure can be used for analysis related to the constraints: limited labor force and limited capacities in the various sectors |20|.

Example 2. The Norwegian Approach

Let the idea of "consistent" and "efficient" comprehensive national planning mean that the various parts of the plan are not contradictory |21| on the one hand, and the organizational aspect of the plan emphasizes the quality of interdepartmental communication and the type of the model the different agencies ought to possess from the point of view of the overall coordination on the other hand |22|. A very brief account on how the Norwegians approach these aspects of planning, with the other examples, may help for the establishment of practical but fruitful avenues for plan appraisal and plan evaluation.

National Budget as a Tool for Planning

(1) Up to about 1960, attempts for improving the coordination of decisions made by the various government agencies controlling the allocation of reaources were based on national budgeting. Causes for deviations between the national budget projections and the corresponding ex post magnitudes were numerically analyzed by Bjerve |23| and were tested by a simple numerical Keynesian model made by Amundsen |23|. This extremely simple model contains two behavioral equations: one describes gross national product as a function of total consumption; and the other describes total import as a function of consumption, gross investment and total exports. The model has one definitional equation: gross national product plus imports equal total consumption plus gross investment plus

exports. This gives a system of three equations in five variables. If two of these variable are determined outside the system, the model determines the remaining three variables. All variables are measured at constant prices. According to Bjerve, there were no other better analytical models available, at that time.

(2) In the process of approximating the unknown system where all the behavioral relations is completely specified with no instrumental variables included, the planner faced a fact. The fact was that regardless of how the endogenous variables, instrument variables, and exogenous variables are estimated, all of these variables must be consistent with the identities of the Norwegian national budgets, which were in the form of a double entry book-keeping system |24|. The aims and concepts of national budgets with their deficiencies and the planning model can be found in detail elsewhere |25|.

The Role of Input-Output

(3) Because of certain defects in using national budgets in planning with respect to consistency and efficiency such as using different assumptions and different forms in the sub-budgets, there was a need for a more formalized budget model. According to Sevaldson |26| with the traditional process, a test for the impact of alternative assumptions may be impractical. Hence, the Norwegians choose the input-output table which can be transformed into a full set of national accounts |24|, and the choice of input-output type model was formed with the accounting principles. It was not explained in terms of general equilibrium analysis nor was it in terms of mathematical programming. The reason for this choice was to construct an operational input-output national budget model.

(4) From input-output analysis, the following relation can be derived from a system of n linear equations and n unknowns.

$$X_i - \sum_{j=1}^{n} a_{ij}X_j = 0 \qquad (i = 1,2 \ldots n)$$

For any solution other than $X_i = 0$ for all i, the equations must be consistent. Taking all X_i s into account, and knowing the value of output level of only one sector, (e.g. the household sector whose value is assumed given) the values of the elements of the table can be determined. Let the given

household sector value be $\bar{X}_n$ and in terms of value-added be $\bar{S}_i$. Then with K equations:

$$X_m - \sum_{j=1}^{n-1} a_{mj} X_j = a_{mn} X_n = \bar{S} \qquad (m = 1, 2 \ldots, K)$$

For consistency, $\sum_{m=1}^{K} a_{mj} = a_{nj}$ and $\sum_{m=1}^{K} X_m = \bar{X}_n$.

This procedure is followed in the Norwegian model |24|.

To keep the flow of discussion free from further mathematical notations, tests for consistency on input-output closed system (all sectors are endogenous) or open system (one or more sectors are predetermined) can be carried on for proving or disproving consistency by applying the theorem of linear algebra. In words, "... a system AX + B = 0 of m linear equations in n unknowns is consistent if and only if the coefficient matrix A and the augmented matrix |A, B| have the same rank" |27|.

