




ARTIKLER

32



**THE STABILITY OF
INPUT-OUTPUT COEFFICIENTS**

By Per Sevaldson

**STABILITET I
KRYSSLØPSKOEFFISIENTER**

OSLO 1970

STATISTISK SENTRALBYRÅ

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Forord

Denne artikkelen gir en del av resultatene fra et større forskningsprosjekt i Statistisk Sentralbyrå. Prosjektet tar sikte på å undersøke empirisk holdbarheten av forutsetningen om faste forholdstall i produksjonssektorene (næringene) mellom produksjonsvolum og innsats av varer og andre produksjonsfaktorer. Dette er en hovedforutsetning i kryssløpsanalysen, slik den nå anvendes i økonomiske planleggingsmodeller i Norge og mange andre land.

Norge, med sitt godt utbygde system av årlige nasjonalregnskap, er et av de svært få land hvor disse viktige undersøkelser kan gjennomføres.

Artikkelen ble lagt fram på den fjerde internasjonale konferanse om kryssløpsanalyseteknikk i Genève i 1968 og er trykt i Proceedings of the Fourth International Conference on Input-Output Techniques, redigert av A. P. Carter og A. Brody (North Holland Publishing Company, Amsterdam 1969). Den gjengis her med samtykke av redaktører og forlag.

Statistisk Sentralbyrå, Oslo, 8. oktober 1969.

Petter Jakob Bjerve

Preface

This publication gives some results from a major research project in the Central Bureau of Statistics. The purpose of the project is to test empirically the assumption of fixed proportions between the volume of production and the input of commodities and other production factors in the production sectors (industries). This assumption is basic in input-output analysis, as it is now applied in economic models for planning in Norway and in many other countries.

With its highly developed system of annual national accounts with extensive input-output detail, Norway is one of the very few countries where studies of this type can be made.

The present report was presented at the fourth international conference on input-output techniques in Geneva in 1968 and is printed in Proceedings of the Fourth International Conference on Input-Output Techniques, edited by A. P. Carter and A. Brody (North Holland Publishing Company, Amsterdam 1969). The report is published in this series with the agreement of the editors and the publisher.

Central Bureau of Statistics, Oslo, 8 October 1969.

Petter Jakob Bjerve

CHAPTER 10

The stability of input-output coefficients

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The Central Bureau of Statistics of Norway

Introduction

We still know relatively little about the fluctuations over time in input-output coefficients and the causes of these fluctuations. This is the subject of a research program carried out by the Central Bureau of Statistics of Norway. One aspect of this program has been the study of time series of input-output accounts at constant prices. The presentation of some results from this study is the topic of the present paper.

It is generally believed that the stability of input-output coefficients depends on the level of sector specification in the data. This problem is discussed in section 1. The data are presented in section 2. In sections 3 and 4 some results of computations in a relatively detailed sector specification are presented, and in section 5 these results are compared to the results of computations in aggregated sector specifications.

1. The effects of aggregation on variability in input-output coefficients

It is generally recognized that aggregation in input-output tables entails tendencies both to make the coefficients more stable and to make them more variable. A cause of increased stability can be found in the fact that when sectors are aggregated, sectors which produce raw

materials that are close substitutes will frequently be combined, and the coefficients for use of the sum of their products may reasonably be expected to be more stable than the coefficients for the use of each of them. A cause of increased instability is the fact that when two or more sectors with different coefficients for the same input are combined, the aggregate coefficient will be an average of the coefficients of the individual sectors, and the average will depend on the relative weight of the production in each sector. When these weights vary, the average coefficient will vary, even if the individual coefficients are constant.

Let us examine the possibilities a little more closely:

We shall use the following notation:

- x_j = production in sector j in the detailed specification, measured in value at constant prices ($j = 1, 2, \dots, n$).
- x_{ij} = input from sector i to sector j ($i = 1, 2, \dots, n; j = 1, 2, \dots, n$).
- a_{ij} = x_{ij}/x_j = input coefficient for use in detailed sector j of products from detailed sector i ($i = 1, 2, \dots, n; j = 1, 2, \dots, n$).
- a_{Ij} = $\sum_{i \in I} a_{ij}$ = input coefficient for use in detailed sector j of products from aggregate sector I ($I = 1, 2, \dots, N; j = 1, 2, \dots, n; n > N$). $\sum_{i \in I}$ means the sum over all those sectors i , which belong to the aggregate sector I .
- A_{IJ} = $\sum_{i \in I} \sum_{j \in J} x_{ij} / (\sum_{j \in J} (x_j - \sum_{i \in J} x_{ij}))$ = input coefficient for use in aggregate sector J of products from aggregate sector I ($I = 1, 2, \dots, N; J = 1, 2, \dots, N; I \neq J$).
- A_{II} = 0 ($I = 1, 2, \dots, N$).
- r_j = $x_j / (\sum_{j \in J} (x_j - \sum_{i \in J} x_{ij}))$ = relative share of total production in detailed sector j in (net) production in aggregate sector J ($j = 1, 2, \dots, n; J = 1, 2, \dots, N$).

var a_{ij} = variance of a_{ij}

var a_{IJ} = variance of a_{IJ}

var A_{IJ} = variance of A_{IJ}

var r_j = variance of r_j

We may now write (for $I \neq J$):

$$A_{IJ} = \sum_{i \in I} \sum_{j \in J} \frac{x_{ij}}{\sum_{j \in J} (x_j - \sum_{i \in J} x_{ij})} = \sum_{j \in J} \frac{\sum_{i \in I} x_{ij}}{x_j} \cdot \frac{x_j}{\sum_{j \in J} (x_j - \sum_{i \in J} x_{ij})} = \sum_{j \in J} r_j \cdot a_{IJ}$$

In principle there may exist interdependency between r_j and a_{Ij} , since they are both derived from items in an input-output table, and the items of such a table are bound together through definitional relationships (identity of column sums with corresponding row sums). However, this is likely to impose only a mild restriction on the free variability of r_j in relation to a_{Ij} . More serious is the possibility of an interdependence due to the fact that x_j occurs in the definition of both r_j and a_{Ij} , and that x_{Ij} is part of x_j (through the relationship $x_j = \sum_i x_{Ij} + \text{value added in sector } j$). Even though the possibility of interdependency cannot, thus, be ruled out, we will disregard it here.

If r_j and a_{Ij} are independent, we have for the variance of the product ($r_j a_{Ij}$):

$$\text{var}(r_j a_{Ij}) = \text{var } r_j \cdot \text{var } a_{Ij} + \bar{a}_{Ij}^2 \text{var } r_j + \bar{r}_j^2 \text{var } a_{Ij},$$

where \bar{a}_{Ij} and \bar{r}_j are the average (expected) values of a_{Ij} and r_j respectively. If, further, the individual products ($r_j a_{Ij}$) are independent for $j \in J$, then

$$\text{var } A_{IJ} = \sum_{j \in J} \text{var}(r_j a_{Ij}) = \sum_{j \in J} \text{var } r_j \text{var } a_{Ij} + \sum_{j \in J} \bar{a}_{Ij}^2 \text{var } r_j + \sum_{j \in J} \bar{r}_j^2 \text{var } a_{Ij}$$

or

$$\text{var } A_{IJ} = \sum_{j \in J} (\bar{r}_j^2 + \text{var } r_j) \text{var } a_{Ij} + \sum_{j \in J} \bar{a}_{Ij}^2 \text{var } r_j$$

The variance of the aggregate coefficient is, in this case, a weighted sum of the variances of the coefficients, and the variances of the relative shares of the individual sectors in production in the aggregate sector.

