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STATISTISK SENTRALBYRÅ

83/9 19. april 1983

A SIMPLE DYNAMIC MODEL USING THE SHORT RUN G.L. FUNCTION

by

Petter Frenger and Jon Rinde

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Introduction

The purpose of this paper is to apply the short run G.L. function in a simple numerical model. The short run G.L. function has been derived and analyzed in Frenger (1982). We are also interested in the possible applicaton of the results to the MSG model, and we therefore start in section 1 by describing the effects of introducing explicitly a scale parameter and technological change in the derivation of the long run and the short run cost function. Section 2 then presents the short run G.L. function, while section 3 presents å simple short run dynamic model. In section 4 we present the dynamic model using the short run G.L. function, and section 5 then presents some numerical results using data from the MSG sector "production of metals".

1. Long and short run cost functions

This section is based on Frenger (1982). The primary purpose here is to show how the explicit introduction into the production function of non-constant returns to scale and technological change affects the expressions for the dual long run and short run cost functions.

The technology of a typical production sector in MSG may be written

$$\sum_{j=1}^{m} x_j^+ = e^{\varepsilon t} [f(x_1,\ldots,x_n)]^{\mu} \quad , \qquad (1.1)$$

where $x^+ = (x_1^+,\ldots,x_m^+)$ and $x = (x_1,\ldots,x_n)$ represent the output and the input vectors repectively. The function f is assumed to be linearly homogeneous, ε is the coeffisient for (Hicks neutral) technological change, t is a parameter representing time, and μ is the return to scale parameter. The outputs are assumed to be produced in fixed proportions, i.e.

$$x_j^+ = a_j^+ y \quad , \qquad j=1,\ldots,m, \tag{1.2}$$

where $a^+ = (a_1^+,\ldots,a_m^+)$ is the vector of constant output coefficients and $y = \Sigma x_j^+$ is the gross output of the sector.

Define the transformed gross output

$$\tilde{y} = (e^{-\varepsilon t} y)^{\frac{1}{\mu}} \quad . \tag{1.3}$$

Then (1.1) may be rewritten

$$\tilde{y} = f(x_1,\ldots,x_n) \quad .$$

Let p denote the exogenous factor prices, and let $\tilde{c}(p)$ represent the unit cost function which is dual to f. Then the cost function of the sector is[1)]

$$C(y,p) = \tilde{y}\,\tilde{c}(p) = (e^{-\varepsilon t}y)^{\frac{1}{\mu}}\,\tilde{c}(p) \quad , \tag{1.4}$$

while the factor demand is, by Shephard's lemma,[2)]

$$x_i(y,p) = (e^{-\varepsilon t}y)^{\frac{1}{\mu}}\,\tilde{a}_i(p) \; , \qquad i=1,\ldots,n, \tag{1.5}$$

where $\tilde{a}_i(p) = \partial\tilde{c}/\partial p_i$. The input coefficients are

$$a_i(y,p) = \frac{x_i(y,p)}{y} = e^{-\frac{\varepsilon}{\mu}t}\, y^{\frac{1-\mu}{\mu}}\,\tilde{a}_i(p) \tag{1.6}$$

$$= \frac{\tilde{y}}{y}\,\tilde{a}_i(p) \quad .$$

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1) This is a basic property of homothetic production functions.
2) Shephard's lemma states that the price derivatives of the cost function equals the factor demand functions.

Consider the cost function (1.4). Marginal cost is given by

$$\frac{\partial C}{\partial y} = \frac{1}{\mu} e^{-\frac{\varepsilon}{\mu}t} y^{\frac{1-\mu}{\mu}} \tilde{c}(p) = \frac{1}{\mu} \frac{\tilde{y}}{y} \tilde{c}(p) \quad . \tag{1.7}$$

Using the fact that $\tilde{c}(p)$ is linear homogeneous in prices, this may be rewritten

$$\frac{\partial C}{\partial y} = \frac{1}{\mu} \frac{\tilde{y}}{y} \sum_{i=1}^{n} p_i \tilde{a}_i(p)$$

$$= \frac{1}{\mu} \sum_{i=1}^{n} p_i a_i(y,p)^{3)} \quad . \tag{1.8}$$

In the short run some of the factors are fixed. We will partition the inputs and the input prices into two sets, i.e. $x = (x_A, x_B)$ and $p = (p_A, p_B)$ and assume that the factors represented by x_A are variable in the short run, while those represented by x_B are fixed. It follows from the skew-conjugacy between the long run and the short run cost function,[4] that the short run cost function $V(y,p_A,x_B)$ is defined by

$$V(y,p_A,x_B) = \tilde{V}(\tilde{y},p_A,x_B) \quad , \tag{1.9}$$

where $\tilde{V}(\tilde{y},p_A,x_B)$ is the skew-conjugate of the long run cost function $\tilde{y}\,\tilde{c}(p)$, i.e.

$$\tilde{V}(\tilde{y},p_A,x_B) = \sup_{p_B} \{\tilde{y}\,\tilde{c}(p_A,p_B) - \langle p_B,x_B \rangle\} \quad . \tag{1.10}$$

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3) This last expression coincides, except for the missing term for sectoral taxes, with the right hand side of eqation (2.1) in Longva, Lorentsen, and Olsen (1981).

4) See Frenger (1982), eq. (1.1).