System of Plans

(5) The national budgeting model based on input-output frame work and extended to include relations for income generation, taxation and private consumption demand, was named MODIS I (a MOdel of DISaggregated type) and used for the first time in 1961. Later it became to be a series of "MODIS" and "PRIM" models, to predict industrial output, prices and income distribution |28|. In these models the major final demand aggregates are predicted exogenously, and coefficients are not estimated by the prediction approach. However, these models are used for short-term planning and policy analysis. In other countries, model of input-output type are used for medium and long term plans. With respect to perspective analysis of 15 -20 years, long term projections were worked out on the basis of a multi-sector model built by Professor Leif Johansen. This model can be classified as a disaggregated neoclassical growth model with over 30 sectors of production. Unlike other countries, this model and other Norwegian planning models are used in the regular administrative planning process by the Ministry of Finance. They are fully integrated in the process of policy-making in Norway |25|.

(6) MODIS IV. It may be defined as a consistent macro-economic scheme uniting several submodels with various assumptions, with intention to fit the whole model into the administrative system of policy making and to replace informal models and heuristic reasoning. The backbone of MODIS IV is its close connection with the national accounting system, in current use in Norway in real and financial flows, MODIS IV combines a disaggregated input-output framework with a number of additional relations, |28| and auxiliary assumptions with emphasis on inter-sectorial and intra-sectorial accounting consistency with attention to institutional structures like tax systems. The main feature of MODIS IV is to analyze the system in terms of commodity-sector-activity for practical use rather than for an academic exercise. The potential advantages of MODIS IV are: (i) to bring detailed information from the unknown system into a consistent and comprehensive accounting frame work, (ii) to assess the combined effect of a great number of detailed assumptions about the functioning of the economy, and (iii) to evaluate the impact on the working of the known and unknown sub-systems.

The Norwegian Co-ordination Decisions

(7) The Norwegian approach to the planning problem is based on splitting the overall problem into main small manageable parts: the quantity submodel, the price sub-model and direct and indirect taxes sub-model. The basic relations for the quantity submodel is:

$$\Lambda A = X$$

where A = a vector of activity level

X = a vector of addition to stocks by commodity

Λ = a matrix in which the typical element a_{ij} gives net output of commodity i per unit of activity level j.

Consistency is achieved when the sum total of addition to stocks equals the sum total of activity levels (with transformation activities excluded).

(8) The basic price equation is:

$$\Lambda' b_x + t = p_A$$

where:

b_x = a column vector of commodity prices in basic values

p_A = a column vector of prices of activity levels in market values

The column vector t is the net value of commodity taxes per unit of activity level. For consistency to be achieved with respect to price input-output model if unit value of output is equal to unit value of commodity input plus primary input, if gross product (in current values) is equal to final demand (in current values), and if interaction between input prices and output prices are taken into account |29|.

(9) Linking the two submodels the commodity and the price together |30|:

(1) $\Lambda A = X$

(2) $\Lambda' b_x = p_A$ (excluding t)

It follows

(3) $b_x' X = p_A' A$

(4) $e' A = e' X$

where (3) is in current values and (4) is in fixed values, and e' indicates element unit. Total values of stocks should equal total values of activity level (net) is required for consistency.

This procedure can be compared easily with that of USSR (see Example 3). MODIS IV used in the preparation of the revised national budget for 1974 has benefited from the accumulated experience in planning and from the improvement in the national accounts |31|. One can give a couple of these benefits without difficulties in analyzing them: (i) in accordance with the new national accounts, the flows of goods and services were grouped in the input-output accounts for production by both industry and commodity, e.g. the T account for production has thousands of grouping commodities. The uses of these commodities were classified and added which had to be equal to the resources (locally produced and imported) for consistency purposes. (ii) MODIS IV show more detailed and precise picture of the Norwegian economy |31|. For example, without having a separate direct tax model, the effects of changes in direct

tax rates, both in real and in financial terms may be analyzed by this model |31|. Since the model can predict both the annual and the four year program data, better coordination is arrived at.