Since both $\sum_{j \in J} (\bar{r}_j^2 + \text{var } r_j)$ and $\sum_{j \in J} \bar{a}_{Ij}^2$ may easily be considerably less than 1, and even their sum may be below 1, we should not be surprised to find that $\text{var } A_{IJ}$ will frequently be less than the typical variances of both the r_j and the a_{Ij} . Moreover, since A_{IJ} will tend to be bigger than the average of the a_{Ij} coefficients (because the sum of weights in $A_{IJ} = \sum_{j \in J} r_j a_{Ij}$ will normally exceed 1) we may expect $\text{var } A_{IJ}$ quite often to be less than the variance of coefficients a_{Ij} or a_{ij} of the same magnitude. These conclusions rest on the previous assumptions about independence but since there are few reasons to expect positive covariation between r_j and the corresponding a_{Ij} or between the various ($r_j a_{Ij}$) in the same aggregate sector, this does not appear to be a serious restriction.

Let us consider the weights of $\text{var } a_{Ij}$ and $\text{var } r_j$ a little more closely: When production is very unevenly distributed among the detailed sectors in an aggregate sector, one \bar{r}_j may be large (near 1) and the others must be limited in number and small. In this case all the $\text{var } r_j$ may be expected to be small since we may generally expect the r_j with small \bar{r}_j to have small variances and the variance of the one r_j with great \bar{r}_j will be approximately equal to the variance of the (small) sum of the small r_j 's. The weight sum $\sum_{j \in J} (\bar{r}_j^2 + \text{var } r_j)$ will be dominated by the square of the largest \bar{r}_j ($j \in J$). We may therefore expect the sum of weights for the coefficient variances to be less than one, even if the dominating sector has an \bar{r}_j as big as 0.8 or 0.9. With such large fractions for one sector, the number of detailed sectors in the aggregate sector will also normally be limited and the weight sum $\sum_{j \in J} \bar{a}_{Ij}^2$ for the terms $\text{var } r_j$ will be small, even for values of \bar{a}_{Ij} up to 0.2 or 0.3, which must be considered to be large. At the same time $\text{var } r_j$ may be expected to be small, as already mentioned. Consequently, we conclude that, in this case, we can normally expect $\text{var } A_{Ij}$ to be less than the typical $\text{var } a_{Ij}$ for $j \in J$.

If the \bar{r}_j are small or moderate in size, then their squares and sum of squares will be well below 1, and even if the $\text{var } r_j$ are added to the squares, their sum is not likely to reach the neighbourhood of 1. Now, if the number of detailed sectors to be aggregated is large, and if the coefficients \bar{a}_{Ij} are large, the sum $\sum_{j \in J} \bar{a}_{Ij}^2$ may become large too, and the sizes of the variances of the production shares gain increasing influence on $\text{var } A_{Ij}$. However, with an increasing number of detailed sectors to be aggregated, their individual shares tend to decrease, and so do the variances of these shares (in absolute terms). Consequently, the weight sum for $\text{var } a_{Ij}$ decreases and the $\text{var } r_j$ themselves decrease, and $\text{var } A_{Ij}$ may still well be less than the typical $\text{var } a_{Ij}$.

We have assumed that the r_j and a_{Ij} are independent for any given j in J . By and large this appears to be a plausible assumption. One possible cause for a negative correlation might appear if there were a random component in the productivity of sector j , which acted so that, in certain periods, the output was high in relation to inputs, and so that the high output also led to a higher than normal share in production, r_j , while in other periods output in relation to inputs and thus share in production was low. It is hard to imagine that such tendencies should dominate

the figures, but if they existed we would expect them to *diminish* and not to increase the variance of the products ($r_j a_{IJ}$).

Our other previous assumption, that the individual products in the sum $\sum_{j \in J} r_j a_{IJ}$ are independent is probably not realistic, since the r_j are likely to be negatively correlated, at least when the number of detailed sectors in each aggregate sector is small. However, this should tend to diminish the variance of the sum.

If we compare the disaggregated coefficients, a_{ij} , and the aggregated coefficients A_{IJ} , we arrive at the following conclusion: The variance of the 'semi-aggregated' coefficients a_{IJ} will in general be less than the variances of the component detailed coefficients a_{ij} ($i \in I$) because of the substitution effect. But it does not necessarily follow that, in general, the variance of 'semi-aggregated' coefficients will be smaller than the variance of detailed coefficients of the *same* size. We shall, therefore, assume that the variance of 'semi-aggregated' coefficients is of the same order of magnitude as the variance of detailed coefficients of the same size. The coefficients A_{IJ} will tend to be greater than the average of the corresponding 'semi-aggregated' coefficients, a_{IJ} ($j \in J$), since the sum of the weights ($\sum_{j \in J} r_j$) will in general exceed 1.

Now we have demonstrated that the variance of A_{IJ} , under quite plausible assumptions, will be less than the variance of the 'semi-aggregated' coefficients a_{IJ} . Consequently, we expect the variance of A_{IJ} , in general, to be smaller than the variance of detailed coefficients a_{ij} of the same size. We have not proved that $\text{var } A_{IJ}$ will always be less than the variance of detailed coefficients a_{ij} of the same size, but we have demonstrated that the opposite will occur only for rather extreme values of the product shares r_j . Let us consider a few numerical examples with an aggregation of four detailed sectors into one aggregate sector. We make the following assumptions:

$$0.1 \leq r_j \leq r$$

$$1 \leq \sum_{j \in J} r_j \leq 1.1$$

$$\text{var } r_j \leq \frac{1}{9} \bar{r}_j^2 \quad \text{i.e. (the standard deviation of } r_j) \leq \frac{1}{3} \bar{r}_j$$

$$a_{IJ} \leq a_I$$

$$\text{var } a_{IJ} \leq \frac{1}{4} a_{IJ}^2 \quad (\leq \frac{1}{4} a_I^2),$$

where r and a_I are maximum values for r_j and a_{Ij} respectively. We now get

$$\begin{aligned} \text{var } A_{IJ} &= \sum_{j \in J} ((\bar{r}_j^2 + \text{var } r_j) \text{var } a_{Ij} + \bar{a}_{Ij}^2 \text{var } r_j) \\ &\leq \sum_{j \in J} \left(\frac{10}{9} \bar{r}_j^2 \cdot \frac{1}{4} a_I^2 + a_I^2 \frac{1}{9} \bar{r}_j^2 \right) = \frac{14}{36} a_I^2 \sum_{j \in J} \bar{r}_j^2 \end{aligned}$$

and

for a_I	=	0.05	0.10	0.20	0.30
var a_{Ij}	≤	0.00062	0.00250	0.01000	0.02250
var $A_{IJ} \leq$	for $r = 0.6$	0.00046	0.00180	0.00731	0.01645
	$r = 0.5$	0.00042	0.00167	0.00669	0.01506
	$r = 0.4$	0.00036	0.00144	0.00576	0.01295
	$r = 0.3$	0.00030	0.00121	0.00482	0.01080

In all these cases we have established an upper bound for $\text{var } A_{IJ}$ that is below the upper bound for $\text{var } a_{Ij}$, whereas the upper bound for A_{IJ} is 1.1 a_I , i.e., 1.1 times the upper bound for a_{Ij} .

Obviously, greater stability in the coefficients of an aggregated table is no argument for the preference of an aggregated model to a more detailed one. A reasonable interpretation is that we should expect greater precision in the estimates of aggregate production levels – even when derived from a model with an aggregate input-output table – than in the estimates of detailed production levels, estimated from a model with a detailed input-output table. But we should expect to get even more accurate estimates of aggregate production levels when they are taken as sums of detailed production levels, estimated from a model with a detailed input-output table.

2. The data

2.1. The detailed input-output accounts

Annual input-output accounts for Norway for the period 1949–1960, in fixed 1955 prices, are specified for 89 production sectors. Ten of these sectors have been excluded from the investigation of input-output relationships, either because they are only dummy sectors with no

real counterparts in the productive activity of the economy, or because they have no inputs of raw materials, so that their value added is identically 100 per cent of gross production in all years. The accounts are given at purchasers' market prices, but, in order to eliminate a possibly unstable element, gross trade margins have been deducted from the value of production of each sector in this investigation. Correspondingly, the inputs from the trade sector covering these margins have been eliminated from the input accounts. With deliveries from each of 83 production sectors and each of 60 import sectors to each of the 79 sectors of the investigation, and one item for value added for each of these 79 sectors, the number of possible items each year is 11297. Actually we had only about 1500 items, the rest being zeroes in all years.