2. The G.L. function

We will now assume that $f(x_1,\ldots,x_n)$ [see (1.1)] is described by a G.L technology, and thus that:

$$\tilde{c}(p) = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\,(p_i p_j)^{\frac{1}{2}} , \qquad b_{ij} = b_{ji} . \tag{2.1}$$

Assume that only the n'th factor is fixed. The short run cost function becomes[1)]

$$V(y,p_n,x_n) = \tilde{V}(\tilde{y},p_A,x_n) = \tilde{y}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \left(b_{ij} + \frac{b_{in}\,b_{jn}}{\frac{x_n}{\tilde{y}} - b_{nn}}\right)(p_i p_j)^{\frac{1}{2}}$$

$$= \tilde{y}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} d_{ij}\,(p_i p_j)^{\frac{1}{2}} , \tag{2.2}$$

where $\tilde{y}$ is given by (1.3) and

$$d_{ij} = b_{ij} + \frac{b_{in}\,b_{jn}}{\frac{x_n}{\tilde{y}} - b_{nn}} . \tag{2.3}$$

The demand for the variable factor is

$$x_i(y,p_A,x_n) = \frac{\partial V(y,p_A,x_n)}{\partial p_i} = \tilde{y}\sum_{j=1}^{n-1} d_{ij}\left(\frac{p_j}{p_i}\right)^{\frac{1}{2}} , \tag{2.4}$$

$$i=1,\ldots,n-1,$$

while the short run input coefficient for the variable factor is

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1) See Frenger (1982) for derivation.

$$a_i(y,p_A,x_n) = \frac{x_i(y,p_A,x_n)}{y} = \frac{\tilde{y}}{y}\sum_{j=1}^{n-1} d_{ij}\left(\frac{p_j}{p_i}\right)^{\frac{1}{2}} . \qquad (2.5)$$

$$i=1,\ldots,n-1$$

The long run demand for the fixed factor is

$$x_n^*(y,p_A,p_n) = \frac{\partial C(y,p_A,p_n)}{\partial p_n} = \tilde{y}\sum_{j=1}^{n} b_{nj}\left(\frac{p_j}{p_n}\right)^{\frac{1}{2}} . \qquad (2.6)$$

For later reference, it may be noted that (2.6) may be written

$$\frac{x_n^*}{\tilde{y}} - b_{nn} = \sum_{j=1}^{n-1} b_{nj}\left(\frac{p_j}{p_n}\right)^{\frac{1}{2}} . \qquad (2.7)$$

Thus x_n^* may be considered as an optimal or desired level for the fixed input.

The short run marginal cost is obtained by differentiating (2.2) with respect to y

$$\frac{\partial V}{\partial y} = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\left(d_{ij} + \tilde{y}\frac{\partial d_{ij}}{\partial \tilde{y}}\right)\frac{\partial \tilde{y}}{\partial y}(p_i p_j)^{\frac{1}{2}}$$

$$= \frac{1}{\mu}\frac{\tilde{y}}{y}\left[\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} d_{ij}(p_i p_j)^{\frac{1}{2}} + \frac{x_n}{\tilde{y}}\left(\frac{\sum_{j=1}^{n-1} b_{jn} p_j^{\frac{1}{2}}}{\frac{x_n}{\tilde{y}} - b_{nn}}\right)^2\right] . \qquad (2.8)$$

Using (2.7) this may be rewriten

$$\frac{\partial V}{\partial y} = \frac{1}{\mu}\frac{\tilde{y}}{y}\left[\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} d_{ij}(p_i p_j)^{\frac{1}{2}} + \frac{p_n x_n}{\tilde{y}}\left(\frac{\frac{x_n^*}{\tilde{y}} - b_{nn}}{\frac{x_n}{\tilde{y}} - b_{nn}}\right)^2\right] . \tag{2.9}$$

The long run function (1.4) is linear homogeneous in $\tilde{y}$. It follows that the short run function $\tilde{V}(\tilde{y},p_A,x_n)$ is linearly homogeneous in $\tilde{y}$ and x_n [2)]. Applying Euler's theorem gives

$$\tilde{V}(\tilde{y},p_A,x_n) = \frac{\partial\tilde{V}}{\partial\tilde{y}}\tilde{y} + \frac{\partial\tilde{V}}{\partial x_n}x_n .$$

Marginal cost may therefore be written

$$\frac{\partial V}{\partial y} = \frac{\partial\tilde{V}}{\partial\tilde{y}}\frac{\partial\tilde{y}}{\partial y} = \frac{1}{\mu}[V(1,p_A,\frac{x_n}{y}) - \frac{x_n}{y}\frac{\partial V}{\partial x_n}] . \tag{2.10}$$

The negative of $\partial V/\partial x_n$ is the shadow price p_S of the fixed factor. Using (2.7), its expression is given by

$$p_S = -\frac{\partial V}{\partial x_n} = -\tilde{y}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial d_{ij}}{\partial x_n}(p_i p_j)^{\frac{1}{2}}$$

$$= \left(\frac{\sum_{i=1}^{n-1} b_{in}p_i^{\frac{1}{2}}}{\frac{x_n}{\tilde{y}} - b_{nn}}\right)^2 = \left(\frac{\frac{x_n^*}{\tilde{y}} - b_{nn}}{\frac{x_n}{\tilde{y}} - b_{nn}}\right)^2 p_n . \tag{2.11}$$

The shadow price is the capital service price which would have to prevail for the existing capital stock to be optimal. We see from (2.11) that

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2) Note that the function $V(y,p_A,x_n)$ is not linearly homogeneous in y and x_n.

if the capital stock is optimal, i.e $x_n = x_n^*$, then $p_S = p_n$.

