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A SHORT TERM MODEL FOR PLANNING IN NORWAY

by

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I. General description

A. Introduction

The Norwegian interindustry-consumption model MODIS II[1),2)] has been developed by the Central Bureau of Statistics for use in economic analysis and planning. The version of this model, which has so far been made operative, is particularly designed for use in the preparation of the annual "national budget", the economic plan for the coming year, which is presented each year in October by the Norwegian government. This version of the model has been formulated in consultations with the Ministry of Finance and the University of Oslo.

The present model, MODIS II, was preceded by a somewhat simpler version (now given the name MODIS I) which was in regular use since 1960 in various types of economic analysis as well as in government economic planning for one- and four-year periods.[3)]

A summary description of the main characteristics of MODIS II may be useful as a background for a formal description of its variables and relationships.

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1) MOdel of DISaggregated type, second version.

2) Arne Øien of the Central Bureau of Statistics was responsible for the work on MODIS II. The present description is based on Mr. Øien's presentation of the model in a Working Paper from the Central Bureau of Statistics (Series no. IO 66/3).

3) See Per Sevaldson: An Interindustry Model of Production and Consumption for Economic Planning in Norway. Income and Wealth. Series X. Edited by Colin Clark and Geer Stuvel. London 1964.

B. Basic philosophy of the model

The model is based on the following assumptions:

1. Constant price input-output coefficients are fixed.
2. Product prices are the sum of input prices, weighted by the fixed coefficients, plus profit rates, subject to modification by indirect taxation.
3. Incomes are determined by production through the input requirements, by prices for labour which again depend on wage and productivity rates and by profit rates.
4. Private consumption is determined by disposable income and prices.

In order to formulate a model on these assumptions the following coefficients are needed:

5. Input-output coefficients for all production sectors.
6. A set of coefficients characterising the relationships between incomes and prices on one hand and private consumption of goods and services on the other.
7. A number of "transformation" coefficients, needed mainly for the transformation of exogenous estimates to the specifications of the model.

In order to make a model based on these assumptions determinate, the following variables must be given as exogenous estimates:

8. For each sector quantity of production or final demand other than private consumption must be given as exogenous estimates.
9. For each sector the product price or the profit rate must be given as exogenous estimates.
10. Prices of labour and other not-produced inputs (e.g. imports) must be given as exogenous estimates.
11. Tax rates must be given exogenously.

The determinate model gives estimates of:

12. Production and all deliveries from each sector of production in quantities and values.
13. All items of private consumption in quantities and values.
14. Requirements of labour, imports and other primary inputs in total and in each sector of production, total value added in fixed prices in each sector and in total.
15. Prices and profit rates for all sectors of production. Price indexes based on fixed or current weights.
16. Incomes shares and tax revenues.

In MODIS II the following choices have been made in regard to exogenous estimates:

17. For one group of production sectors it is assumed that the volume of production can most easily be estimated on the basis of exogenous information. Then, for some sectors in this group, supplementary imports are assumed to fill in a remaining gap between demand and domestic production. For other sectors at least one item in final demand is determined by the model, and finally, for some sectors it is assumed that intermediate demand for the products of a sector is determined as the difference between exogenous estimates of production and exogenous estimates of final demand, the assumption of fixed input-output coefficients for the use of these products being abandoned. Sectors for which the volume of production is exogenously estimated are typically sectors where production levels are relatively independent of short run shifts in demand, e.g. agriculture, some sectors where production is mainly determined by capacity and some which are under direct government control.
18. For the remaining production sectors it is assumed that production is determined by demand, and that final demand other than private consumption is determined by exogenous estimates.
19. For one group of production sectors, defined independently of the above grouping, it is assumed that their prices are given by world market conditions or by policy decisions and thus have to be estimated outside the model. This will be the case for all export sectors, and for many sectors competing with imports in the domestic market, as well as for agriculture, fisheries and a number of sectors dominated by public enterprises. In these sectors profit rates will be residually determined.
20. For the remaining sectors it is assumed that profit rates per unit produced is determined by conditions in the domestic economy, much in the same way as wage rates. These profit rates must be estimated exogenously. Then input costs, profit rates and indirect taxation will determine product prices.

C. Data requirements and specifications

a. Data requirements

The Norwegian model has been formulated in such a way that:

1. Coefficients of the consumption relationships could be estimated on the basis of national accounts and consumer budget studies for the period 1952-1962.

2. National accounts and supplementary statistics for a "base year" provide the basis for all other data in the model (except, of course, for the exogenous estimates). Input-output coefficients, for instance, are computed on this basis. (When the model is used in national budgetting the base year is the year 2 years before the budget year.)
3. Model estimates are given in the form of estimated changes a) from the base year to the "current" year and b) from the "current" year to the "budget" year.[1] In order to find the absolute values for the budget year, therefore, accounts figures for the base year and estimates of exogenous variables both for the "current" year and for the "budget" year are needed.
4. The model has been programmed on an electronic computer (Univac 1107) in such a way that when national accounts for the base year and, in addition, up to ten alternative sets of exogenous variables for the current year and the budget year are fed into the machine, the results are produced in an integrated operation in the form of a booklet containing the ordered and texted tables in the form which is most convenient for the user. Unfortunately, this requires relatively much machine time (10 hours, 7 hours if only one alternative is computed). Nevertheless, results can be produced from one day to the next.
5. Work on a simplified model for rough "in between" calculations is in progress.

b. Specifications in MODIS II

1. Internal specifications

The model distinguishes between:

165 sectors of production

30 "import sectors" or groups of import commodities

9 categories of income shares (depreciation, direct and indirect taxes, subsidies, wages and profit by type of organisation). (These income shares can again be specified by sector or by group of sectors.)

2. Specifications in exogenous estimates

(i) price estimates

59 estimates of product prices for Norwegian sectors of production

2 estimates of the rate of change of profit rates in the remaining Norwegian sectors of production

(cont.)