Since the very small coefficients were considered to be of limited interest for the analysis, all input items which were less than 2 per cent of gross production in a sector in all the years, and less than 1 per cent in at least one year, were lumped together into one item for each sector, called 'small unspecified'. These inputs (75 altogether) are not analysed in the same detail as the others. The specified input items were classified into five main groups, namely:

- Norwegian competitive (161 items)
- Norwegian non-competitive (153 items)
- Imports, competitive (137 items)
- Imports, non-competitive (26 items)
- Gross value added (79 items).

For each sector the following sums were taken, and treated as separate inputs in the analysis: the sum of each item of Norwegian competitive and the corresponding competitive imports (225 items); the sum of electricity and all fuel inputs (53 items); the sum of the principal input and all inputs which could be expected to be relatively close substitutes for it (53 items); and the sum of all imported inputs (68 items).

The inputs were classified according to whether the receiving sector was an extractive or a service producing industry (37 sectors), or a commodity processing industry (42 sectors). The inputs were also classified by type into the following categories: direct materials (455 items), auxiliary materials (233 items), service inputs (79 items) and packaging materials (41 items).

2.2. *The aggregated input-output accounts*

The aggregated accounts are based on the detailed accounts and cover the same series of annual data. The aggregations give a 14-sector and a 5-sector specification, and were designed for purposes other than the present investigation. (The figures have been published in 'National Accounts Classified by Fourteen and Five Industrial Sectors 1949-1961', Vol. 1, Central Bureau of Statistics of Norway, Oslo 1965). Since coefficients had already been calculated on the basis of values at purchasers' prices, including trade margins, no deduction from the value of gross production was made in these series.

3. *Characteristics of the ordinary proportional input-output coefficient*

Our computations give estimates of this coefficient in two alternative forms, namely

$$(1) \quad \frac{x_{ij}(t)}{x_j(t)} = a_{ij} + u_{ij}(t)$$

$$(2) \quad \frac{x_{ij}(t)}{x_j(t)} = b_{ij} + d_{ij}t + v_{ij}(t)$$

Here

$x_{ij}(t)$ = amount of item i absorbed in sector j in absolute, constant (1955) prices (kroner) in year t (purchasers' prices);

$x_j(t)$ = total production in sector j in absolute, constant (1955) prices (kroner) in year t (producers' prices);

t = year t , $t = 1$ for 1949, $t = 2$ for 1950, ..., $t = 12$ for 1960.

a_{ij} , b_{ij} and d_{ij} = constants estimated over the period.

$u_{ij}(t)$ and $v_{ij}(t)$ = residual terms, when the constants are determined so that

$$\sum_{t=1}^{12} u_{ij}(t) = \sum_{t=1}^{12} v_{ij}(t) = 0$$

$$\sum_{t=1}^{12} [u_{ij}(t)]^2 = \text{minimum}$$

$$\sum_{t=1}^{12} [v_{ij}(t)]^2 = \text{minimum}$$

We have $0 \leq a_{ij} < 1$ and $0 \leq b_{ij} < 1$ for all i and j . The standard deviations of $u_{ij}(t)$, 'standard deviation about the average coefficient', and of $v_{ij}(t)$, 'standard deviation about the trend in the coefficients', are given in table 1.*

TABLE 1

Standard deviations about average coefficients and coefficient trends for main categories of inputs

Type of inputs	Number of coefficients	Standard deviation	
		about average	about trend
Norwegian competitive	161	0.017	0.013
Norwegian non-competitive	153	0.008	0.006
Imports, competitive	137	0.017	0.012
Imports, non-competitive	26	0.014	0.011
All specified inputs	477	0.014	0.010
Competitive inputs combined	225	0.018	0.012
Fuels combined	53	0.006	0.005
Substitution groups	53	0.031	0.023
Import sums	68	0.025	0.020
Small unspecified	75	0.008	0.006
Gross value added	79	0.032	0.026

The standard deviation about the average coefficient is of the order of 1 to 3 per cent of production and the standard deviation about a linear trend through the period is roughly three quarters of the standard deviation about the average. Since the specified coefficients vary in average size from around 2 per cent up to 100 per cent (and, in a special case even above 100) it will be important to ascertain whether the standard deviation varies with the size of the coefficient. This is done in table 2. Table 2 indicates a clear correlation between the average size of the coefficients and their variance. The average standard deviation about the

* In comparing the standard deviation about the average and about the trend, note that the estimates of standard deviation about the trend have been adjusted for degrees of freedom, whereas no such adjustment has been made in the estimates of standard deviations about the average. The adjustment would increase the estimated standard deviations about the average by 4.45 per cent.

TABLE 2
Standard deviations about average coefficients and coefficient trends for main categories of inputs,
classified by average size of the coefficient

	Average size of coefficient																	
	0.02 and less			0.02-0.05			0.05-0.10			0.10-0.25			0.25-0.50			Over 0.50		
	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*
Norwegian competitive	55	0.007	0.006	53	0.013	0.009	23	0.016	0.011	17	0.032	0.025	8	0.064	0.054	5	0.050	0.034
Norwegian non-competitive	64	0.005	0.004	55	0.006	0.004	23	0.015	0.011	9	0.018	0.015	1	0.023	0.025	1	0.029	0.031
Imports, competitive	54	0.009	0.007	39	0.011	0.009	23	0.020	0.015	14	0.032	0.024	5	0.054	0.036	2	0.084	0.050
Imports, non-competitive	9	0.007	0.006	9	0.008	0.006	1	0.019	0.020	4	0.014	0.007	2	0.031	0.023	1	0.090	0.089
All specified inputs	182	0.007	0.006	156	0.010	0.007	70	0.017	0.012	44	0.028	0.021	16	0.054	0.043	9	0.060	0.044
Competitive inputs combined	57	0.008	0.006	77	0.011	0.008	36	0.015	0.011	32	0.027	0.018	14	0.060	0.040	9	0.043	0.033
Fuels combined	38	0.002	0.002	9	0.008	0.005	3	0.017	0.011	2	0.021	0.019	-	-	-	1	0.058	0.057
Substitution groups	3	0.002	0.002	4	0.008	0.004	4	0.010	0.006	12	0.023	0.017	14	0.041	0.023	16	0.045	0.039
Import sums	8	0.006	0.006	13	0.010	0.009	6	0.017	0.014	26	0.028	0.022	11	0.037	0.031	4	0.074	0.061
Small unspecified	22	0.004	0.003	30	0.009	0.006	21	0.009	0.008	1	0.010	0.008	-	-	-	-	-	-
Gross value added	-	-	-	-	-	-	2	0.033	0.033	6	0.058	0.048	27	0.042	0.036	44	0.022	0.016

* Standard deviation.

average coefficient varies from less than 1 per cent, for coefficients of 2 per cent or less, to 5 per cent and more for the largest coefficients. The notable exception is the coefficient for gross value added. This coefficient is by definition equal to 1 minus the sum of all other coefficients, and its standard deviations might, as well, be expected to vary in inverse proportion to its size. This appears to be the case for the three upper size groups. In the lower size groups there are only two observations (one with high standard deviations, 0.052 and 0.056 respectively, and one with low, 0.014 and 0.009).

The frequency distributions of the standard deviations about the average coefficient for the 477 specified input items show that particularly for the small coefficients, the distributions are quite concentrated, with a few cases of extreme variability. Table 3 illustrates how peaked these distributions are, in comparison to the normal distribution.

TABLE 3
Characteristics of size distributions of standard deviations
about the average coefficient for the 477
specified input items

Group average coefficient (per cent)	Percentage of values of more than the average plus or minus		
	3 times the standard deviation of the distribution	2 times	1 times
0- 2.00	2.2	3.3	5.0
2.01- 5.00	2.5	2.5	12.3
5.01-10.00	2.9	2.9	14.3
10.01-25.00	2.3	6.8	27.3
25.01 and over	0.0	4.0	12.0
The normal distribution	0.3	4.6	31.7

If we compare the first four categories of specified input coefficients (table 2), the variance for the non-competitive items appears to be slightly less than for the competitive items, but it does not seem to

reduce the variance appreciably to combine corresponding domestic and foreign competitive items.