Using the expression for the shadow price and (2.5), permits us to write the expression (2.9) for the marginal cost as:

$$\frac{\partial V}{\partial y} = \frac{1}{\mu}\left[\sum_{i=1}^{n-1} p_i a_i + p_S \frac{x_n}{y}\right] . \qquad (2.12)$$

Equation (2.10) implies that this expression is valid also when the technology is not described by a G.L. cost function.

3. A dynamic one-sector model

We will now present a simple dynamic model of an exporting sector which we will then use to experiment with the short run G.L. function presented in the previous section. The sector is assumed to be a price taker in all its input markets, and its capital stock is fixed in the short run. Its technology is represented by the short run cost function $V(y,p_A,K)$, where K is the level of the capital stock (we will use K instead of x_n in the rest of the paper). The producer sets his price q equal to marginal cost

$$q_t = V_y(y_t, p_{At}, K_t) , \qquad (3.1)$$

and sells all his output on the export market where he is faced with a demand function

$$y_t = h\left(\frac{q_t}{q_{Wt}}, R_t\right) , \qquad (3.2)$$

where q_W is the price of competing goods and R is an income variable. For given p_{At}, q_{Wt}, K_t, and R_t, (3.1) and (3.2) determine the endogenous variables q_t and y_t.

Over time, the capital stock is variable, and the sector adjusts the capital stock according to the rule

$$\dot{K}_t = \gamma\,(y_t a^*_{Kt} - K_t) , \qquad o < \gamma \leq 1 , \tag{3.3}$$

where $y_t a^*_{Kt}$ is the optimal level of capital stock. The optimal coefficient a^*_{Kt} is given by the long run unit factor demand

$$a^*_{Kt} = \frac{\partial}{\partial p_K} C(1,p_{At},p_{Kt}) . \tag{3.4}$$

We are here implicitly assuming that the long run technology is linearly homogeneous[1]. The investment theory implies static expectations about output and prices. Note that the specification of the technology enters the model via the short run cost function in (3.1) and the long run cost function in (3.4).

The short run model is represented by (3.1), (3.2) and (3.4), while its dynamic behavior is described by (3.3). The model is asymptotically stable about the equilibrium point as the following argument will show. Setting (3.4) into (3.3) and differentiating the resulting system totally with respect to the endogenous variables gives:

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1) This implies in particular that the scale paramcter μ of the previous section is assumed equal to one.

$$dq = V_{yy}\, dy + V_{yK}\, dK \;, \tag{3.5}$$

$$dy = \eta \frac{y}{q}\, dq \;, \tag{3.6}$$

$$\dot{dK} = \gamma\,(a_K^*\, dy - dK) \;, \tag{3.7}$$

where $\eta = [\partial y/\partial q/q_W)]/[y/(q/q_W)]$ is the price elasticity of demand. The marginal cost function V_y is homogeneous of degree zero in y and K (since C is linear homogeneous in y), i.e.

$$V_{yK} = -\frac{y}{K} V_{yy} = -\frac{V_{yy}}{a_K^*} \;, \tag{3.8}$$

Since $a_K^* = K/y$ at long run equlibrium. Using (3.8) and setting (3.5) into (3.6) we get

$$dy = -\frac{\eta \frac{y}{q} \frac{V_{yy}}{a_K^*}}{1 - \eta \frac{y}{q} V_{yy}}\, dK \;, \tag{3.9}$$

and setting this into (3.7) gives the effect of a small displacement from the long run equilibrium

$$\frac{\dot{dK}}{dK} = -\frac{\gamma}{1 - \eta \frac{y}{q} V_{yy}} < 0 \;. \tag{3.10}$$

The short run cost function is convex in y, and the own second derivative V_{yy} is therefore non-negative. The elasticity of demand η is non-positive. It follows that the expression for $\dot{dK}/dK$ is negative and that the model is asymptotically stable about the equilibrium point. The speed of adjustment about the equilibrium point will increase if γ increases, and it will decrease if $|\eta|$ or V_{yy} increase. Note that the limit of $d\dot{K}/dK$ is 0 as $\eta \to -\infty$.

Consider what happens when the elasticity of demand η is infinite, i.e. the sector is a price taker. The demand function (3.2) will not hold, but we will instead have the world price determining the domestic price, i.e. $q_t = q_{Wt}$. The short run output will then be determined by (3.1). But what happens to the stability of the model and to its long run behavior? Let us note that equality of long run and short run marginal cost, i.e. $V_y = C_y$ is a necessary condition for equilibrium. Since an equilibrium will only occur if $ya_K^* = K$, we may set this into the expression for short run marginal cost and obtain $V_y(y,p_A,ya_K^*(p_A,p_K)) = C_y$ [2).]

But the long run marginal cost is determined by the factor prices through the unit cost function, i.e. $C_y = c(p_A,p_K)$. We must now distinguish three cases depending on whether world price p_W exceeds, equals, or is less than long run unit cost of production.

i) $c(p_A,p_K) < q_W$: In the short run $V_y(y,p_A,K) = q_W$ and this determines y. But from $C_y < V_y$ it follows that $p_S > p_K$, i.e. $a_K > a_K^*$. The actual capital stock is larger than the desired capital stock and the sector will disinvest. But this will not improve the situation and the disinvestment will continue until $y = K = 0$.