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1) The terms "current" year and "budget" year here refer to the use of the model in national budgetting. In other uses of the model any set of "analysis" years may of course be studied, their relevant characteristics given by the particular values choosen for the exogenous estimates and by the requirement that structural relationships and data from the "base" year be applicable.

(i) price estimates (cont.)

29 estimates of import prices

13 estimates of wage rate changes in groups of Norwegian sectors of production

12 estimates of productivity changes in groups of Norwegian sectors of production

(ii) tax rate estimates

20 estimates of indirect tax- and subsidy rates

6 estimates of rates for direct taxes, social insurance and direct subsidies

(iii) quantity estimates

49 estimates of production volumes in the same number of Norwegian sectors

380 estimates of final demand for the same number of types of goods

Of these:

7 estimates of exogenously determined items of private consumption

18 estimates of government consumption

182 estimates of export items

70 estimates of inventory investments

103 estimates of other investment items

3. Specifications in results:

When the model is used for planning, the purpose is to obtain a consistent set of estimates of key national accounting variables for the plan (budget) year, irrespective of whether these are endogenously or exogenously estimated. The model therefore produces as its end result a complete set of tables of such variables. These variables are to some extent more aggregated than they would be if the full internal specifications were maintained, and they are not systematically divided into exogenous and endogenous variables. The tables give figures in current and fixed prices, both absolute figures for the base year, the current year and the budget year, and figures for the changes between each pair of years, absolutely and in per cent. Price indexes are given for all items.

4. The main tables are:

(I) Tables of gross national product and its components

(i) Domestic product by category of expenditure, 16 items

(ii) Export by delivering group of sectors, 14 items

(iii) Gross investment by type of capital goods, 13 items

(iv) Government consumption, 11 items

(v) Private consumption by type of goods, 17 items

(vi) Import by sector, 32 items

(vii) Gross national product by sector of production, 72 sectors and sector groups

(II) Tables of balance of payments and income accounts

(i) Balance of payments, 6 items

(ii) Income accounts: specifications of incomes earned in each of 13 groups of sectors, alltogether 120 items

(III) Tables of disposable incomes and savings. Tables giving income and savings figures in total as well as separately for persons, corporations, central and local governments and social security.

D. Price determination in the model

The set of assumptions on which MODIS II is based makes prices independent of market conditions in the product markets: Prices are either exogenously determined, or determined by unit costs together with exogenously given profit rates.

The prices which are not determined entirely outside the model are determined by the exogenous prices through the assumption of fixed input coefficients in production, and by the assumptions about tax, wage and profit rates. The model cannot take care of the possibility that entrepreneurs may vary profit rates in order to change prices in response to changes in demand. (Such a possibility must be handled outside the model, e.g. by iterative adjustments in the assumed profit rates.)

Formally, price determination is represented by a submodel, which may be solved separately, independent of the quantity estimates, when the exogenous price estimates and estimates of tax, wage (and productivity) and profit rates are given.

In the model prices influence the volume and distribution of production and imports through their influence on the volume and distribution of private consumption. Adjustments to prices through substitutions in the production sectors are excluded by the assumption of fixed input-output coefficients.

Prices are also decisive for the level and distribution of private incomes and public revenues.

II. The formal framework

A. A simplified model

The system of relationships describing MODIS II is in principle quite simple, since the model is based on a rather simplified representation of a limited number of economic relationships. However, due to the high degree of specifications, and a relatively large number of cases which are given special treatment in the actual formulation of the model, a symbolic representation nevertheless becomes rather cumbersome. The best way of introducing the model therefore seems to be to start from a simplified formulation, representing the basic principles, and then to introduce the complicating features one by one as expansions and modifications in the basic model. A simple input-output consumption model could be described by the following set of relationships:

$$(1) \quad x = A_x x + c_x + y_x$$

$$(2) \quad b = A_b x + c_b + y_b$$

$$(3) \quad r = \hat{p}_w W x$$

$$(4) \quad c = c^o + D(\frac{1}{\pi} r - r^o) + N(\frac{1}{\pi} p - i_c)$$

$$(5) \quad p_x' = p_x' A_x + p_b' A_b + p_w' W$$

$$(6) \quad \pi = p' \frac{1}{\gamma} c^o$$

$$(7) \quad \gamma = i_c' c^o$$

$$(8) \quad y = \text{given}$$

$$(9) \quad p_b = \text{given}$$

$$(10) \quad p_w = \text{given}$$

Here:

x = a column vector of production levels measured in constant price values. The dimension n_x of this vector is equal to the number of production sectors (industries) in the model.

c_x = a column vector of sector deliveries to private consumption, measured in constant price values and of dimension n_x.

y_x = a column vector of sector deliveries to final uses other than private consumption (exports, gross investment and government consumption) measured in constant price values and of dimension n_x.

b = a column vector of imports measured in constant price values. The dimension, n_b, of this vector is equal to the number of commodity types in the specification of imports in the model.

c_b = a column vector of import deliveries for private consumption, measured in constant price values and of dimension n_b.

y_b = a column vector of import deliveries to final uses other than private consumption, measured in constant price values and of dimension n_b.

r = a column vector of primary income shares (wages, depreciation, charges, indirect taxes and entrepreneurial incomes) measured in current price values. The dimension n_r of this vector is equal to the number of primary income shares (specified by category and sector groups) in the model.

A_x = a n_x by n_x dimensional matrix of "input-output coefficients", assumed to be constant.

A_b = a n_b by n_x dimensional matrix of "import-coefficients", assumed to be constant.

W = a n_r by n_x dimensional matrix of "income share coefficients", estimated in the base year.

p_x = a column vector of price indices for all production sectors (dimension n_x).

p_b = a column vector of price indices for all import commodity types (dimension n_b).

p_w = a column vector of "price indices" for all income shares (dimension n_r). For each income share the price index is defined as current income in all sectors of production divided by the sum for all sectors of base year income times the index of production for the sector concerned.