(10) Evaluation: MODIS IV has advantages and also it has disadvantages. Besides what it has already been mentioned, the model is useful in handling different problems such as 1973 currency revaluation, annual and four year projections, and income settlement. However, the determinants of private investment and aspects of technical change are analyzed outside the model |30|. MODIS IV like other MODIS's contain far more variables than relationships. Also, each new MODIS has more new variables which are greater than more new structural relationships |31|. Nevertheless, the model is flexible with respect of handling exogenous variables such as wage rates and labor productivity. This provide some link between the planner and the decision-makers based on the compromise approach. The purpose of the model is to taste government decisions with respect to consistency and efficiency, and to replace the informal procedures which can be used in planning but not to substitute value-judgment. Of-course, these remarks are incomplete to cover all aspects. For an excellent reference see "Trends in Norwegian Planning, 1945-1975" by Petter Jakob Bjerve in "National Accounts Models and Analysis" in honor of Odd Aukrust Sixtieth Birthday, Oslo, 1975. Also, a written discussion with Norwegian planners is under way.

Example 3. General structure of an intersectoral model USSR

The purpose of calculating the intersectoral balance at the initial stage of long term planning is to evaluate the consistency of the projections made by the sectors |32|.

Consider first the distribution of goods produced:

$$A_t Z + PX + Y = Z \qquad (1)$$

where

A_t = matrix of normatives describing technological inputs into production of the commodity (product by product);

Z = volume of production n groups of goods in physical terms;

P = normative of input of all products per domestic currency unit of the sector's output; and

Y = end product

X = volume of production in value terms.

Consider secondly the production equation: Formation of volumes of production in value terms

$$QZ + A_o X = X \qquad (2)$$

where

Q = average prices of the commodities produced by each sector; and

A_o = a diagonal matrix each of whose elements represents the the proportion of the sector's output not included in Z.

Equation (2) determines X, and:

$$X = (E - A_o)^{-1} QZ \qquad (3)$$

can be obtained.

Where

E = unit matrix of order (mxm) for sectors.

Inserting the value of (3) in (1):

$$Z = \left[I - \left[A_t Z + P(E - A_o)^{-1} Q\right]^{-1}\right] Y \qquad (4)$$

where

I = Unit matrix of order (nxn) for quantities.

Or

$$Z = [\text{total input coefficients}]\, Y$$

This physical-value intersectoral balance, which is the basic model, is designed for use in preparing plans of long-terms, five year and annual.

If there is a single resource constrained, for example, man power ecpressed by:

$$\sum_{j=1}^{n} \ell_j (X_{jt}) \leq L_t \qquad (5)$$

where:

ℓ_j = the amount of labor $\ell_j(X_{jt})$ required to produce a gross output of X_{jt} in sector j

L_t = a function of limited man power in the national economy as a whole in year t.

The system of equations (4) and (5) makes it possible to calculate coordinated sectoral decisions taken into account the limitations of man power resources and at the same time to perform the coordination of sectoral decision from the production standpoint |32|.

Example 4. The Hungarian Co-ordination Decisions

The National Office of Materials and Prices has prepared, among other models, short term planning models and the price models |34|.

For the short-term planning models, at the present (1974) only input-output models are applied in short-term national economic planning. They are used regularly for analyzing the initial conditions of planning (second type of the planning problem) and for the final coordination of the plans (compromise) approach.

The medium-term price models are in two variants. The first presents price development in 15 branches on the assumption that prices will change parallelly with costs and costs will as they have over the past decade. The second model variant allows not only for changes in costs but also consider modifications in the incentives and their influence on prices. Certain optimality conditions have been incorporated in the model and the cinsistency of valume and value indicators has also been ensured. Shadow prices obtained with different objective functions in the programming model are used for medium and long-term price planning, thus creating the conditions for the consistent evaluation of changes at the two levels.

Example 5. The USSR Co-ordination Decisions

The USSR uses models of intersectoral links which are constructed on the basis of the balance ratios governing the distribution of the gross output of the sectors. These models provide the coordination of sectoral decisions from the stand point of the national economy.