The combination of all fuels (energy sources) into one item appears to reduce the variability slightly as compared to that of other items. If we compare the combined fuels items with the specified fuels items, we get the following picture:

	Average size of coefficient											
	0.02 and less			0.02-0.05			0.05-0.10			Greater than 0.50		
	Standard deviation about			Standard deviation about			Standard deviation about			Standard deviation about		
	Number	Average	Trend	Number	Average	Trend	Number	Average	Trend	Number	Average	Trend
Fuels combined	38	0.002	0.002	9	0.008	0.005	3	0.017	0.011	1	0.058	0.057
Specified fuel items	14	0.008	0.006	7	0.016	0.011	6	0.013	0.010	1	0.073	0.064

There appears to be a definite reduction in variability when the fuels items are combined instead of being treated as separate items.

If we group the coefficients according to the size of delivery in absolute value (kroner) we obtain the figures in table 4. The standard deviation is still in general larger for big items than for small. However, the correlation with coefficient size is far less pronounced.

Table 5 gives a joint distribution by size of coefficient and by input size. There appears to be a slight tendency for the standard deviations to be somewhat smaller when the coefficient is calculated for an input which is large in absolute (kroner) value than when the input is small in absolute value. The important difference, however, is between the standard deviations for small and big coefficients.

In this study the 79 production sectors were divided into two groups: Group 0, extractive industries and service industries and Group 1, commodity processing industries. Table 6 gives the same information as table 2 for each of the two groups separately. Surprisingly, the coefficients appear to be more stable in the extractive and service sectors than in the commodity processing sectors, over the period investigated.

TABLE 4
Standard deviations about average coefficients and coefficient trends for main categories of inputs,
classified by average size of the input item

	Average size of input item in millions of 1955-kroner																				
	0-10.0			10.1-50.0			50.1-100.0			100.1-250.0			250.1-500.0			500.1-1000.0			1000.1 and over		
	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*
Norwegian competitive	85	0.011	0.009	43	0.018	0.013	13	0.026	0.020	10	0.048	0.033	9	0.018	0.013	1	0.044	0.032	-	-	-
Norwegian non-competitive	108	0.008	0.006	36	0.008	0.007	3	0.005	0.005	4	0.003	0.002	2	0.016	0.017	-	-	-	-	-	-
Imports, competitive	69	0.011	0.008	50	0.020	0.015	9	0.031	0.024	7	0.012	0.009	2	0.082	0.034	-	-	-	-	-	-
Imports, non-competitive	15	0.006	0.006	6	0.017	0.010	3	0.017	0.014	1	0.090	0.089	-	-	-	-	-	-	1	0.018	0.010
All specified inputs	277	0.010	0.008	135	0.016	0.012	28	0.024	0.019	22	0.030	0.023	13	0.028	0.017	1	0.044	0.032	1	0.018	0.010
Competitive inputs combined	93	0.011	0.009	75	0.018	0.012	24	0.023	0.014	19	0.034	0.024	14	0.026	0.017	-	-	-	-	-	-
Fuels combined	39	0.003	0.003	14	0.014	0.011	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Substitution groups	3	0.053	0.045	14	0.031	0.021	9	0.030	0.022	9	0.035	0.022	12	0.031	0.025	5	0.017	0.013	1	0.013	0.014
Import sums	16	0.013	0.010	25	0.027	0.023	10	0.024	0.021	12	0.033	0.028	3	0.054	0.031	1	0.009	0.006	1	0.018	0.011
Gross value added	3	0.082	0.073	13	0.043	0.038	13	0.035	0.028	31	0.023	0.019	10	0.025	0.015	5	0.035	0.030	4	0.025	0.013

* Standard deviation.

TABLE 5

Standard deviations about average coefficients and coefficient trends for main categories of inputs, classified by average size of the coefficients and average size of the input item (in kroner)

Coefficient, % Input, mill. kr.	0-10.0 0-50			Over 10.0 0-50			0-10.0 Over 50			Over 10.0 Over 50		
	Num- ber	Aver- age*	Trend*	Num- ber	Aver- age*	Trend*	Num- ber	Aver- age*	Trend*	Num- ber	Aver- age*	Trend*
Norwegian competitive	119	0.012	0.008	9	0.045	0.041	12	0.010	0.006	21	0.043	0.032
Norwegian non-compet- itive	135	0.007	0.005	9	0.019	0.016	7	0.002	0.002	2	0.021	0.022
Imports, competitive	110	0.012	0.010	9	0.045	0.035	7	0.014	0.011	11	0.039	0.025
Imports, non-compet- itive	18	0.008	0.007	3	0.017	0.008	1	0.002	0.002	4	0.040	0.035
All specified items	382	0.010	0.007	30	0.035	0.029	27	0.009	0.006	38	0.040	0.030
Competitive inputs combined	153	0.011	0.008	15	0.043	0.032	18	0.008	0.005	39	0.037	0.024
Fuels combined	50	0.004	0.003	3	0.033	0.031	-	-	-	-	-	-
Substitution groups	6	0.008	0.005	11	0.049	0.036	5	0.006	0.002	31	0.033	0.025
Import sums	26	0.012	0.011	15	0.038	0.030	2	0.009	0.006	25	0.032	0.026
Gross value added	2	0.033	0.033	14	0.053	0.046	-	-	-	63	0.027	0.021

* Standard deviation.

TABLE 6

Standard deviations about average coefficients and about coefficient trends for main categories of inputs, classified by average size of coefficient and type of sector

	0.02 and less			0.02-0.05			Average size of coefficient						0.25-0.50			Over 0.50		
	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*
<i>Extractive and service sectors</i>																		
Norwegian competitive	13	0.004	0.003	6	0.012	0.008	4	0.008	0.004	2	0.021	0.007	-	-	-	1	0.033	0.029
Norwegian non-competitive	25	0.003	0.003	40	0.006	0.004	16	0.012	0.007	8	0.018	0.014	1	0.023	0.025	-	-	-
Imports, competitive	9	0.008	0.005	4	0.007	0.005	3	0.019	0.011	2	0.032	0.020	-	-	-	-	-	-
Imports, non-competitive	3	0.005	0.005	5	0.005	0.005	1	0.019	0.020	2	0.016	0.003	2	0.031	0.023	-	-	-
All specified inputs	50	0.004	0.003	55	0.006	0.004	24	0.012	0.008	14	0.020	0.012	3	0.028	0.024	1	0.033	0.029
Competitive inputs combined	9	0.007	0.005	12	0.011	0.007	7	0.012	0.006	6	0.021	0.014	-	-	-	1	0.031	0.029
Fuels combined	8	0.002	0.002	2	0.014	0.008	1	0.024	0.011	-	-	-	-	-	-	-	-	-
Substitution groups	3	0.002	0.002	2	0.006	0.002	2	0.009	0.004	4	0.012	0.011	-	-	-	-	-	-
Import sums	8	0.006	0.006	8	0.009	0.006	2	0.018	0.010	6	0.019	0.014	2	0.032	0.029	-	-	-
Gross value added	-	-	-	-	-	-	-	-	-	-	-	-	4	0.030	0.024	33	0.019	0.014
<i>Commodity processing sectors</i>																		
Norwegian competitive	42	0.008	0.007	47	0.013	0.009	19	0.017	0.013	15	0.034	0.028	8	0.064	0.054	4	0.055	0.036
Norwegian non-competitive	39	0.006	0.005	15	0.008	0.007	7	0.021	0.019	1	0.020	0.022	-	-	-	1	0.029	0.031
Imports, competitive	45	0.009	0.007	35	0.011	0.009	20	0.021	0.016	12	0.032	0.025	5	0.054	0.036	2	0.084	0.050
Imports, non-competitive	6	0.008	0.006	4	0.011	0.007	-	-	-	2	0.013	0.012	-	-	-	1	0.090	0.089
All specified inputs	132	0.008	0.006	101	0.011	0.009	46	0.019	0.015	31	0.030	0.025	13	0.060	0.047	8	0.063	0.045
Competitive inputs combined	48	0.008	0.006	65	0.011	0.008	29	0.016	0.012	26	0.028	0.019	14	0.060	0.040	8	0.045	0.033
Fuels combined	30	0.002	0.002	7	0.007	0.005	2	0.014	0.011	2	0.021	0.019	-	-	-	1	0.058	0.057
Substitution groups	-	-	-	2	0.011	0.006	2	0.012	0.008	8	0.029	0.020	13	0.043	0.025	16	0.045	0.039
Import sums	-	-	-	5	0.012	0.013	4	0.016	0.016	20	0.031	0.024	9	0.038	0.031	4	0.074	0.061
Gross value added	-	-	-	-	-	-	2	0.033	0.033	6	0.058	0.049	23	0.044	0.038	11	0.030	0.020

* Standard deviation.