D = a $(n_x + n_b)$ by n_r dimensional matrix of "marginal propensities to consume".

N = a $(n_x + n_b)$ by $(n_x + n_b)$ dimensional matrix of price coefficients for consumption.

π = a price index for consumers goods (a scalar).

i_c = a $(n_x + n_b)$ dimensional column vector with all elements = 1.

$$c = \begin{pmatrix} c_x \\ c_b \end{pmatrix}, \quad p = \begin{pmatrix} p_x \\ p_b \end{pmatrix}, \quad y = \begin{pmatrix} y_x \\ y_b \end{pmatrix} \quad \text{etc.}$$

A "prime" (') denotes transposition.

(cont.)

Superscript o denotes base year values.

A "cap" (^) denotes that a vector is written as a diagonal matrix. This will also be denoted by a $\bigwedge$ and the symbol of the vector written within parenthesis, i.e.: $\hat{p}_w = \bigwedge(p_w)$.

Here the first set of equations, (1), gives the levels of production in all production sectors, (x), as the sum of deliveries to production sectors, $(A_x x)$, deliveries to private consumption, (c_x), and deliveries to final uses other than private consumption, (y_x). Deliveries to production sectors, $(A_x x)$, are assumed to be proportionate to production in the receiving sectors. The elements of A_x can be estimated on the basis of accounts for a base year.

The second set of equations, (2), gives the levels of demand for all import commodity types, (b) as the sum of deliveries to production sectors, $(A_b x)$, deliveries to private consumption, (c_b), and deliveries to final uses other than private consumption, (y_b). This implies that all imports are treated as "structural". The simple model has no provision for substitution between domestic and imported goods. The elements of A_b can be estimated on the basis of accounts for a base year.

The third set of equations, (3), gives the primary income shares in current values as determined by the production volumes in all production sectors. The income shares as fractions of total production in each production sector in a base year, W, can be established on the basis of accounts. Now the wage payment per unit produced in a given sector will change as a result of changes in productivity and wage rates. The indirect tax payment per unit produced in a given sector will change with the change in the tax rate calculated on a per unit basis. The depreciation charges per unit produced are assumed to be constant in volume, and will consequently change in proportion to a price index for these charges. Price indexes expressing these types of changes are assumed to be given. If we also assume changes in entrepreneurial income per unit produced in each sector to be known, then we have all the elements of p_w.

The fourth set of equations, (4), gives private consumption of products from each sector as the sum of consumption of the same product in the base year plus a set of terms depending on the change from the base year in "real income" $(\frac{1}{\pi} r - r^o)$, - real income defined as income deflated by a general price index (π) - plus a set of terms depending on the change from the base year in relative prices. c^o can be found from accounts for a base year. The elements of D and N must be estimated by statistical studies of consumer behaviour.

The fifth set of equations, (5), gives the price relationships which must obtain in the model. The price index for products from a given production sector equals the weighted sum of the price indexes for all inputs and income shares in the sector, the weights being the fraction each element constituted of total production in the base year. These fractions are given in the matrices A_x, A_b and W.

The sixth group of equations, (6) and (7), consists of only two. Taken together they give a formula for computing a general consumers' price index on the basis of base year weights.

By (8), (9) and (10) we assume final demand other than private consumption, (y), import prices, (p_b), and "prices" for income shares, (p_w), to be given.

The system is solved by first solving (5) for p_x, then π may be computed by (6) and (7), and x, r and c are determined by simultaneously solving (1), (3) and (4). Finally b is computed from (2).

Having solved the equation system (1)-(10) we can also compute the vectors of current price values: $\hat{p}_x x$, $\hat{p}_b b$, $\hat{p} c$ and $\hat{p} y$.

In order to make it more easy to keep track of subsequent partitionings of variable vectors and coefficient matrices, we may write the equations (1-3) in the following form:

$$\begin{pmatrix} x \\ b \\ r \end{pmatrix} = \begin{pmatrix} A_x \\ A_b \\ \hat{p}_w W \end{pmatrix} x + \begin{pmatrix} c_x \\ c_b \\ 0 \end{pmatrix} + \begin{pmatrix} y_x \\ y_b \\ 0 \end{pmatrix}$$

and the equations (5) in the following form

$$p'_x = (p'_x, p'_b, p'_w) \begin{pmatrix} A_x \\ A_b \\ W \end{pmatrix} = (p'_x, p'_b, i'_r) \begin{pmatrix} A_x \\ A_b \\ \hat{p}_w W \end{pmatrix}$$

i_r = a n_r-dimensional column vector with all elements = 1.

The last form of (5) illustrates the close relationship between the production-income-equations (1-3) and the price equations (5), through the identity of the coefficient matrices. This identity applies in all but the last of the following modifications of the model, and even in the last modifications the differences are not very striking.

B. Specifications of final demand. "Commodity converters"

We assume all variables for goods delivered in the model to be measured in (current or base year) producers' market prices. This implies that trade and transportation margins on all goods delivered are entered as parallel deliveries from the trade and transportation sectors. The vector c, for instance, contains deliveries to private consumption from each sector at producers' prices, and separate items of the trade and transportation margins on these deliveries.