The following model outlines the main procedure [33]. Let the national economy be a set of n interrelated sectors:

$$(1) \quad X_{it} = \sum_{j=1}^{n} f_{ij}(X_{jt}) + U_{it} \qquad (i=1, \ldots n)$$

where:

X_{it} = gross output

f_{ij} = sectoral cost functions

U_{it} = fund of nonproductive consumption of the output of sector i.

The system of balance equations can be written as the operator:

$$(2) \quad X_t = Q(X_t) + U_t$$

where:

X_t = the vector of gross output

U_t = the vector of nonproductive consumption

$Q(X_t)$ = non-linear operator

Consider the ith component of the nonlinear operator $Q(X_t)$:

$$(3) \quad Q_i(X_t) = \sum_{j=1}^{n} f_{ij}(X_{jt})$$

(a) Data of the $f_{ij}(X_{jt})$ are obtained from the models of the individual sectors

(b) If monotonic of its argument, then the operator $Q(X_t)$ is also monotonic.

(c) The convergence of the method of successive approximation is established when the conditions of non-negativity are met.

Example 6. Plan Evaluation Silent Models

(6-1) A Simple Evalution Test (USSR)|35|

$$Y_t = Y_o + e_t \cdot \Sigma T_t$$

$$Y_t = C_t + T_t \cdot b_t$$

$$Y_t = P_t \cdot L_t$$

where

Y_t = the net domestic product

e_t = the efficiency of capital investment; the ratio of net national product increment during the period under review to the volume of capital investment

T_t = the annual volume of capital investment

C_t = personal consumption, public consumption non-industrial investment and non-industrial capital repairs

b_t = the relationship between capital investment and net national product not used for personal consumption, it is equal to $\frac{(Y_t - C_t)}{T_t}$

P_t = labor productivity

L_t = the number of persons engaged in production

e_t and P_t are the efficiency indicators

(6-2) The Intersectoral Balance Models (USSR)|35|

(1) The balances of production and distribution in physical terms:

$$X_i = \sum_j \sum_1 a^1_{ij} x^1_j + \sum_1 a_{i1} \hat{x}_1 + Y_i$$

(2) The distribution by sectors; mininstry, department:

$$X^1_j = W^1_j X_j$$

(3) The coordination for production indicators in monetary and physical terms

$$\Sigma X_j^1 P_j^1 + X_1^n r_1 = X_1^n$$

where

X_i = the output of type i products in physical terms in the national economy as a whole;

X_i^1 = idem in sector (ministry, department) 1;

a_{ij}^1 = the direct input coefficient in physical terms in sector 1; fixed for type j product;

X_1^n = the gross output of sector 1 in money terms

a_{i1} = the product i direct input coefficient (in physical terms) per a currency unit of sector 1 gross output

Y_i = the end product of type i;

X_j^1 = the share of sector 1 in total national output of product j

P_j^1 = the price of product j produced by sector 1

r_1 = the share of other types of product, not among the items of the quantity-value balance, in the gross output of sector 1.

(6-3) Use of the Concept of mathematical separability (FRANCE)

The frame work of a general optimization model for removing the main difficulty raised by the sectoral approach (i.e. to reconcile the general criterion of efficient resource allocation and compliance with the constraints of macroeconomic equilibrium) is as follows:

Target function of the allocation office

$F\ (1^\infty \ldots i^\infty \ldots)$

Constraints

(i) The relationships of the sectoral models

Sectoral model 1	Sectoral model i
$\psi_1\ (1^\infty \ldots i^\infty \ldots) = 0$	$\psi_i\ (1^\infty \ldots i^\infty \ldots) = 0$

(ii) Macro-economic cinstraints

$$
\begin{array}{ll}
\pi_1 & X_1^{*} = X_1 (1^{\infty} \ldots i^{\infty} \ldots) \\
\vdots & \vdots \\
\pi_h & X_h^{*} (1^{\infty} \ldots i^{\infty} \ldots) \\
\vdots & \vdots
\end{array}
$$

The variables X_h^{*} denote the economic policy objectives in the medium term projections, e.g. growth rate of GDP which is determined from the sectoral variable by aggregation.