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2) A consequence of the conjugacy correspondence between V and C.

ii) $c(p_A, p_K) = q_W$: In this case the long run equilibrium condition $V_y = C_y = q_W$ will hold also in the short run. Any capital stock will be an equilibrium capital stock.

iii) $c(p_A, p_K) > q_W$: This is the converse of i) and the sector will go on expanding indefinitely.

4. A dynamic G.L. model

Let us now assume that the technology is described by a G.L. function as presented in section 2, and that the scale parameter $\mu=1$ [see (1.1)] and that $\tilde{y} = e^{-\varepsilon t}y$ [see (1.3)]. The short run model of section 3 may be written

$$a_K = \frac{K_{-1}}{y}, \qquad (4.1)$$

$$d_{ij} = b_{ij} + \frac{b_{iK}b_{jK}}{e^{\varepsilon t}a_K - b_{KK}}, \qquad i,j=1,\dots,n-1, \quad \text{[see (2.4)]} \qquad (4.2)$$

$$a_i = e^{-\varepsilon t}\sum_{j=1}^{n-1} d_{ij}\left(\frac{p_j}{p_i}\right)^{\frac{1}{2}}, \qquad i=1,\dots,n-1, \quad \text{[see (2.5)]} \qquad (4.3)$$

$$a_K^* = e^{-\varepsilon t}\sum_{j=1}^{n} b_{Kj}\left(\frac{p_j}{p_K}\right)^{\frac{1}{2}}, \qquad \text{[see (2.6)]} \qquad (4.4)$$

$$p^S = \left(\frac{e^{\varepsilon t}a_K^* - b_{KK}}{e^{\varepsilon t}a_K - b_{KK}}\right)^2, \qquad \text{[see (2.11)]} \qquad (4.5)$$

$$q = \sum_{i=1}^{n-1} p_i a_i + p_S a_K , \qquad \text{[see (2.12)]} \tag{4.6}$$

$$y = \left(\frac{q}{q_W}\right)^{\eta} R \quad . \tag{4.7}$$

Equations (4.1) through (4.6) are essentially just a convenient way of writing the price equals marginal cost condition (3.1). This condition is formally given in (4.6), but (4.1) through (4.5) defines the endogenous variables in terms of which (4.6) is expressed. (4.1) defines the short run capital coefficient, and (4.2) defines the set of short run G.L coefficients (note that $d_{ij} = d_{ji}$). (4.3) and (4.4) gives the input coefficients for the short run variable inputs and the optimal capital stock respectively, and (4.5) defines the shadow price of capital.

It should be noted that we have also defined the optimal capital coefficient a_K^* in the process of defining the short run function [see also (3.4)]. Equation (4.7) is the demand function for exportables.

The dynamics of the model is provided by the equation

$$K = \gamma\, y\, a_K^* + (1-\gamma)\, K_{-1} , \tag{4.8}$$

which represents the change in the capital stock and is the descrete time equivalent of (3.3). K represents the capital stock at the end of the period.

The model will converge towards a long run equilibrium, i.e. a point at which $K^* = K$. This set of equilibrium points defines the long run model, which will have a recursive structure. The output price q will be determined by the cost side [see (1.4) and (2.1)]

$$q = \frac{\partial C(y,p)}{\partial y} = e^{-\varepsilon t} \Sigma\Sigma\, b_{ij}\, (p_i p_j)^{\frac{1}{2}} \quad , \tag{4.9}$$

the double summations being over all $i,j=1,\ldots,n-1,K$. The output is determined by the demand equation (4.7), while the capital stock is

$$K = y\, a_K^*(p) \quad . \tag{4.10}$$

5. Some numerical simulations

To illustrate the model, we have taken some data from MSG sector 43 "production of metals" with three variable inputs and fixed capital stock in the short run. The variable inputs are labor (L), intermediate inputs (M), and energy inputs (U). The elements of the long run G.L. coefficient matrix are

$b_{MM} = 0.272$			
$b_{UM} = 0.117$	$b_{UU} = -0.070$		
$b_{LM} = 0.852$	$b_{LU} = 0.226$	$b_{LL} = -2.106$	
$b_{KM} = 0.065$	$b_{KU} = -0.060$	$b_{KL} = 0.836$	$b_{KK} = 0.485$

The exogenous input prices are the same as those used in the base year of MSG, i.e.

$p_M = 1.000$ $\qquad$ $p_L = 0.050$

$p_U = 1.000$ $\qquad$ $p_K = 0.122$,

and are assumed constant.

The example illustrates the effect of using different values of the elasticity of demand η. We assume that, because of some natural disaster, the capital stock has been reduced by 1/3 from its historical 1979 value of 15 472 mill. kroner to $K_{1979} = K_0 =$ 10 315[1], and that the capital stock adjustment coefficient $\gamma = 0.25$. We then simulate the model with $\eta = 0.0$, -1.0, and -5.0. The results are presented in figures 2 to 5 below.