Our first modification of the simple model will be to introduce "commodity converter matrices", which convert a vector of commodity classes demanded for private consumption and a vector of commodity classes demanded for final uses other than private consumption, both specified in (constant) purchasers' prices, into vectors of deliveries in constant producers' prices from production sectors and import classes.

$$(1') \quad x = A_x x + C_x z + F_x u$$

$$(2') \quad b = A_b x + C_b z + F_b u$$

$$(3') \quad r = \hat{p}_w W x$$

$$(4') \quad z = z^o + D(\frac{1}{\pi} r - r^o) + N(\frac{1}{\pi} p - i_z)$$

$$(5') \quad p'_x = p'_x A_x + p'_b A_b + p'_w W$$

$$(6a') \quad p'_z = p'_x C_x + p'_b C_b$$

$$(6b') \quad p'_u = p'_x F_x + p'_b F_b$$

$$(6c') \quad \pi = p'_z \frac{1}{\gamma} z^o$$

$$(7') \quad \gamma = i'_z z^o$$

$$(8') \quad u = \text{given}$$

$$(9') \quad p_b = \text{given}$$

$$(10') \quad p_w = \text{given}$$

Here:

z = a column vector of deliveries of consumers' goods measured in constant purchasers' price values. Dimension n_z.

u = a column vector of deliveries of final demand goods other than private consumers' goods, measured in constant purchasers' price values. Dimension n_u.

C_x = a n_x by n_z matrix of coefficients characterizing the composition of each consumers' good in terms of the fractions delivered by each production sector.

F_x = a n_x by n_u matrix of coefficients characterizing the composition of each final good other than consumers' goods in terms of the fractions delivered by each production sector.

C_b = a n_b by n_z matrix of coefficient characterizing the composition of each consumers' good in terms of the fractions delivered from each import commodity type.

F_b = a n_b by n_u matrix of coefficients characterizing the composition of each final good other than consumers' goods in terms of the fractions delivered from each import commodity type.

i_z = a n_z dimensional column vector with all elements = 1.

C_x, F_x, C_b and F_b are assumed to be constant and estimated on the basis of base year accounts.

The equation of private consumption, (4), must be reformulated in terms of z instead of in terms of c.

The procedure of solution will be the same as for the simplified model:

We can first compute prices, p_x, by solving (5'), then p_z, p_u and π can be computed by (6a'), (6b'), (6c') and (7'). x, r and z are determined by simultaneously solving (1'), (3') and (4') and b is computed by (2').

Equations (1' - 3') now may be written:

$$\begin{pmatrix} x \\ b \\ r \end{pmatrix} = \begin{pmatrix} A_x & C_x & F_x \\ A_b & C_b & F_b \\ \hat{p}_w W & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ z \\ u \end{pmatrix}$$

and equations (5', 6a', 6b'):

$$(p'_x, \; p'_z, \; p'_u) = (p'_x, \; p'_b, \; p'_w) \begin{pmatrix} A_x & C_x & F_x \\ A_b & C_b & F_b \\ W & 0 & 0 \end{pmatrix}$$

C. Indirect taxes

We will then consider in some more detail the treatment of indirect taxes in the model. Indirect taxes will be specified as a subvector of r. Since we assume that the sector deliveries are given in producers' market prices indirect taxes collected from the producers are included in the value of production of each sector, and the amount of each tax collected per unit value produced in the base year is characterized by an element in W. However, indirect taxes collected in trade are not included in the product deliveries from the ordinary production sectors, but will be included in the deliveries from the trade sectors to the receivers of the goods on which the taxes are levied. Thus, a general sales tax on consumers' goods will be represented as deliveries from the trade sectors into private consumers' goods. The revenue from such a tax will be represented in the matrix W by an item giving the revenue as a fraction of the total "deliveries" from the relevant trade sector in the base year. If we subdivide the trade sector functionally in such a way that the collection of each of the types of indirect taxes which are collected in trade is considered a separate subsector of trade, then this fraction must be identically one.

Now, indirect taxes may be collected on a quantity or on a value basis, and some indirect taxes, in particular subsidies, which we may consider as negative taxes, are levied in amounts which are independent both of volume and value of operations. If indirect taxes are collected on a quantity basis, and if all rates for a given tax are changed in the same proportion, then this change may be expressed by an index giving the (average) rate per quantity unit in a given year divided by the (average) rate per quantity unit in the base year. Such indexes will then be elements of the vector p_w. They will not make necessary any reformulations in the simple model, neither for the indirect taxes collected from the producers, nor for those collected in trade. If the introduction of a new quantity tax, which was not collected in the base year, is contemplated, then it will be necessary to try to compute what this tax would have amounted to, if it had been collected on base year qyantities at the contemplated rate. On this basis, items in an expanded r^o vector and W matrix can be computed. The price index to be employed on this last element of r will be identically one.

For taxes collected on a value basis the case will be different: These items must be related to price or current value figures and not to quantity (constant price value) figures. In order to account for these taxes we will partition the vector of income shares, r, into three subvectors:

r_1, r_2 and r_3, and correspondingly we subdivide p_w into p_{w1}, p_{w2}, and p_{w3}, and W into W_1, W_2 and W_3, in such a way that p_{w1} are the elements of p_w and W_1 are the rows of W corresponding to r_1 a.s.o.

In r_1 we keep all the incomes shares, which are not the revenues of taxes collected on a value basis. Thus we still have:

$$(3a'') \quad r_1 = \hat{p}_{w1} W_1 x$$

In r_2 we keep the revenues from taxes collected from the production sectors on the basis of the value of production. For r_2 we write:

$$(3b'') \quad r_2 = \hat{v} W_2 (\hat{p}_x x)$$

Here:

v is a vector of "tax rate indexes" expressing the relative change in rates from the base year. It replaces the vector p_w in (3'). We will assume v to be given by government decree.[1] Finally the vector x in (3') is in this subset substituted by $(\hat{p}_x x)$, i.e. the volume figures are substituted by current price values. (New taxes may be taken into account by expanding the r_2-vector with items for the revenue of the new tax, and by entering the new tax coefficients in the places corresponding to the places of $(\hat{v} W_2)$ in the corresponding equations (3b'')).