The problem is then to find a sectoral choice criterion which would be consistent with both economic efficiency (function F) and the pursuit of economic policy objectives $X_h(\ldots) = X_h^{*}$.

In another form

$$\text{Max} \quad F(\alpha_1, \ldots \alpha_i \ldots) + \sum_h \pi_h X_h(\alpha_i \ldots)$$

Subject to constraints:

$$\psi_i(\alpha_i) = 0$$

where π_h being the dual variable linked to the constraint $X_h^{*} = X_h(\ldots)$. For further analysis and conclusions see |36|.

Concluding Remarks

The above brief account of actual cases of planning help to give supporting background for the various ways to appraise and evaluate plans. In some countries in the early stages of planning, an aggregated model projecting the main social and economic indicators might be used as a test for consistency and efficiency of a plan based on administrative planning procedures. The results of which may be used also later in more detailed sectoral models, whose results again could be used as a test for consistency and efficiency for a plan based on detailed sectoral models and they would be used also by various branch sub-sectoral models later on.

RECOMMENDATIONS

In my search for a consistent and efficient plan during my stay in Oslo, Norway for the first quarter of 1976, I have reached the following guidelines. Though the period seems to be too short to cover all aspects of planning, but with the assistance of the adminstrative staff at the Institute of Economics, University of Oslo for the first three weeks and at the Central Bureau of Statistics for the remainimg of the period, much time was saved. I am very thankful to the assistence received from all of these agencies and in particular the Norwegian Agency for International Development (NORAD). The following recommendations were based on my independent study as well as on oral disscussions with high power planners at the Institutes and at the Bureau. Also, several visits were made to the Ministry of Finance, the Planning Division and one to the Economic Division. I have also visited the Chr. Michelsens Institutt at Bergen.

Recommendation I

(a) Decompose the Planning Problem

The planning problem should be decomposed into smaller and managable parts. The decomposition is to be on two levels. One is vertical: Center-Sector-Project with highly aggregated plans, intermediate plans, and the smallest detailed plans. The second level is horizontal: One problem is partitioned into smaller problems side by side on the same line, say, among different sectors or ministeries. For this recommendation to has an effective results, certain requirements are to be met such as decentralizing the power and the quality of exchange of information. The former means that each sector has the power to regulate certain definite processes affecting the structure of the economy. The latter indicate that each sector has the ability to revise the tentative solutions of the problem. Hence, the planning problem may take the first type form at lower level and the second type form at the higher level.

(b) Decompose the Planning Time

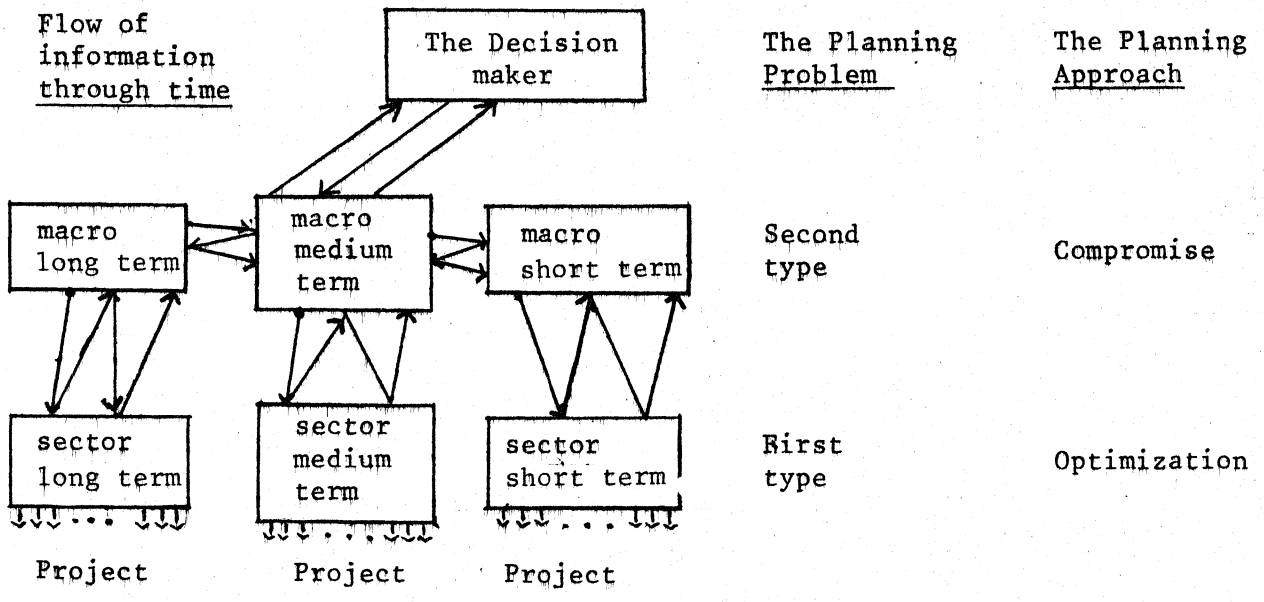
Horizon. The familiar time dimension: short-term, medium term, and long-term planning is important for comparing plans at a macro stage, sector stage and project stage. The lower level of planning project and