To illustrate what happens, it will be convenient to utilize the familiar demand and supply diagram of introductory textbooks. Figure 1 below illustrates, for each of the three alternative demand functions, the situation immediately after the capital stock has been reduced to K_0. The horisontal line is the long run marginal cost $C_y = q^*$. After the reduction in capital stock output larger than y_0 must be produced at a cost larger than q^*. This is shown by the increasing marginal cost function $V_y(K_0)$. The intersection between the marginal cost and the demand functions determines the output volume and price. We see from figure 1 that

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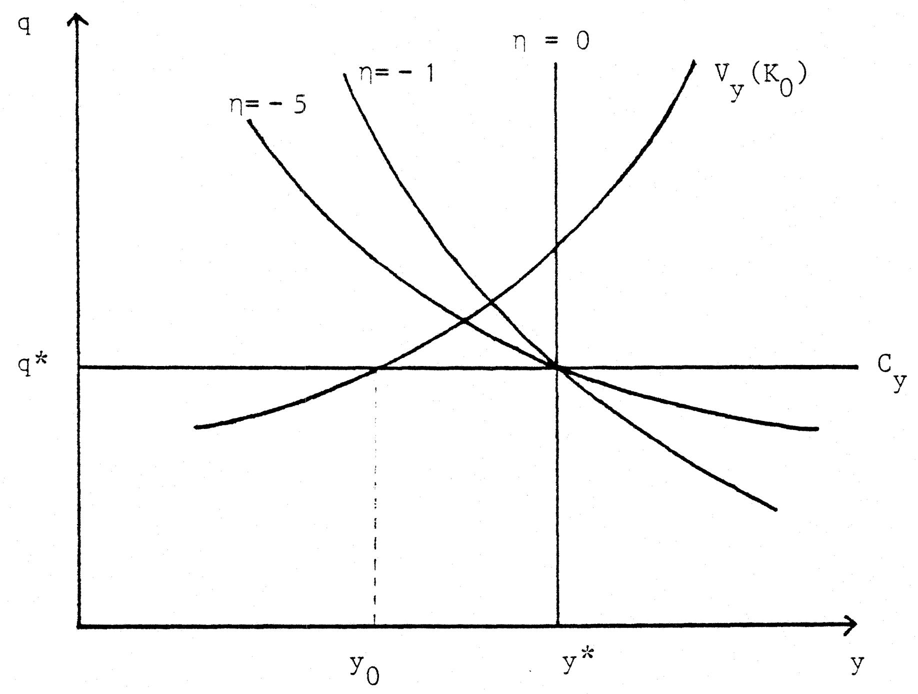
1) It follows from (2.3) or (4.5) that K_t/y_t must be larger than $b_{KK} = 0.485$, since the short run cost function is not defined otherwise. With y exogenous, i.e. $\eta = 0$, this implies that the starting value of K must be larger than 7 275 mill kroner. But the condition will always be satisfied if $\eta < 0$ since the output price and the output itself will then adjust.

Figure 1: Demand and supply curves



the output price is higher the smaller $|\eta|$ is, since the output price must assume more of the burden of adjustment. This is also seen from figure 2 and 3 where the output (y) and the output price (q) is presented. With $\eta = 0$, y is exogenous and kept constant at the equilibrium level (15 000 mill. kroner), as illustrated by the vertical demand function in figure 1. Eventually all three alternatives will reach the equilibrium point. The adjustment process means that the short run marginal cost function shifts downwards as the capital stock increases. Figure 4 shows how the capital stock behaves, while figure 5 shows how the ratio of the shadow price to the (exogenous) capital price converges to the long run value of unity.

It was pointed out in section 3 that the speed of adjustment of K about the equilibrium point will decrease as $|\eta|$ increases. This is brought out by figure 4, where the curve for $\eta = 0$ approaches the equilibrium level much

Figure 2: Output (y), 1000 million 1978 kroner.

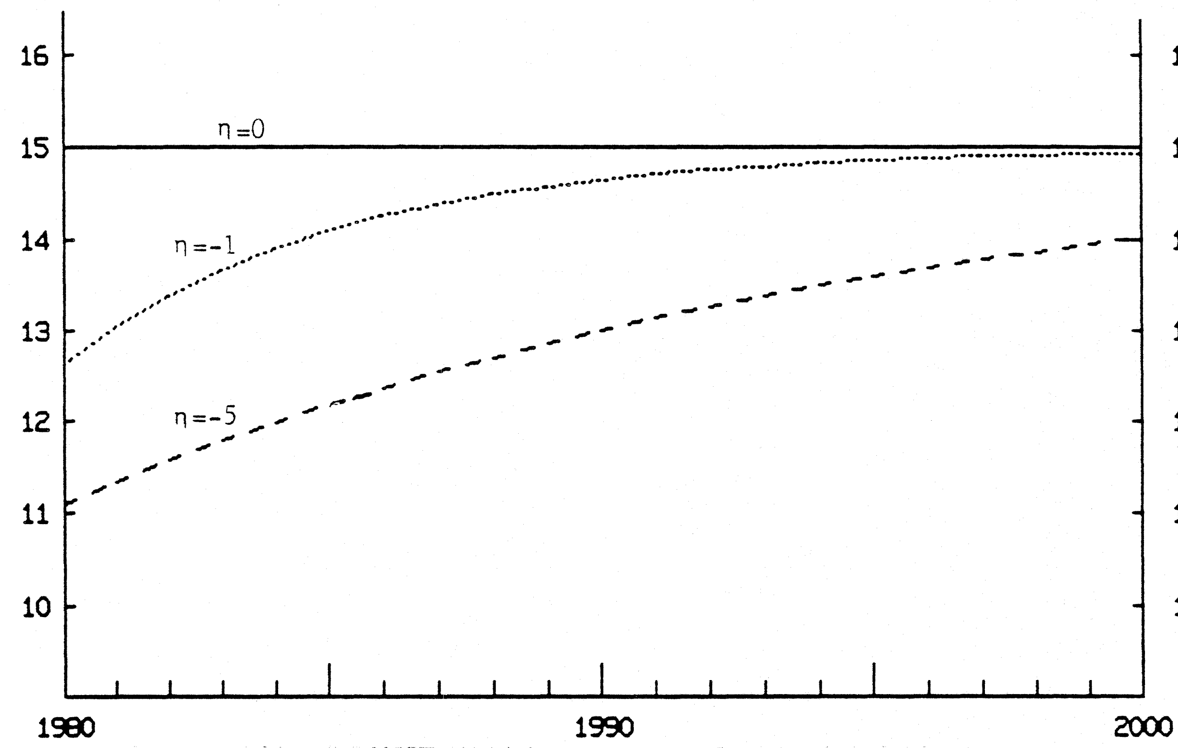


Figure 3: Output price (q).