TABLE 7

Distribution of ratios of trend coefficients to standard deviations*

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Ratio	Norwegian competitive	Norwegian non-competitive	Imports, competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
-16.01 to -17.00	-	1	-	-	1	-	-	-	-	-
-15.01 to -16.00	-	-	-	-	-	-	-	-	-	-
-14.01 to -15.00	-	-	-	-	-	1	-	-	-	-
-13.01 to -14.00	-	1	-	-	1	-	-	-	-	-
-12.01 to -13.00	-	1	-	-	1	-	-	-	-	1
-11.01 to -12.00	-	-	-	-	-	1	-	1	-	-
-10.01 to -11.00	-	2	-	-	2	-	-	-	-	-
-9.01 to -10.00	-	3	-	-	3	1	1	-	-	-
-8.01 to -9.00	2	3	2	-	7	2	-	1	-	2
-7.01 to -8.00	2	4	1	1	8	3	-	1	1	-
-6.01 to -7.00	3	7	4	-	15	6	4	2	-	6
-5.01 to -6.00	6	2	1	1	9	9	1	3	2	1
-4.01 to -5.00	15	6	11	4	36	16	3	1	3	2
-3.01 to -4.00	13 (1)	5 (1)	6 (1)	1	25 (3)	11 (2)	3	5	2	6 (1)
-2.01 to -3.00	11 (5)	8 (5)	10 (4)	1	30 (14)	15 (6)	4 (2)	3 (2)	5 (3)	7 (2)
-1.01 to -2.00	16 (22)	16 (20)	6 (18)	4 (4)	42 (64)	16 (30)	8 (7)	6 (7)	5 (9)	9 (10)
-0.01 to -1.00	12 (52)	16 (51)	13 (45)	4 (9)	45 (157)	17 (73)	- (17)	5 (17)	7 (22)	7 (26)
-0.00 to 0.99	17 (53)	9 (52)	18 (46)	2 (9)	46 (158)	18 (74)	8 (18)	6 (18)	11 (22)	6 (27)
1.00 to 1.99	12 (22)	14 (20)	14 (18)	3 (4)	43 (64)	28 (30)	8 (7)	3 (7)	9 (9)	7 (10)
2.00 to 2.99	16 (5)	15 (5)	11 (4)	3	45 (14)	20 (6)	4 (2)	3 (2)	4 (3)	12 (2)
3.00 to 3.99	12 (1)	17 (1)	13 (1)	1	43 (3)	16 (2)	2	3	8	4 (1)
4.00 to 4.99	6	11	11	-	28	7	4	2	3	2
5.00 to 5.99	6	8	6	1	21	12	3	1	6	2
6.00 to 6.99	6	1	4	-	11	11	-	1	-	5
7.00 to 7.99	2	1	3	-	6	3	-	-	1	-
8.00 to 8.99	1	1	2	-	4	5	-	1	1	-
9.00 to 9.99	1	-	-	-	1	2	-	1	-	-
10.00 to 10.99	2	1	1	-	4	2	-	2	-	-
11.00 to 11.99	-	-	-	-	-	1	-	-	-	-
12.00 to 12.99	-	-	-	-	-	-	-	2	-	-
13.00 to 13.99	-	-	-	-	-	1	-	-	-	-
14.00 to 14.99	-	-	-	-	-	1	-	-	-	-
Total	161 (161)	153 (153)	137 (137)	26 (26)	477 (477)	225 (225)	53 (53)	53 (53)	68 (68)	79 (79)

* Figures in parentheses are the figures that would be expected if the distributions were *t*-distributions with 10 degrees of freedom.

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TABLE 8

Trend characteristics of coefficients by size groups (number of coefficients)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.02 and less</i>										
Clear positive trend	8	12	15	-	35	15	7	-	2	-
Moderate positive trend	6	10	4	1	21	5	2	-	-	-
No trend	26	28	26	5	85	27	19	3	6	-
Moderate negative trend	3	2	5	-	10	4	3	-	-	-
Clear negative trend	12	12	4	3	31	6	7	-	-	-
Total	55	64	54	9	182	57	38	3	8	-
<i>Coefficient size 0.02-0.05</i>										
Clear positive trend	11	12	11	1	35	18	1	3	3	-
Moderate positive trend	5	7	2	2	16	5	1	-	2	-
No trend	19	20	18	5	62	29	3	-	7	-
Moderate negative trend	3	2	-	-	5	7	-	-	1	-
Clear negative trend	15	14	8	1	38	18	4	1	-	-
Total	53	55	39	9	156	77	9	4	13	-
<i>Coefficient size 0.05 and over</i>										
Clear positive trend	12	7	12	-	31	24	1	9	11	11
Moderate positive trend	5	2	6	-	13	8	-	3	4	9
No trend	22	16	10	4	52	31	3	20	20	36
Moderate negative trend	2	2	7	1	12	5	1	3	5	7
Clear negative trend	12	7	9	3	31	23	1	11	7	16
Total	53	34	44	8	139	91	6	46	47	79
<i>All coefficients</i>										
Clear positive trend	31	31	38	1	101	57	9	12	16	11
Moderate positive trend	16	19	12	3	50	18	3	3	6	9
No trend	67	64	54	14	199	87	25	23	33	36
Moderate negative trend	8	6	12	1	27	16	4	3	6	7
Clear negative trend	39	33	21	7	100	47	12	12	7	16
Total	161	153	137	26	477	225	53	53	68	79

TABLE 9
Trend characteristics of coefficients (percentage distribution)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.02 and less</i>										
Clear positive trend	15	19	28	-	19	26	18	-	25	-
Moderate positive trend	11	15	8	11	12	9	6	-	-	-
No trend	47	44	48	56	47	47	50	100	75	-
Moderate negative trend	5	3	9	-	5	7	8	-	-	-
Clear negative trend	22	19	7	33	17	11	18	-	-	-
Total	100	100	100	100	100	100	100	100	100	-
Number of coefficients	55	64	54	9	182	57	38	3	8	-
<i>Coefficient size 0.02-0.05</i>										
Clear positive trend	21	22	28	11	23	23	11	75	23	-
Moderate positive trend	9	13	5	22	10	7	11	-	15	-
No trend	36	36	46	56	40	38	33	-	54	-
Moderate negative trend	6	4	-	-	3	9	-	-	8	-
Clear negative trend	28	25	21	11	24	23	45	25	-	-
Total	100	100	100	100	100	100	100	100	100	-
Number of coefficients	53	55	39	9	156	77	9	4	13	-
<i>Coefficient size 0.05 and over</i>										
Clear positive trend	23	21	27	-	22	26	17	19	23	14
Moderate positive trend	9	6	14	-	9	9	-	7	9	11
No trend	41	47	23	50	38	34	50	43	42	46
Moderate negative trend	4	6	16	12	9	6	16	7	11	9
Clear negative trend	23	20	20	38	22	25	17	24	15	20
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	53	34	44	8	139	91	6	46	47	79
<i>All coefficients</i>										
Clear positive trend	19	20	28	4	21	25	17	23	23	14
Moderate positive trend	10	12	9	11	10	8	6	6	9	11
No trend	42	42	39	54	42	39	47	43	49	46
Moderate negative trend	5	4	9	4	6	7	7	6	9	9
Clear negative trend	24	22	15	27	21	21	23	22	10	20
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	161	153	137	26	477	225	53	53	68	79

4. Trends in the input-output coefficients

Tables 1–6 all indicate that the standard deviation about a trend in the coefficients is somewhat smaller than the standard deviation about the average coefficient for the whole period, and we shall now examine more closely the existence of trends in the coefficients. Trends can appear quite easily in series of only 12 items. Technical change may be expected to register as coefficient trends, since the change will usually take effect more quickly in some establishments than in others, and even within an establishment the switch to new techniques will often be gradual.