In r_3 we collect the revenue from indirect taxes collected on a value basis in trade. We will return to r_3 shortly. With the partitioning of W into W_1, W_2 and W_3 price equations (5') can now be written

$$p_x' = p_x' A_x + p_b' A_b + p_{w1}' W_1 + v' W_2 \hat{p}_x + p_{w3}' W_3$$

This may be written

$$p_x' = p_x' A_x + p_b' A_b + p_{w1}' W_1 + p_x' \Lambda(v' W_2) + p_{w3}' W_3$$

or

$$p_x' (I - A_x - \Lambda(v' W_2) = p_b' A_b + p_{w1}' W_1 + p_{w3}' W_3$$

Here: I stands for a diagonal unit matrix of appropriate dimension (here n_x by n_x).

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1) A complication occurs when not all the deliveries from a given sector are taxed. In this case the indexes of tax rate changes, v, will not be the actual change in rates for that part of production which is taxed, but have to be derived from these rates. We shall not go into these computations here.

Taxes collected in trade on goods delivered to final uses will in the base year be represented by elements of C_x and F_x whether they are collected on a quantity or on a value basis. If they are collected on a value basis it makes little sense to compute the revenue on a "constant price" basis and to compute "price indexes" for these concepts.

In order to be able to form the price equations for final goods, let us instead assume that C_x and F_x can be partitioned horizontally into C_1 and C_2 and F_1 and F_2 respectively by collecting in C_2 and F_2 those rows of the respective matrixes which relate to indirect taxes on value. With a corresponding partitioning of p_x we now write instead of the price equations (6a') and (6b').

$$(6a'') \qquad p_z' = p_1' C_1 + s' C_2 \hat{p}_z + p_b' C_b = p_1' C_1 + p_z' \bigwedge(s' C_2) + p_b' C_b$$

$$(6b'') \qquad p_u' = p_1' F_1 + s' F_2 \hat{p}_u + p_b' F_b = p_1' F_1 + p_u' \bigwedge(s' F_2) + p_b' F_b$$

Here s is a vector of indexes of changes in the value tax rates. We will assume s to be given, in the same way as v.

Corresponding to the partitioning of C_x and F_x we get a partitioning of x into x_1 and x_2, where x_2 represents the "volume of production" in those functionally defined trade sectors which are charged with the (sole) task of collecting these taxes. Correspondingly, p_x will be partitioned into p_1 and p_2. Here x_2 and p_2 are of little interest separately, but the current value of the tax revenue will be given by

$$p_2 x_2 = \hat{s} C_2(\hat{p}_z z) + \hat{s} F_2(\hat{p}_u u)$$

Since the whole activity in this group of sectors is the collection of the tax and since the taxes collected belong to the income share vector, these will also be the remaining elements of r, i.e. r_3.

$$(3c'') \qquad r_3 = \hat{s} C_2(\hat{p}_z z) + \hat{s} F_2(\hat{p}_u u) \quad (= \hat{p}_{w3} W_3 x)$$

Corresponding to the partitioning of x into x_1 and x_2 we may also partition W_1, W_2 and W_3 vertically into W_{11}, W_{12}, W_{21}, W_{22}, W_{31} and W_{32}, to get

$$W_1 x = W_{11} x_1 + W_{12} x_2$$

$$W_2 x = W_{21} x_1 + W_{22} x_2$$

$$W_3 x = W_{31} x_1 + W_{32} x_2$$

Here all the elements of W_{31} must be identically zero, since the collection of taxes belonging to r_3 is by definition restricted to the sectors corresponding to x_2. W_{12} and W_{22} must both be zero matrixes and W_{32} must be the identity matrix, I, since the corresponding sectors are functionally defined to have the sole task of collecting the taxes, r_3.

We have already mentioned that p_2 is without interest, and we can therefore write the price equations:

$$p_1' = p_1' A_{x1} + p_b' A_{b1} + p_{w1}' W_{11} + v' W_{21} \hat{p}_1$$

$$(5'') \qquad p_1' (I - A_{x1} - \bigwedge(v' W_{21})) = p_b' A_{b1} + p_{w1}' W_{11}$$

A_{x1} and A_{b1} are submatrixes of A_x and A_b. Partitioning of A_x and A_b in correspondence with the partitioning of x and p_x must give:

$$A_x = \begin{pmatrix} A_{x1}, & 0 \\ 0, & 0 \end{pmatrix} \text{ and } A_b = \begin{pmatrix} A_{b1}, & 0 \end{pmatrix}$$

(New taxes on the value of final deliveries and collected in trade can be handled in a way which is quite similar to the treatment of new taxes on the value of production.)

We have not discussed the treatment of value taxes collected in trade, and levied on the use in production of certain raw materials. Since the revenue from such a tax per unit produced in the sector using the taxed raw material would depend only on the quantity of the raw material used per unit of production and the price of the raw material, it might easily have been taken into account. Nevertheless, in MODIS II it was thought more convenient to treat all taxes collected in trade on the use of raw materials as if they were levied on a quantity basis. If this approximation is taken into account when the "price indexes" for these taxes are computed, the consequent errors in calculations will be negligible.

Also subsidies and taxes levied independently of value and volume are roughly recomputed on a per unit basis in the Norwegian model.