sector should satisfy conditions determined by the macro stage with respect to time dimension. The long-term lower level planning should be compatible with the long-term macro plan. This will satisfy the necessary conditions for exchange of information and mutual adjustment through iterations.

(c) Decompose the Planning Approach

The planning approach should be splitted in at least two major approaches. The optimization approach is more relevant to project and sectoral plans through which a survey of the society initial situation could be provided to the leadership regarding the best possible basis on which to build its decisions. The compromise approach is more relevent to the macro stage, where the trade-off in the national preference function could be established e.g. between accelerating economic growth and transformation to socialism through time.

The links between what was just suggested and the type and the approach of the planning problem can be seen as follows:



Recommendation 2

a) Develop an adequate national accounting system (or material balances). This is essential for all planners. For the statistician he needs this system for fundamental consistency controls. For the model-builder, he needs the system for analyzing the interrelationships among the planning variables. For the decision-maker, he needs this system in making consistent set of goals. In the absence of adequate time-series data, this system will serve the backbone for planmaking. Data collection should aim at maximum efficiency.

b) Develop an acceptable methodology for widening the national accounting system for further uses. All types of budgets: State, national, and fiscal is deduced from such system. Social indicators, which make it possible to establish the link between end results are related to this system. One example is the link between, as it is well stated by the Iraqi national preference, the level and structure of production (accelerating economic growth) on the one hand, and income distribution with other social indicators such as health or educational status, etc. (transformation into socialism) on the other.

c) Develop an institutional dialogue between national and regional authorities in the course of planning and decision-making process. The national accounting system should play the common language providing such links. These remarks are incomplete but they give the essential ideas which stress the complexity of socio-economic reality and the lack of unifying concepts necessary for the integration of different fields. This kind of dialogue is essential in planning, based on different approaches, for the linkage's convergency. The number of iteration is important. The number of iteration should be minimized but increasing the number of iteration improves the accuracy of calculations and the convergence of the decicion process.

Recommendation 3

a) Construct (or improve) a traditional input-output model which transforms the objectives and the final requirement of the utilization of the resources in terms of objectives and production requirements of goods and services. This could be achieved by a set of matrices which explain production in final demand, taking into account the intermediate requirements determined by the intersectorial productive process.

b) Construct a dynamic input-output model which is a special case of linear activity analysis model, developed by Koopmans, 1951, 1957 |37, 38|. In such a model there is a one-to-one correspondance between commodities and activities and each actitity can be regarded as the activity of producing the corresponding commodity. This model has several versions, it can be presented for a situation in which there is no joint production. Also, it may have fixed input proportions for the production of any one commodity. For the input requirements; flow requirements are distringuished from stock requirements. It would be better to think about them in physical terms. There is also a need for certain variables to be determined outside this model, by using some submodels, for example; income submodel, price sub-model, foreign trade submodel, etc.