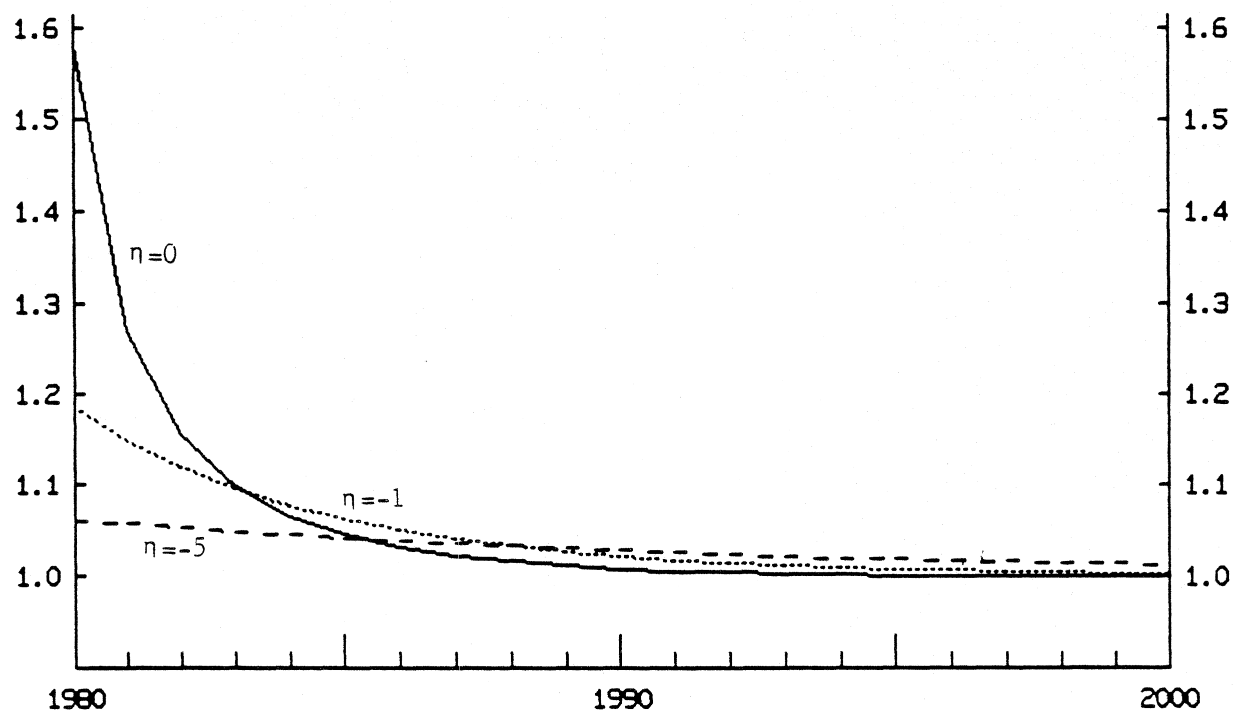
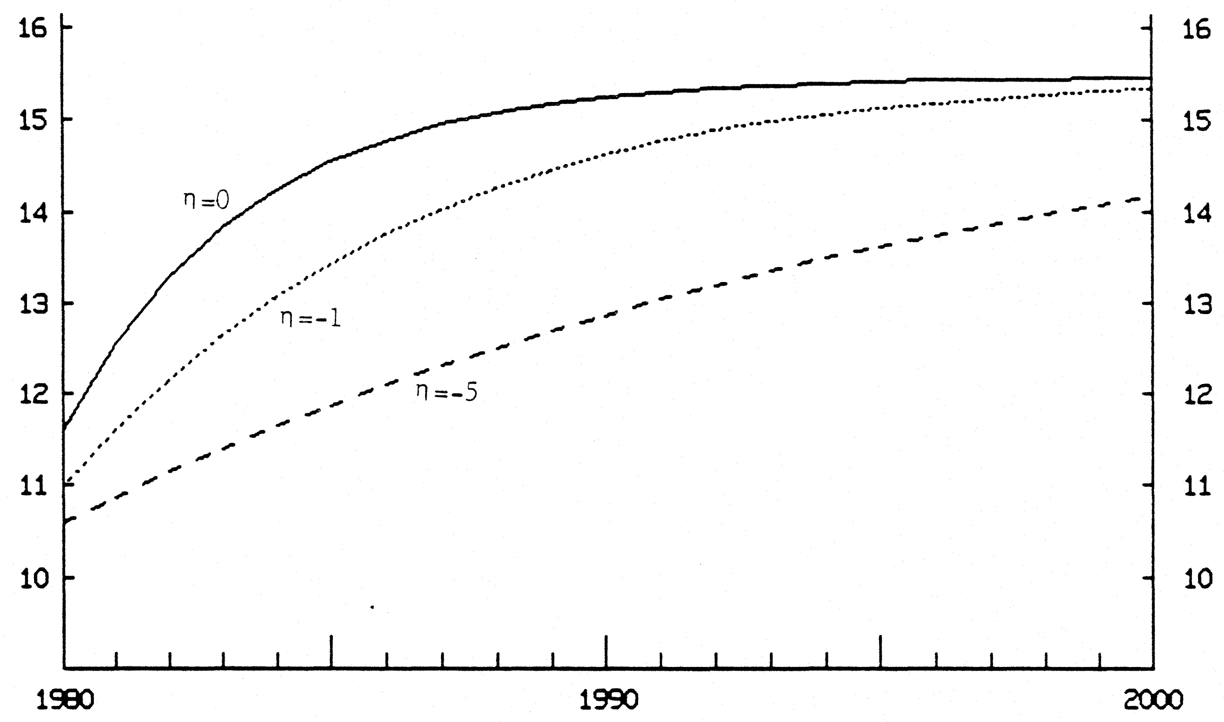