Writing out the modified system of equations we now have:

$$(1'') \qquad x_1 = A_{x1} x_1 + C_1 z + F_1 u$$

$$(2'') \qquad b = A_{b1} x_1 + C_b z + F_b u$$

$$(3a'') \qquad r_1 = \hat{p}_{w1} W_{11} x_1$$

$$(3b'') \qquad r_2 = \hat{v} W_{21} (\hat{p}_1 x_1)$$

$$(3c'') \qquad r_3 = \hat{s} C_2(\hat{p}_z z) + \hat{s} F_2(\hat{p}_u u)$$

(Cont.)

$$(4'') \qquad z = z^{o} + D\left(\frac{1}{\pi} r - r^{o}\right) + N\left(\frac{1}{\pi} p - i_z\right)$$

$$(5'') \qquad p_1' \left(I - A_{x1} - \wedge(v' W_{21})\right) = p_b' A_{b1} + p_{w1}' W_{11}$$

$$(6a'') \qquad p_z' \left(I - \wedge(s' C_2)\right) = p_1' C_1 + p_b' C_b$$

$$(6b'') \qquad p_u' \left(I - \wedge(s' F_2)\right) = p_1' F_1 + p_b' F_b$$

$$(6c'') \qquad \pi = p_z' \frac{1}{\gamma} z^{o}$$

$$(7'') \qquad \gamma = i_z' z^{o}$$

$$(8'') \qquad u = \text{given}$$

$$(9'') \qquad p_b = \text{given}$$

$$(10a'') \qquad p_{w1} = \text{given}$$

$$(10b'') \qquad v = \text{given}$$

$$(10c'') \qquad s = \text{given}$$

Equations (1'' - 3c'') can be written:

$$\begin{pmatrix} x_1 \\ r_3 \\ b \\ r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} A_{x1} & C_1 & F_1 \\ 0 & \hat{s}\, C_2\, \hat{p}_z & \hat{s}\, F_2\, \hat{p}_u \\ A_{b1} & C_b & F_b \\ \hat{p}_{w1}\, W_{11} & 0 & 0 \\ \hat{v}\, W_{21}\, \hat{p}_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ z \\ u \end{pmatrix}$$

and equations (5'' - 6b''):

$$(p_1', p_z', p_u') = (p_1', s', p_b', p_w', v') \begin{pmatrix} A_{x1} & C_1 & F_1 \\ 0 & C_2\, \hat{p}_z & F_2\, \hat{p}_u \\ A_{b1} & C_b & F_b \\ W_{11} & 0 & 0 \\ W_{21}\, \hat{p}_1 & 0 & 0 \end{pmatrix}$$

D. Endogenous and exogenous prices

For the next modification in our model we take as a starting point the equation (5''), written in the following form:

$$p_1' = p_1' A_{x1} + p_1' \wedge(v' W_{21}) + p_b' A_{b1} + p_{w1}' W_{11}$$

We now partition p_1 into p_I and p_{II}, i.e. $p_1' = (p_I', p_{II}')$. Correspondingly we partition A_{x1} into $A_{I\,I}$, $A_{I\,II}$, $A_{II\,I}$ and $A_{II\,II}$, A_{b1} into A_{bI} and A_{bII}, W_{11} into W_{1I} and $W_{1\,II}$ and W_{21} into $W_{2\,I}$ and $W_{2\,II}$, i.e.

$$A_{x1} = \begin{pmatrix} A_{I\,I} & A_{I\,II} \\ A_{II\,I} & A_{II\,II} \end{pmatrix}$$

$$A_{b1} = (A_{b\,I} \quad A_{bII})$$

$$\begin{pmatrix} W_{11} \\ W_{21} \end{pmatrix} = \begin{pmatrix} W_{1I} & W_{1\,II} \\ W_{2I} & W_{2\,II} \end{pmatrix}$$

We can then write:

$$(5a''') \qquad p_I' = p_I' A_{I\,I} + p_{II}' A_{II\,I} + p_I' \wedge(v' W_{2I}) + p_b' A_{bI} + p_{w1}' W_{1\,I}$$

$$p_{II}' = p_I' A_{I\,II} + p_{II}' A_{II\,II} + p_{II}' \wedge(v' W_{2\,II}) + p_b' A_{bII} + p_{w1}' W_{1\,II}$$

Finally, we substitute zeroes for the coefficients on the line(s) corresponding to entrepreneurial incomes in $W_{1\,II}$ (but not in $W_{1\,I}$) and add a vector e of the same dimension as p_{II}, in the second equation. Writing $\overline{W}_{1\,II}$ for the matrix with the zero line(s) we get

$$(5b''') \qquad p_{II}' = p_{II}' A_{I\,II} + p_{II}' A_{II\,II} + p_{II}' \wedge(v' W_{2\,II}) + p_b' A_{bII} + p_{w1}' \overline{W}_{1\,II} + e'$$

The vector e now represents the entrepreneurial incomes per unit produced in each of those production sectors for which the product prices are given by p_{II}.

The incomes equation (3a'') must now be replaced by:

$$(3a''') \qquad r_1 = \hat{p}_{w1} W_{1\,I}\, x_I + \hat{p}_{w1} W_{1\,II} x_{II} + E\, \hat{e}\, x_{II}$$

Here E is a matrix which adds up the elements of $e\, x_{II}$ and puts them into the right category of incomes shares in r_1. x_I and x_{II} are the subvectors of x_1 resulting from a partitioning of x_1 corresponding to the partitioning of p_1 into p_I and p_{II}.

We will assume p_{II} to be given exogenously:

$$(10d''') \qquad p_{II} = \text{given}$$

The price indexes of p_{II} may represent prices of goods that compete on the world market and prices which are determined directly by the government.

p_I will now be determined uniquely by (5a''').

E. Wages and productivity

We will discuss in some more detail those elements of p_{wl} which represent the indexes of labour cost per unit produced. It is obvious that we must have at least as many labour income elements in r_1 as we want to have possibilities for assuming differences in the indexes of changes in labour cost per unit produced in the various sectors.

An index of unit labour cost will be determined by the average changes in the wage rate and the average change in labour productivity in the sectors concerned.

Let

p_k be the subvector of p_{wl} giving the indexes of unit labour costs.

Let

k be the vector of wage indexes and let g be the vector of productivity changes, then

(10e''') $p_k = (\hat{g})^{-1} k$

We will assume g and k to be given.

F. Price index of depreciation charges

In the Norwegian model entrepreneurial incomes are computed net of depreciation charges. This means that depreciation charges is a separate element in r_1 and that a price index is needed for this element.

The index used is computed on the basis of the prices for investment goods, i.e. some of the elements in p_u. We have

(6d''') $p_d = d' p_u$

p_d will be an element of the vector of price indexes for incomes shares, p_{wl}.