c) Avoid elaborating a national development plan which is based on complicated models for which the data available are far too weak to approximate the economy. These weak data may not be able to specify or to implement a structurally and behaviorally sound representation. In such a case, most hypothesis testing such as best linear unbiased estimaters, and the like are then wholly irrelevent. Alan S. Manne has presented a comparative analysis om multi-sector models for development planning |39|. Manne table is reproduced here in an appendix. Manne table shows, among other things, that with no explicit objective function, an open consistency model, dynamic input-output has been used in many countries. According to Wassily Leontief: Norway makes the best use of the input-output table among 120 countries.

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|2| Kornai, J., "The Place of Mathematical Planning in the Control of the Economic Systems", First Seminar on Mathematical Methods and Computer Techniques, Economic Commission for Europe, Varna 1970.

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|17| Replace $X_{ij} = a_{ij}X_j$ by $X_{ij} = f_{ij}$ where i,j =. (1,2 ... n) i.e. replacing the assumption of constant input-output coefficient by non-linearity. Follow the procedure to get

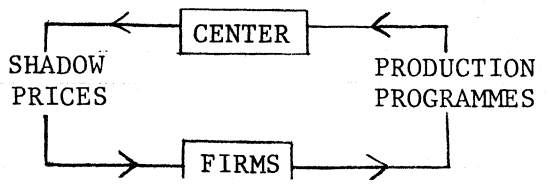
$$X_1^{(S)} = \sum_{j=1}^{n} f_{ij} \left(X_j^{(S-1)}\right) + Y_1$$

$$\vdots$$

$$X_n^{(S)} = \sum_{j=1}^{n} f_{nj} \left(X_j^{(S-1)}\right) + Y_n$$

Johnsen has shown the procedure in detail.

|18| Heal, G.M., "The Theory of Economic Planning", see |2|, has shown the use of shadow prices in "The Malinvaud process" as follows:



where the center announces the shadow prices taken from a set of Lagrange multipliers. These shadow prices represent the marginal social values of the various goods in accordance with production possibilities. Each firm responds by informing the center. The firm would act as a profit maximizer at these prices.

|19| Johansen, Leif shows how the CPA exchanges information with consumers when $Y_i = C_i + U_i$ where C_i represents private consumption demand and U_i represents other final demand. The latter is determined by the CPA while C_i depends on total income.

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- See |22|

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Aukrust, Odd "PRIM I A Model of the Price and Income Distribution Mechanism of an Open Economy", Artikler 35 from the Central Bureau of Statistics, Oslo 1970.

|29| And also in fixed values

|30| See Bjerkholt and Longva as in |28|.

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Appendix. This table has been reproduced from Alan S. Manne: "Multi-Sector models for Development Planning: A Survey". Technical report no. 91, march 1973. Institute for Mathematical Studies in the Social Sciences, Standford University, California.

Table 1. Some characteristics of 18 development planning models.

Reference	Country	Objective function, basic case	Open- or closed-loop	Solution technique (if other than linear programming)	Problem size				Labor substitution possibilities; human capital formation
					Number of time periods	Length of planning horizon (years)	Number of input-output sectors or commodities	Number of labor skill classes	
Adelman (1966)	Argentina	Alternatives: (1) maximize discounted GNP; (2) maximize GNP, terminal year; (3) minimize discounted sum of net foreign capital inflows.	Closed		4	20	9	6	Substitution between educational levels for each labor skill type; human capital formation for all labor skills.
Adelman et al. (1969)	South Korea	No explicit objective function.	Open	Consistency model, dynamic input-output.	7	7	43	1	
Blitzer (1971)	Turkey	Maximize GNP, terminal year.	Closed		5	15	8	6	Human capital formation for all labor skills
Bruno et al. (1970)	Israel	Maximize discounted consumption plus value of terminal capital stocks.	Open	Static input-output matrix was inverted to reduce the number of rows and columns in the linear programming matrix.	6	15	30	2	
Celasun (1971)	Turkey	Maximize GNP, terminal year.	Closed		3	8	13	0	
Clark and Taylor (1972)	Chile	No explicit objective function.	Open	Consistency model, dynamic input-output. Terminal year output targets obtained through static linear programming, applying a stock-flow conversion factor for investment.	10	10	17	8	Partial substitutability between labor types.
Eckaus and Parikh (1968)	India	Maximize discounted consumption, subject to consumption growth constraints.	Open		10	30	11	0	