Figure 4: Capital stock (K), 1000 million 1978 kroner.



Figur 5: Ratio of shadow price to actual price of capital (p_K/p_S).

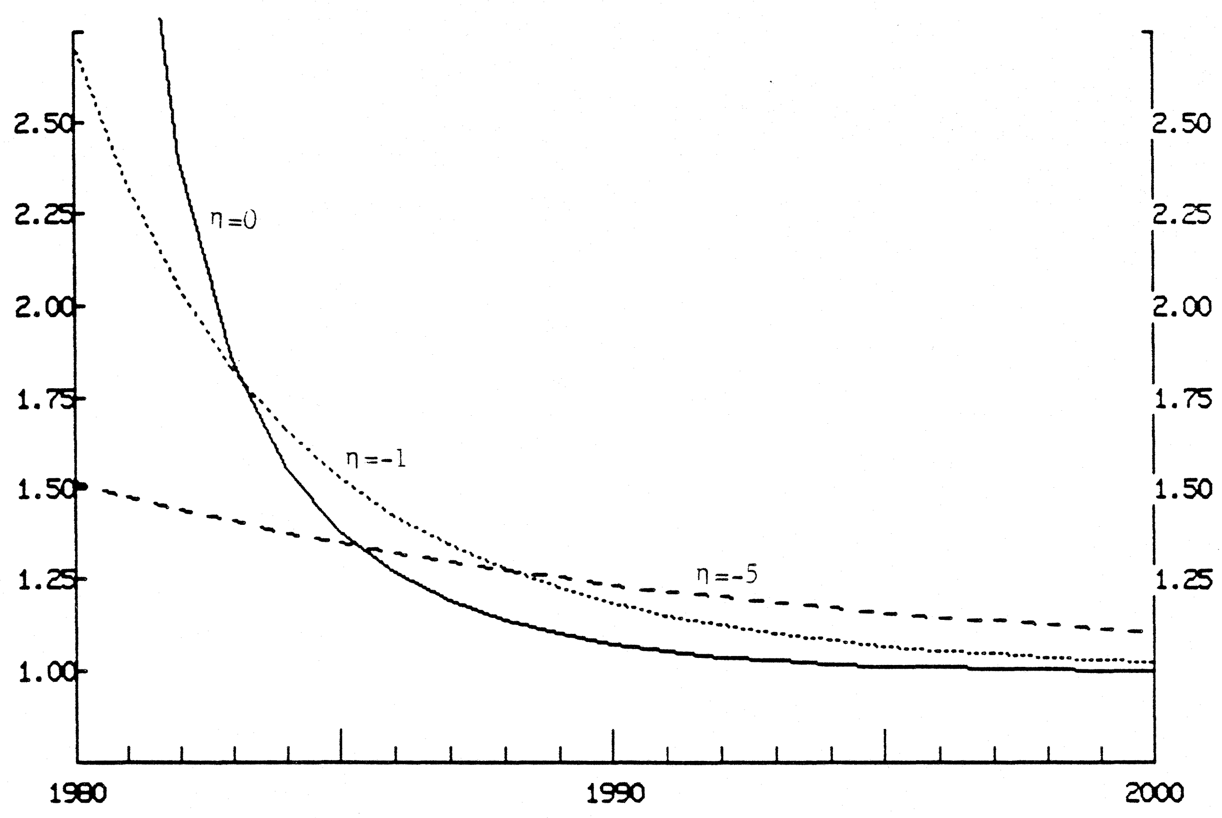
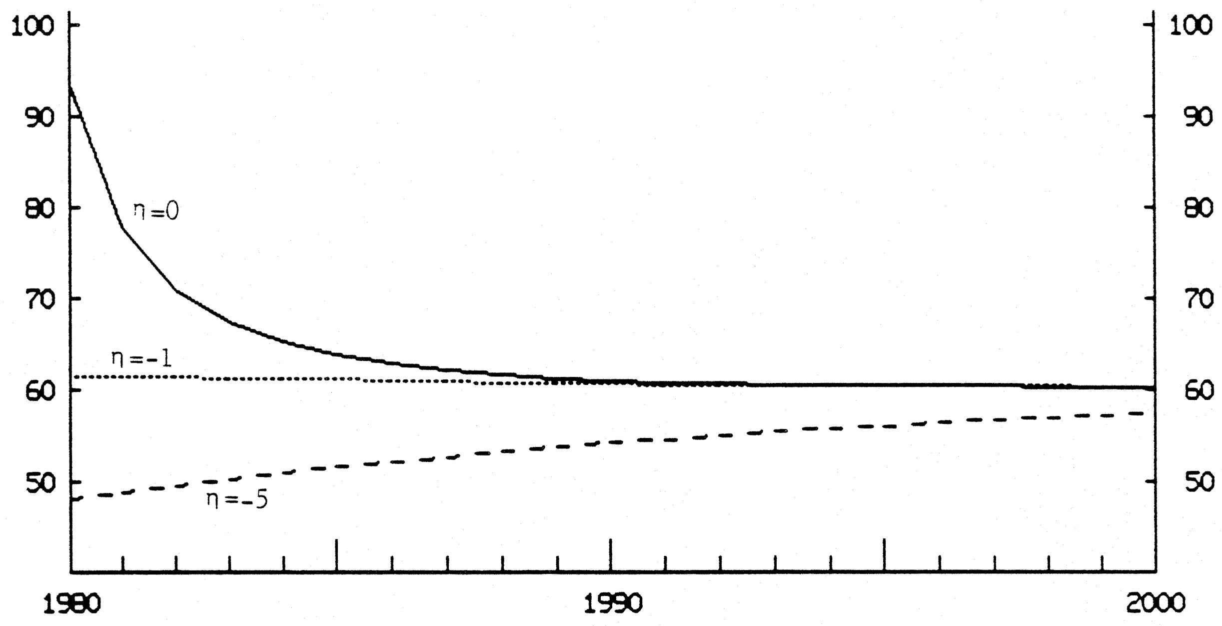