G. The system of price equations

Writing p_{wo} for the remaining vector of p_{wl}, when p_k and p_d are removed, writing W_{kI}, W_{kII} and W_{dI} and W_{dII} for the rows of W_{lI} and $\overline{W}_{lII}$

corresponding to p_k and p_d respectively (the "labour coefficients" and the "depreciation coefficients") and writing W_{oI} and W_{oII} for the remaining rows of $W_{1\,I}$ and $\overline{W}_{1\,II}$, we can now write the price equations of the system:

$$(5a^{iv}) \qquad p'_I = p'_I A_{I\,I} + p'_{II} A_{II\,I} + p'_I \Lambda(v' W_{2I}) + p'_b A_{bI} + p'_{wo} W_{oI} + p'_k W_{kI} + p_d W_{dI}$$

$$(5b^{iv}) \qquad p'_{II} = p'_I A_{I\,II} + p'_{II} A_{II\,II} + p'_{II} \Lambda(v' W_{2II}) + p'_b A_{bII} + p'_{wo} W_{oII} + p'_k W_{kII} + p_d W_{dII} + e'$$

$$(6a^{iv}) \qquad p'_z (I - \Lambda(s' C_2)) = p'_1 C_1 + p'_b C_b$$

$$(6b^{iv}) \qquad p'_u (I - \Lambda(s' F_2)) = p'_1 F_1 + p'_b F_b$$

$$(6c^{iv}) \qquad \pi = p'_z \frac{1}{\gamma} z^o$$

$$(6d^{iv}) \qquad p_d = d' p_u$$

$$(7^{iv}) \qquad \gamma = i'_z z^o$$

$$(9^{iv}) \qquad p_b = \text{given}$$

$$(10a^{iv}) \qquad p_{wo} = \text{given}$$

$$(10b^{iv}) \qquad v = \text{given}$$

$$(10c^{iv}) \qquad s = \text{given}$$

$$(10d^{iv}) \qquad p_{II} = \text{given}$$

$$(10e^{iv}) \qquad p_k = (\hat{g})^{-1} k$$

$$(11a^{iv}) \qquad g = \text{given}$$

$$(11b^{iv}) \qquad k = \text{given}$$

$$(12^{iv}) \qquad p_1 = \begin{pmatrix} p_I \\ p_{II} \end{pmatrix}$$

This is a determinate subsystem of the entire set of equations. It makes possible the determination of p_I, p_k, p_d, p_z, p_u and π from a set of given price-, wage-, tax- and productivity variables.

This subsystem also gives a representation of the price relationships in the Norwegian model MODIS II.

Using again the matrix form for equations ($5a^{iv}$- $6b^{iv}$) we get:

$$(p_I', (p_{II}- e)', p_z', p_u') =$$

$$= (p_I', p_{II}', s', p_b', p_{wo}', p_k', p_d, v') \begin{pmatrix} A_{I\,I} & A_{I\,II} & C_I & F_I \\ A_{II\,I} & A_{II\,II} & C_{II} & F_{II} \\ 0 & 0 & C_2\hat{p}_z & F_2\hat{p}_u \\ A_{bI} & A_{bII} & C_b & F_b \\ W_{oI} & W_{oII} & 0 & 0 \\ W_{kI} & W_{kII} & 0 & 0 \\ W_{dI} & W_{dII} & 0 & 0 \\ W_{2I}\hat{p}_I & W_{2II}\hat{p}_{II} & 0 & 0 \end{pmatrix}$$

H. Exogenous estimates of production levels

In writing out the quantity and income relationships we will now add a new column vector $j = (j_I', j_{II}')'$, so that we have:

($1a^{iv}$) $\quad x_I = A_{I\,I}\, x_I + A_{I\,II}\, x_{II} + C_I\, z + F_I\, u + j_I$

($1b^{iv}$) $\quad x_{II} = A_{II\,I}\, x_I + A_{II\,II}\, x_{II} + C_{II}\, z + F_{II}\, u + j_{II}$

(2^{iv}) $\quad b = A_{bI}\, x_I + A_{bII}\, x_{II} + C_b\, z + F_b\, u + K_{bI}\, j_I + K_{bII}\, j_{II}$

($3a^{iv}$) $\quad r_o = \hat{p}_{wo}\, W_{oI}\, x_I + \hat{p}_{wo}\, W_{oII}\, x_{II} + E\,\hat{e}\, x_{II} + K_{oI}\,\hat{p}_I\, j_I + K_{oII}\,\hat{p}_{II}\, j_{II}$

($3b^{iv}$) $\quad r_k = \hat{p}_k\, W_{kI}\, x_I + \hat{p}_k\, W_{kII}\, x_{II}$

($3c^{iv}$) $\quad r_d = p_d\, W_{dI}\, x_I + p_d\, W_{dII}\, x_{II}$

($3d^{iv}$) $\quad r_2 = \hat{v}\, W_{2I}\,\hat{p}_I\, x_I + \hat{v}\, W_{2II}\,\hat{p}_{II}\, x_{II}$

($3e^{iv}$) $\quad r_3 = \hat{s}\, C_2\,\hat{p}_z\, z + \hat{s}\, F_2\,\hat{p}_u\, u$

($8a^{iv}$) $\quad y_I = F_I\, u + M_I\, j_I$

($8b^{iv}$) $\quad y_{II} = F_{II}\, u + M_{II}\, j_{II}$

($8c^{iv}$) $\quad u =$ given

The elements of j fall into two categories: Either they are identically zero, in which case the equations ($1a^{iv}$) and ($1b^{iv}$) are used to determine the corresponding elements of $x = (x_I', x_{II}')'$, or they are unknowns, to be determined by the equations ($1a^{iv}$) and ($1b^{iv}$). The elements of x corresponding to non-zero elements of j must be exogenously given. They are the production levels that are considered to be determined by conditions of nature, or by the availability of factors of production.