In our computations the trend effect in the coefficients was tested in the form

$$\frac{x_{ij}(t)}{x_j(t)} = b_{ij} + d_{ij}t + v_{ij}(t).$$

We then computed the standard deviation of d_{ij} , σ_{ij}^\dagger and d_{ij}/σ_{ij} . If d_{ij} is normally distributed about 'true' value of 0, with standard deviation σ_{ij} , then d_{ij}/σ_{ij} will be distributed according to the t -distribution with 10 degrees of freedom.

The distributions for the specified input items are given in table 7. The corresponding t -distributions are also shown. The deviations from the t -distributions are very marked and indicate the existence of negative as well as positive trends in the coefficients. For the t -distribution, 5 per cent of the observations will deviate from zero by more than 2.2 times the standard deviation and 1 per cent will deviate by at least 3.2 times the standard deviation, if the true value of the coefficient is zero (no trend). Accordingly, we have grouped the coefficients into the following groups (tables 8 and 9):

- | | |
|----------------------------|---|
| 1. Clear positive trend | $\frac{d_{ij}}{\sigma_{ij}} > 3.2$ |
| 2. Moderate positive trend | $3.2 \geq \frac{d_{ij}}{\sigma_{ij}} > 2.2$ |
| 3. No trend | $\geq 2.2 \frac{d_{ij}}{\sigma_{ij}} \geq -2.2$ |

† With $(12 - 2) = 10$ degrees of freedom.

4. Moderate negative trend $-2.2 > \frac{d_{ij}}{\sigma_{ij}} \geq -3.2$
5. Clear negative trend $-3.2 > \frac{d_{ij}}{\sigma_{ij}}$

For all coefficient groups, about 40 per cent fall in the no trend category, 10 per cent in each of the categories of moderate trends, and 20 per cent in each of the extreme groups. As might be expected, the no trend category is slightly greater for the smallest coefficients than for the other size groups.

The notable exceptions are the various import coefficients. The larger competitive import coefficients had relatively fewer cases of no trend and smaller competitive import coefficients fewer cases of negative trends than other input categories, with a corresponding overrepresentation of positive trends for all size groups. Non-competitive import coefficients had relatively more cases of no trend and negative trends than usual, and relatively fewer cases of positive trends, for all size groups. The sum coefficients for imports had relatively more positive and less negative trends than normal.

TABLE 10

Standard deviations about average coefficients and about trends for input coefficients, classified by trend characteristics

	Average size of coefficient								
	0.02 and less			0.02-0.05			0.05-0.10		
	Num-ber	Aver-age*	Trend*	Num-ber	Aver-age*	Trend*	Num-ber	Aver-age*	Trend*
<i>All specified inputs</i>									
No trend	85	0.006	-	62	0.008	-	21	0.015	-
Moderate trend	31	0.008	0.006	21	0.009	0.007	14	0.016	0.013
Clear trend	66	0.007	0.004	73	0.012	0.007	35	0.019	0.011
<i>Competitive inputs combined</i>									
No trend	27	0.007	-	29	0.008	-	14	0.010	-
Moderate trend	9	0.010	0.007	12	0.012	0.010	6	0.011	0.009
Clear trend	21	0.008	0.005	36	0.013	0.009	16	0.022	0.011
<i>Gross value added</i>									
No trend	-	-	-	-	-	-	1	0.052	-
Moderate trend	-	-	-	-	-	-	-	-	-
Clear trend	-	-	-	-	-	-	1	0.014	0.009

* Standard deviation.

The standard deviation about the average coefficient is generally greater for the coefficients with a moderate trend than for those with no trend and greater for those with a clear trend than for those with no trend. But when the standard deviation is taken about the trend value for coefficients with trend, the size of the standard deviation does not appear to increase as we move from no trend coefficients to moderate trend and clear trend coefficients (table 10).

A comparison of manufacturing sectors with primary and service sectors with respect to trend characteristics of input coefficients does not indicate strong systematic differences. There is some indication of a greater proportion of no trend coefficients for inputs in manufacturing (tables 11 and 12). Also when we compare the trend characteristics of the various types of input coefficients, there do not seem to be systematic differences between direct materials, auxiliary materials, service inputs and packaging materials (tables 13 and 14).

We shall finally consider the slopes of the trend lines (tables 15 and 16). We have grouped the coefficients with clear or moderate negative or positive trends according to the numerical size of the estimated annual

0.10-0.25			Average size of coefficient 0.25-0.50			Greater than 0.50		
Number	Average*	Trend*	Number	Average*	Trend*	Number	Average*	Trend*
21	0.018	-	6	0.049	-	4	0.045	-
8	0.026	0.020	2	0.042	0.036	1	0.073	0.064
15	0.041	0.025	8	0.061	0.038	4	0.072	0.038
9	0.019	-	5	0.045	-	3	0.036	-
5	0.017	0.014	-	-	-	2	0.035	0.028
18	0.034	0.019	9	0.067	0.037	4	0.053	0.033
2	0.061	-	15	0.036	-	17	0.016	-
2	0.069	0.055	6	0.058	0.050	8	0.022	0.017
1	0.029	0.011	6	0.042	0.025	19	0.027	0.014

TABLE 11

Trend characteristics of coefficients by size groups and type of sector
(number of coefficients)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.02 and less</i>										
Extractive and service sectors										
Clear positive trend	2	4	4	-	10	3	1	-	2	-
Moderate positive trend	-	6	1	-	7	1	-	-	-	-
No trend	7	11	4	3	25	5	4	3	6	-
Moderate negative trend	1	-	-	-	1	-	1	-	-	-
Clear negative trend	3	4	-	-	7	-	2	-	-	-
Total	13	25	9	3	50	9	8	3	8	-
Commodity processing sectors										
Clear positive trend	6	8	11	-	25	12	6	-	-	-
Moderate positive trend	6	4	3	1	14	4	2	-	-	-
No trend	19	17	22	2	60	22	15	-	-	-
Moderate negative trend	2	2	5	-	9	4	2	-	-	-
Clear negative trend	9	8	4	3	24	6	5	-	-	-
Total	42	39	45	6	132	48	30	-	-	-
<i>Coefficient size 0.02-0.05</i>										
Extractive and service sectors										
Clear positive trend	3	7	-	-	10	5	-	2	3	-
Moderate positive trend	1	6	-	2	9	-	1	-	1	-
No trend	1	12	2	3	18	5	-	-	4	-
Moderate negative trend	-	1	-	-	1	1	-	-	-	-
Clear negative trend	1	14	2	-	17	1	1	-	-	-
Total	6	40	4	5	55	12	2	2	8	-
Commodity processing sectors										
Clear positive trend	8	5	11	1	22	13	1	1	-	-
Moderate positive trend	4	1	2	-	6	5	-	-	1	-
No trend	18	8	16	2	34	24	3	-	3	-
Moderate negative trend	3	1	-	-	1	6	-	-	1	-
Clear negative trend	14	-	6	1	10	17	3	1	-	-
Total	47	15	35	4	73	65	7	2	5	-