Figure 6: Labor input (L), 100 million man hours.



faster than the other two curves. The same pattern will be recognised also in figures 2,3 and 5. A smaller $|\eta|$ means that actual output is closer to equilibrium output, and therefore desired capital stock is closer to the equilibrium capital stock. Therefore "better" signals to the investment decisions are given. This is shown in figure 4. Faster adjustment of the capital stock also means faster downward shift of the short run cost function and therefore faster adjustment of the output price. As a consequence the output price paths in the three alternatives cross each other in figure 3. The same explanation may be given in figure 5 where the ratio of shadow price to actual price of capital is presented.

Figure 6 shows how the demand for one of the variable inputs, in this case labor, changes. In the beginning a large quantity of labor input is needed per init of output to compensate for the loss of the capital stock. As the capital stock is rebuilt, the input coefficient for labor (and other variable factors) is reduced until it reaches its long run equilibrium level. But the demand for labor is determined both by the input coefficient and the output level. When $\eta = 0$ and the output is constant we obtain temporarily a large increase in the demand for labor. A smaller increase is also obtained when $\eta = -1$, but when $\eta = -5$, the drop in output is so large that it results temporarily in a decreased demand for labor.

Appendix: Printout of model

This appendix reproduces the equations of the model as they were implemented on the TROLL system. The following list establishes the correspondence between the TROLL equations and equations (4.1) - (4.7) in chapter 4.

TROLL	chapter 4
1	4.1
3 - 8	4.2
9 - 11	4.3
12	4.4
14	4.5
15	4.6
16	4.7
2, 13, 17, 18, 19	auxiliary variables

Table A: Printout of TROLL model.

```
1:      AK43 = K43(-1)/Y43

2:      N43 = EPS43*AK43-CKK43

3:      DLL43 = CLL43+CLK43*CLK43/N43

4:      DLM43 = CLM43+CLK43*CMK43/N43

5:      DLU43 = CLU43+CLK43*CUK43/N43

6:      DMM43 = CMM43+CMK43*CMK43/N43

7:      DMU43 = CMU43+CMK43*CUK43/N43

8:      DUU43 = CUU43+CUK43*CUK43/N43

9:      AL43 = 1/EPS43*(DLL43+DLM43*(PM43/PL43)**0.5+DLU43*(PU43/PL43)**
        0.5)

10:     AM43 = 1/EPS43*(DMM43+DLM43*(PL43/PM43)**0.5+DMU43*(PU43/PM43)**
        0.5)

11:     AU43 = 1/EPS43*(DUU43+DLU43*(PL43/PU43)**0.5+DMU43*(PM43/PU43)**
        0.5)

12:     AKD43 = 1/EPS43*(CKK43+CKL43*(PL43/PK43)**0.5+CKM43*(PM43/PK43)**
        0.5+CKU43*(PU43/PK43)**0.5)

13:     ND43 = EPS43*AKD43-CKK43

14:     PS43 = PK43*(ND43/N43)**2

15:     Q43 = (PL43*AL43+PM43*AM43+PU43*AU43+PS43*AK43)/W

16:     Y43 = R43*(Q43/QW)**ETA43

17:     K43 = GAM43*Y43*AKD43+(1-GAM43)*K43(-1)

18:     PKS43 = PS43/PK43

19:     L43 = AL43*Y43
```

The majority of the variables are designated by the same symbols as in the text, except that we have used C to designate the coefficients of the (long run) G.L. function instead of b used in the text. The number 43 represents the code of the sector "production of metals".

Equations 2, 13, and 17 define auxiliary variables which facilitate the writing of the other equations, equation 18 computes ratio of the shadow price to the actual price of capital which is presented in fig. 5, and equation 19 computes the demand for labor which is presented in figure 6. The variable EPS represents the term $e^{\varepsilon t}$ and has been introduced as an exogenous variable. The variable W in equation 15 is a nomalization constant which insures that the equilibrium value for q is one.

References:

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