The existence of non-zero items of j implies that the remaining terms on the right hand sides of equations ($1a^{iv}$) and ($1b^{iv}$) do not account for the total of domestic production, the balancing items, j, are interpreted in three alternative ways:

1) Some of them are considered to be adjustments in the exogenous estimates of inventory changes. In these cases final demand other than private consumption is not entirely exogenously determined, as indicated by the equations ($8a^{iv}$) and ($8b^{iv}$). (M_I and M_{II} are diagonal matrices with 1's and 0's in the diagonals.

2) Some non-zero elements of j are considered to represent import substitution, so that a positive element in j represents surplus supply from domestic production which is available for replacement of a corresponding amount of imports, and a negative element in j represents the need for supplementary import in order to cover demand, which, if output could have been further expanded, would have been supplied from domestic production. This interpretation must have consequences for the import estimates. The matrices K_{bI} and K_{bII} in equation (2^{iv}) pick out those elements of j, which are to be considered as affecting imports, and account for them in the import estimates of the model.

3) The third alternative is to consider the non-zero elements of j to be adjustments to the estimates of demand for inputs in the production sectors implied by the fixed coefficients, A. This interpretation implies that one or more of the production sectors use more or less of the product in question than it would have done if input-output coefficients were fixed. The corresponding saving or dissaving in production costs must be reflected in entrepreneurial incomes. Since we do not try to identify the sectors in which the change of coefficients occur, also the corresponding adjustment in entrepreneurial income can not be specified by sector. (If we want to stick to our exogenous estimates of entrepreneurial income per unit produced in sectors with endogenous price determination, we must assume that the adjustments discussed here only affect sectors with exogenous price determination, where entrepreneurial income is residually determined.) The matrices $K_{oI}\,\hat{p}_I$ and $K_{oII}\,\hat{p}_{II}$ compute the cost savings and add them to those elements in r_o which represent entrepreneurial incomes.

Writing out the quantity and income equations in matrix form, we have:

$$\begin{pmatrix} x_I \\ x_{II} \\ r_3 \\ b \\ r_o \\ r_k \\ r_d \\ r_2 \end{pmatrix} = \begin{pmatrix} A_{I\ I} & A_{I\ II} & C_I & F_I & I & 0 \\ A_{II\ I} & A_{II\ II} & C_{II} & F_{II} & 0 & I \\ 0 & 0 & \hat{s}C_2\hat{p}_z & \hat{s}F_2\hat{p}_u & 0 & 0 \\ A_{bI} & A_{bII} & C_b & F_b & K_{bI} & K_{bII} \\ \hat{p}_{wo}W_{oI} & (\hat{p}_{wo}W_{oII}+E\,\hat{e}) & 0 & 0 & K_{oI}\hat{p}_I & K_{oII}\hat{p}_{II} \\ \hat{p}_{wk}W_{kI} & \hat{p}_{wk}W_{kII} & 0 & 0 & 0 & 0 \\ p_{wd}W_{dI} & p_{wd}W_{dII} & 0 & 0 & 0 & 0 \\ \hat{v}\,W_{2I}\hat{p}_I & \hat{v}\,W_{2II}\hat{p}_{II} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_I \\ x_{II} \\ z \\ u \\ j_I \\ j_{II} \end{pmatrix}$$

I. Disposable income

The final modification is in the consumption equation (4, 4', 4'' respectively). Here we now substitute disposable income for gross income. Only wages, r_k, and entrepreneurial incomes, r_o, are used in the consumption function, and we can write:

$$(4a^{iv}) \qquad z = z^o + D_k\left(\frac{1}{\pi}(r_k - T_k) - (r_k^o - T_k^o)\right) + D_o\left(\frac{1}{\pi}(r_o - T_o) - (r_o^o - T_o^o)\right) + N\left(\frac{1}{\pi}p - i_z\right)$$

T_k = direct taxes on labour income (wages)

T_o = direct taxes on entrepreneurial incomes

In Norway the tax system is such that taxes on entrepreneurial incomes actually to be collected in the course of a given year are determined by the end of the preceding year, namely as the preliminary "advance tax" for the year plus/minus the adjustments due to final settlements for the preceding year. For wage and salary income, preliminary taxes are depending on the current level of income. We have:

$$(4b^{iv}) \qquad T_k = \tau_m\cdot(\hat{k}-I)'(\hat{g})^{-1} W_k x^o + \tau_g\left(\hat{p}_k W_k (x-x^o) + ((\hat{g})^{-1} - I) W_k x^o\right) + T_{kc}$$

Here

τ_m = an estimate of the marginal tax rate for wage and salary earners (in the "budget year"). It is applied to the increase in wage and salary income which would occur with the assumed changes in wage rates and productivity, if production in all sectors remained unchanged. This may be considered an approximation to the change in wage and salary income due to a change in average income per employed worker.

τ_g = an estimate of the average tax rate for wage and salary earners (in the "budget year"). It is applied to the remainder of the change in wage and salary incomes, which may be considered an approximation to the income change due to the change in employment.

T_{kc} = the tax revenues that would occur if present rates (for the "budget" year) were applied to realized wage and salary incomes in the base year. T_{kc} can be estimated with a comparatively high degree of accuracy.

K. Incremental form of the equations

The equation ($4b^{iv}$) is written in incremental form in x.

It is easily seen that the equation ($4a^{iv}$), which is in incremental form in z, can be written in incremental form in x by reformulations of ($3a^{iv}$) and ($3b^{iv}$) and insertion.

Finally ($1a^{iv}$), ($1b^{iv}$) and (2^{iv}) can be written in incremental form in x_I, x_{II}, z, u and b with x_I^o, x_{II}^o, z^o, u^o and b^o disappearing if the same equations are assumed to be valid for the base year with j identically zero.