Table 11 (cont'd.)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.05 and over</i>										
Extractive and service sectors										
Clear positive trend	4	6	3	—	13	5	—	2	2	5
Moderate positive trend	1	—	—	—	1	3	—	1	1	5
No trend	1	12	—	2	15	2	—	4	2	14
Moderate negative trend	—	—	1	1	2	1	—	—	1	3
Clear negative trend	1	7	1	2	11	3	1	—	4	10
Total	7	25	5	5	42	14	1	7	10	37
Commodity processing sectors										
Clear positive trend	8	1	9	—	11	19	1	7	9	6
Moderate positive trend	4	2	6	—	8	5	—	2	3	4
No trend	21	4	10	2	21	29	3	16	18	22
Moderate negative trend	2	2	6	—	9	4	1	3	4	4
Clear negative trend	11	—	8	1	14	20	—	11	3	6
Total	46	9	39	3	63	77	5	39	37	42
<i>All coefficients</i>										
Extractive and service sectors										
Clear positive trend	9	17	7	—	33	13	1	4	7	5
Moderate positive trend	2	12	1	2	17	4	1	1	2	5
No trend	9	35	6	8	58	12	4	7	12	14
Moderate negative trend	1	1	1	1	4	2	1	—	1	3
Clear negative trend	5	25	3	2	35	4	4	—	4	10
Total	26	90	18	13	147	35	11	12	26	37
Commodity processing sectors										
Clear positive trend	22	14	31	1	58	44	8	8	9	6
Moderate positive trend	14	7	11	1	28	14	2	2	4	4
No trend	58	29	48	6	115	75	21	16	21	22
Moderate negative trend	7	5	11	—	19	14	3	3	5	4
Clear negative trend	34	8	18	5	48	43	8	12	3	6
Total	135	63	119	13	268	190	42	41	42	42

TABLE 12
Trend characteristics of coefficients by size groups and type of sector
(percentage distributions)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.02 and less</i>										
Extractive and service sectors										
Clear positive trend	15	16	45	-	20	33	13	-	25	-
Moderate positive trend	-	24	10	-	14	11	-	-	-	-
No trend	54	44	45	100	50	56	50	100	75	-
Moderate negative trend	8	-	-	-	2	-	12	-	-	-
Clear negative trend	23	16	-	-	14	-	25	-	-	-
Total	100	100	100	100	100	100	100	100	100	-
Number of coefficients	13	25	9	3	50	9	8	3	8	-
Commodity processing sectors										
Clear positive trend	14	20	24	-	19	25	20	-	-	-
Moderate positive trend	14	10	7	17	11	8	7	-	-	-
No trend	45	44	49	33	45	46	50	-	-	-
Moderate negative trend	5	5	11	-	7	8	7	-	-	-
Clear negative trend	22	21	9	50	18	13	16	-	-	-
Total	100	100	100	100	100	100	100	-	-	-
Number of coefficients	42	39	45	6	132	48	30	-	-	-
<i>Coefficient size 0.02-0.05</i>										
Extractive and service sectors										
Clear positive trend	50	17	-	-	18	42	-	100	37	-
Moderate positive trend	17	15	-	40	16	-	50	-	13	-
No trend	17	30	50	60	33	42	-	-	50	-
Moderate negative trend	-	3	-	-	2	8	-	-	-	-
Clear negative trend	16	35	50	-	31	8	50	-	-	-
Total	100	100	100	100	100	100	100	100	100	-
Number of coefficients	6	40	4	5	55	12	2	2	8	-
Commodity processing sectors										
Clear positive trend	17	33	31	25	30	20	14	50	-	-
Moderate positive trend	9	7	6	-	8	8	-	-	20	-
No trend	38	53	46	50	47	37	43	-	60	-
Moderate negative trend	6	7	-	-	1	9	-	-	20	-
Clear negative trend	30	-	17	25	14	26	43	50	-	-
Total	100	100	100	100	100	100	100	100	100	-
Number of coefficients	47	15	35	4	73	65	7	2	5	-

Table 12 (cont'd.)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups	Import sums	Gross value added
<i>Coefficient size 0.05 and over</i>										
Extractive and service sectors										
Clear positive trend	58	24	60	-	31	36	-	39	20	14
Moderate positive trend	14	-	-	-	2	21	-	14	10	13
No trend	14	48	-	40	36	15	-	57	20	38
Moderate negative trend	-	-	20	20	5	7	-	-	10	8
Clear negative trend	14	28	20	40	26	21	100	-	40	27
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	7	25	5	5	42	14	1	7	10	37
Commodity processing sectors										
Clear positive trend	17	11	23	-	18	25	20	18	24	14
Moderate positive trend	9	22	15	-	13	6	-	5	8	10
No trend	46	45	26	67	33	38	60	41	49	52
Moderate negative trend	4	22	15	-	14	5	20	8	11	10
Clear negative trend	24	-	21	33	22	26	-	28	8	14
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	46	9	39	3	63	77	5	39	37	42
<i>All coefficients</i>										
Extractive and service sectors										
Clear positive trend	35	19	39	-	22	37	9	33	27	14
Moderate positive trend	7	13	6	15	12	11	9	8	8	13
No trend	35	39	33	62	39	34	37	59	46	38
Moderate negative trend	4	1	6	8	3	6	9	-	4	8
Clear negative trend	19	28	16	15	24	12	36	-	15	27
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	26	90	18	13	147	35	11	12	26	37
Commodity processing sectors										
Clear positive trend	16	22	26	8	22	23	19	20	21	14
Moderate positive trend	11	11	9	8	10	7	5	5	10	10
No trend	43	46	41	46	43	40	50	39	50	52
Moderate negative trend	5	8	9	-	7	7	7	7	12	10
Clear negative trend	25	13	15	38	18	23	19	29	7	14
Total	100	100	100	100	100	100	100	100	100	100
Number of coefficients	135	63	119	13	268	190	42	41	42	42

TABLE 13

Trend characteristics of coefficients by input types (number of coefficients)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups
<i>Direct materials</i>								
Clear positive trend	26	6	26	-	58	35	-	10
Moderate positive trend	13	3	12	-	28	17	-	3
No trend	52	8	34	5	99	58	-	18
Moderate negative trend	7	2	10	-	19	11	-	3
Clear negative trend	30	1	16	3	50	34	-	12
Total	128	20	98	8	254	155	-	46
<i>Auxiliary materials</i>								
Clear positive trend	5	12	12	1	30	20	9	2
Moderate positive trend	1	7	-	1	9	1	3	-
No trend	7	23	20	3	53	22	25	2
Moderate negative trend	1	1	2	-	4	3	4	-
Clear negative trend	5	14	5	3	27	7	12	-
Total	19	57	39	8	123	53	53	4
<i>Service inputs</i>								
Clear positive trend	-	12	-	-	12	-	-	-
Moderate positive trend	-	9	-	2	11	-	-	-
No trend	-	28	-	6	34	-	-	-
Moderate negative trend	-	1	-	1	2	-	-	-
Clear negative trend	-	16	-	1	17	-	-	-
Total	-	66	-	10	76	-	-	-
<i>Packaging materials</i>								
Clear positive trend	-	1	-	-	1	2	-	-
Moderate positive trend	2	-	-	-	2	-	-	-
No trend	8	5	-	-	13	7	-	-
Moderate negative trend	-	2	-	-	2	2	-	-
Clear negative trend	4	2	-	-	6	6	-	-
Total	14	10	-	-	24	17	-	-

TABLE 14

Trend characteristics of coefficients by input types (percentage distribution)