Table 1 (continued)

Reference	Country	Objective function, basic case	Open- or closed-loop	Solution technique (if other than linear programming)	Problem size				Labor substitution possibilities; human capital formation
					Number of time periods	Length of planning horizon (years)	Number of input-output sectors or commodities	Number of labor skill classes	
Goreux (1973)	Ivory Coast	Maximize discounted utility of consumption from individual commodities, assuming want independence.	Open		5	25	3	3	Import of labor; diminishing marginal productivity of unskilled urban labor in the traditional service sector.
Gupta (1971)	India	Maximize discounted consumption	Open		10	10	4	2	
Johansen (1968)	Norway	No explicit objective function.	Open	Consistency model, dynamic input-output	9	27	28	1	Capital-labor substitution in all 28 sectors through Cobb-Douglas form of production function.
Keesing and Manne (1973)	Mexico	Maximize consumption, subject to gradualist growth constraints.	Open		6	18	15	5	Capital-labor substitution in agriculture only; human capital formation for all labor skills.
Kendrick and Taylor (1971)	Korea	Maximize discounted utility of consumption.	Open	Conjugate gradient method of optimal control.	30	30	4	1	Capital-labor substitution in all 4 sectors through CES form of production function.
Kornai (1969)	Hungary	Alternatives:[a] (1) maximize consumption; (2) maximize foreign exchange surplus with: a) capitalist or b) socialist countries; (3) minimize gross investment; (4) minimize labor inputs; (5) minimize prime costs.	Open	Multi-level programming; an approximation to Dantzig-Wolfe decomposition.	1	5	491	1	Capital-labor substitution through changes in the mix of each item's output from 3 vintages of equipment: old, modernized and new.
Manne and Weisskopf (1970)	India	Maximize consumption, subject to gradualist growth constraints.	Open	Preliminary data processing to reduce number of rows and columns in the linear programming matrix.	4	8	30	0	

Table 1 (continued)

Reference	Country	Objective function, basic case	Open- or closed-loop	Solution technique (if other than linear programming)	Problem size: Number of time periods	Problem size: Length of planning horizon (years)	Problem size: Number of input-output sectors or commodities	Problem size: Number of labor skill classes	Labor substitution possibilities; human capital formation
Martens and Pindyck (1973)	Tunisia	Minimize the weighted sum of squares of deviations from a "nominal" path for each state and control variable.	Closed	Linear quadratic tracking method of optimal control.	15	15	5	1	
Murakami et al. (1970)	Japan	Maximize capital stocks, terminal year.	Closed	Composition of terminal capital stocks computed through characteristic vector associated with largest characteristic root of dynamic Leontief matrix.	8	8	10	0	
Ujlaki (1971)	Hungary	Alternatives:[a/] (1) maximize discounted consumption; (2) maximize capital stocks, terminal year; (3) maximize discounted foreign exchange surplus.	Open		4	20	20	1	Capital-labor substitution through changes in the mix of each item's output from 3 vintages of equipment: old, modernized and new.
Westphal (1971)	Korea	Maximize discounted utility of consumption.	Open	Mixed integer programming.	5	10	11	0	

Note:

a/ In each case, there are predetermined bounds on the macroeconomic variables. These bounds are targets derived from official Hungarian five-year plans, and are based upon traditional planning methods, not upon mathematical programming.