	Norwegian competitive	Norwegian non-competitive	Imports competitive	Imports non-competitive	All specified inputs	Competitive inputs combined	Fuels combined	Substitution groups
<i>Direct materials</i>								
Clear positive trend	20	30	27	-	23	23	-	22
Moderate positive trend	10	15	12	-	11	11	-	7
No trend	41	40	35	63	39	37	-	39
Moderate negative trend	6	10	10	-	7	7	-	6
Clear negative trend	23	5	16	37	20	22	-	26
Total	100	100	100	100	100	100	-	100
Number of coefficients	128	20	98	8	254	155	-	46
<i>Auxiliary materials</i>								
Clear positive trend	27	21	31	13	25	38	17	50
Moderate positive trend	5	12	-	13	7	2	6	-
No trend	37	40	51	37	43	41	47	50
Moderate negative trend	5	2	5	-	3	6	7	-
Clear negative trend	26	25	13	37	22	13	23	-
Total	100	100	100	100	100	100	100	100
Number of coefficients	19	57	39	8	123	53	53	4
<i>Service inputs</i>								
Clear positive trend	-	18	-	-	16	-	-	-
Moderate positive trend	-	14	-	20	14	-	-	-
No trend	-	42	-	60	45	-	-	100
Moderate negative trend	-	2	-	10	3	-	-	-
Clear negative trend	-	24	-	10	22	-	-	-
Total	-	100	-	100	100	-	-	100
Number of coefficients	-	66	-	10	76	-	-	3
<i>Packaging materials</i>								
Clear positive trend	-	10	-	-	4	12	-	-
Moderate positive trend	14	-	-	-	9	-	-	-
No trend	57	50	-	-	54	41	-	-
Moderate negative trend	-	20	-	-	8	12	-	-
Clear negative trend	29	20	-	-	25	35	-	-
Total	100	100	-	-	100	100	-	-
Number of coefficients	14	10	-	-	24	17	-	-

change in trend value. The coefficients classified as having no trend have been kept apart. We find that trends which change a coefficient as much as 1 percentage point† per year, i.e., by 10 percentage points or more over a period of 10 years, are quite rare. In the entire 79 sector input-output matrix, with potentially more than 11000 coefficients, and with 1500 registered non-zero coefficients, only 6 intermediate input coefficients, 11 import coefficients and 11 gross value-added coefficients showed changes in trend values of 1 percentage point or

TABLE 15
Size distribution of trend coefficients (number of coefficients)

	No trend	Annual change in trend value*					Total
		0-0.49	0.50-0.99	1.00-1.99	2.00-2.99	3.00-3.99	
Norwegian competitive	67	69	19	4	1	1	161
Norwegian non-competitive	65	81	7	-	-	-	153
Imports, competitive	54	61	11	10	1	-	137
Imports, non-competitive	14	9	3	-	-	-	26
All specified inputs	200	220	40	14	2	1	477
Competitive inputs combined	88	97	21	16	2	1	225
Fuels combined	25	25	3	-	-	-	53
Substitution groups	23	12	8	9	1	-	53
Import sums	34	15	12	4	3	-	68
Gross value added	36	16	16	9	2	-	79

* Per cent of output in receiving sector

more per annum. For changes of $\frac{1}{2}$ or more percentage points per annum, the figures are 32 intermediate input coefficients, 25 import coefficients and 27 gross value-added coefficients. (The latter figure is of course

† Per cent of output in receiving sector.

more than $\frac{1}{3}$ of all the value-added coefficients and not negligible.) Considering that 87 of the intermediate input coefficients, 52 of the import coefficients and all the 79 gross value-added coefficients were in size groups with average standard deviations about the trend value of more than 1 percentage point, trends in the coefficients do not appear

TABLE 16
Size distribution of trend coefficients (percentage distribution)

	No trend	Annual change in trend value*					Total
		0-0.49	0.50-0.99	1.00-1.99	2.00-2.99	3.00-3.99	
Norwegian competitive	41.7	42.8	11.8	2.5	0.6	0.6	100.0
Norwegian non-competitive	42.5	52.9	4.6	-	-	-	100.0
Imports, competitive	39.5	44.5	8.0	7.3	0.7	-	100.0
Imports, non-competitive	53.8	34.6	11.6	-	-	-	100.0
All specified inputs	41.9	46.2	8.4	2.9	0.4	0.2	100.0
Competitive inputs combined	39.1	43.1	9.3	7.1	0.9	0.5	100.0
Fuels combined	47.2	47.2	5.6	-	-	-	100.0
Substitution groups	43.4	22.6	15.1	17.0	1.9	-	100.0
Import sums	50.0	22.1	17.6	5.9	4.4	-	100.0
Gross value added	45.6	20.3	20.2	11.4	2.5	-	100.0

* Per cent of output in receiving sector

to be a major source of instability in input-output coefficients over moderate time intervals. This conclusion is confirmed by the relatively moderate reduction in average standard deviation for the coefficients, when it is taken about the trend value instead of about the arithmetic average.

From the data for table 2 we obtain the figures for table 17.

TABLE 17

Average standard deviations about trend as percents of average standard deviation about the mean*

	Average size of coefficients					
	0.02 and less	0.02– 0.05	0.05– 0.10	0.10– 0.25	0.25– 0.50	Over 0.50
Norwegian competitive	78	68	69	74	82	(65)
Norwegian non-competitive	75	70	70	80	(104)	(103)
Imports, competitive	72	75	70	73	64	(57)
Imports, non-competitive	80	70	(101)	(48)	(71)	(95)
All specified inputs	72	71	70	73	76	70
Competitive inputs, combined	75	67	66	64	64	72
Fuels, combined	85	62	(59)	(86)	–	(94)
Substitution groups	(96)	(43)	(53)	70	55	83
Import sums	92	84	81	74	80	(79)
Small unspecified	82	63	80	(77)	–	–
Gross value added	–	–	(94)	80	82	69

* Here both the standard deviation about the trend and the standard deviation about the mean have been adjusted for degrees of freedom. Figures based on five observations or less have been put in parentheses.

5. *Stability of coefficients in detailed and aggregated tables*

We have previously demonstrated the plausibility of having the variance of the coefficients decrease with progressive aggregation in the data. Comparing the standard deviations in tables 2 and 18 makes it possible to confront this hypothesis with the data. We find that, as we move from the 79 sector table to the 14 sector table, there is a drastic reduction in the standard deviations about the average coefficient, and for most of the coefficient classes there are further reductions as we move on from 14 to 5 sectors. Since all tables are derived from the same set of data, the stability in the aggregate tables must be due to substantial stability in the shares of individual detailed sectors' contributions to the aggregate sectors. Also since most coefficients are quite small, each detailed sector is normally a small fraction of each aggregate sector.

TABLE 18

Standard deviations about average coefficients for main categories of inputs, classified by average size of coefficients for 14 and 5 sector aggregations

	Average size of coefficient					
	0.02 and less		0.02-0.05		0.05-0.10	
	Num-ber	Standard deviation	Num-ber	Standard deviation	Num-ber	Standard deviation
A. 14 sector aggregation						
Norwegian competitive	12	0.003	10	0.005	1	0.003
Norwegian non-competitive	7	0.002	9	0.004	3	0.004
Imports, competitive	2	0.002	12	0.006	5	0.010
Imports, non-competitive	-	-	1	0.008	-	-
All specified inputs	21	0.002	32	0.005	9	0.007
Import sums	2	0.003	-	-	2	0.008
Gross value added	-	-	-	-	-	-
B. 5 sector aggregation						
Norwegian competitive	2	0.002	2	0.004	3	0.004
Norwegian non-competitive	1	0.002	3	0.002	-	-
Imports, competitive	-	-	4	0.003	1	0.003
Imports, non-competitive	-	-	-	-	-	-
All specified inputs	3	0.002	9	0.003	4	0.004
Import sums	-	-	1	0.003	-	-
Gross value added	-	-	-	-	-	-
	0.10-0.25		0.25-0.50		Greater than 0.50	
A. 14 sector aggregation						
Norwegian competitive	5	0.014	-	-	-	-
Norwegian non-competitive	7	0.009	-	-	-	-
Imports, competitive	3	0.014	1	0.031	-	-
Imports, non-competitive	-	-	1	0.014	-	-
All specified inputs	15	0.012	2	0.022	-	-
Import sums	6	0.013	3	0.018	-	-
Gross value added	-	-	5	0.014	9	0.016
B. 5 sector aggregation						
Norwegian competitive	2	0.005	1	0.006	-	-
Norwegian non-competitive	-	-	-	-	-	-
Imports, competitive	3	0.012	-	-	-	-
Imports, non-competitive	-	-	1	0.014	-	-
All specified inputs	5	0.009	2	0.010	-	-
Import sums	3	0.013	1	0.014	-	-
Gross value added	-	-	2	0.011	3	0.015

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