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**IMPORT SHARE FUNCTIONS
IN INPUT-OUTPUT ANALYSIS**

IMPORTANDELSFUNKSJONER I KRYSSLØPSMODELLER

BY
PETTER FRENGER

**STATISTISK SENTRALBYRÅ
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PREFACE

In the Norwegian planning models developed by the Central Bureau of Statistics the bulk of imports is determined by means of an import share matrix of constant coefficients with exogenous adjustments. The model presented in this paper generalizes that approach by making each element of the import share matrix a function of the relative prices of imports and the competing domestic products. This makes it possible to take explicitly into consideration the effect that changing prices of domestically produced and imported inputs have on the import shares and on the volume of imports. We also estimate the import price elasticities of selected sectors and commodities using data from the national accounts for the years 1949 - 1969.

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Petter Jakob Bjerve

FØRØRD

De norske planleggingsmodellene MODIS og MSG bruker en importandelsmatrise for å bestemme importnivået. Elementene i denne matrisen er, med unntak av mulige eksogene endringer, antatt konstante. Modellen som presenteres i denne rapporten generaliserer denne framgangsmåten ved at hvert element i importandelsmatrisen er en funksjon av forholdet mellom importprisen og prisen på konkurrerende hjemme produserte varer. Dette gjør det mulig å analysere virkningen av det endrede relative prisforholdet på importandeler og på importvolumet. Vi har også estimert importpris-elasticiteten for utvalgte varer og sektorer på grunnlag av nasjonalregnskapet for 1949 - 1969.

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Petter Jakob Bjerve

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1. INTRODUCTION

In this paper we develop a model for the demand for imports for Norway by explicitly representing the demand for imports of the individual sectors of the economy for each commodity.¹⁾ The demand for imports by the country as a whole will then be the sum of the demand of the individual sectors. The present analysis is limited to the production sectors, but the approach is also meant to be valid for the final demand sectors. And obviously, all sectors must be included for a national demand function to result. A few sectoral import demand functions are estimated in the latter part of the paper.

In econometric models, the usual procedure for determining x_i^B , the import of commodity i , is to relate it to the macroeconomic activity y and/or the total demand x_i of the i 'th commodity, to the relative price p_i^A/p_i^B , where p_i^A and p_i^B are the prices of domestically produced and imported commodity i respectively, and possible other variables z .²⁾ This gives a set of commodity import demand functions of the form:

$$x_i^B = f^i(y, x_i, p_i^A/p_i^B, z). \quad i = 1, \dots, n \quad (1.1)$$

The reader should be advised that the functional representations in this introduction is only intended as a concise representation of the verbal argument; no rigorous analysis is intended and all necessary symbols will be reintroduced in the main part of the paper. The approach (1.1) ignores the fact that it is the level of activity y_k in each sector k rather than some macroeconomic measure of economic activity which determines the import x_{ik}^B of commodity i to sector k . Relative prices may also differ between sectors, as may the set of "other" factors and the parameters of the demand functions. A sector's demand for the import of commodity i may be written:

$$x_{ik}^B = f^{ik}(y_k, x_{ik}, p_{ik}^A/p_{ik}^B, z_k), \quad (1.2)$$

while total import is the sum of the import to the individual sectors:

$$x_i^B = \sum_k x_{ik}^B. \quad (1.3)$$

Input-output analysis has utilized both of these approaches, either by specifying an import share vector m^B thereby giving the import vector:

$$x^B = \hat{m}^B A y, \quad (1.4)$$

where A is the input-output matrix and y the vector of gross output, or alternatively by formulating an import share matrix M^B . In the latter case the import demand vector is given by:³⁾

$$x^B = (M^B \circ A) y \quad (1.5)$$

The latter approach is currently used in the Norwegian planning models MODIS AND MSG.⁴⁾ But neither (1.4) nor (1.5) allows for substitution between domestically produced and imported inputs, for example as a consequence of changing relative prices as outlined in (1.1) and (1.2) above.

- 1) A related paper was presented at the Sixteenth General Conference of the International Association for Research in Income and Wealth in August 1979. The paper, entitled "Relative Prices and Import Substitution, Sectoral Analysis on Norwegian Data for the Period 1949-1969" gives greater attention to data construction and aggregation, and presents further empirical results. It will be published in the Review of Income and Wealth.
- 2) Commonly used other variables are capacity utilization, cyclical factors, and the ubiquitous time trend.
- 3) The \circ represents an element by element multiplication of the two matrices.
- 4) See Bjerkholt and Longva (1975, 1979), and Lorentsen and Skoglund (1976), respectively.

Using data from the Norwegian national accounts for the period 1949 - 1969 and an aggregation level of about 30 commodities and 30 production sectors we will estimate the price response in (1.2) for the production sectors of the Norwegian economy. We will assume a priori functional separability of the production structure and then estimate the import ratio functions:

$$\frac{x_{ik}^A}{x_{ik}^B} = \gamma_{ik} \left(\frac{p_{ik}^A}{p_{ik}^B} \right) \quad (1.6)$$

The explicit introduction of price variables in (1.2), estimated using (1.6) generalizes (1.5) in the same way as the usual treatment of import represented by (1.1) generalizes (1.4).¹⁾ Section 2 presents a general production model for the economy, and introduces separability to allow estimation of the import share functions independently of other assumptions about the rest of the production structure. We then choose, in section 3, a functional form (CES) for the import share functions, and specify their dynamic and stochastic formulation. In section 4 we briefly explain the data: the national accounts covering the period 1949 - 1969. Section 5 presents single equation estimates and various tests of our specifications, while we, in section 6, take explicit account of the correlation of the residuals and estimate a multivariate model which also makes it possible to test assumptions about price responsiveness across sectors. A brief summary of our conclusions is given in section 7.

5) It does not seem meaningful to combine information on the detailed import share matrix M^B with demand functions like (1.1). The use of an import share matrix necessitates introducing substitution into each element of the matrix, though one may still assume some of the elements to be constant.

2. IMPORT SHARE FUNCTIONS IN A SEPARABLE TECHNOLOGY

We will start with a general description of the production function and the behavior of the producer in an arbitrary sector k , and then derive the demand for imports as factor demand equations.¹⁾ Let x_{ik} be the input of commodity i in sector k . Sector k can purchase commodity i either on the domestic or on the foreign market: x_{ik}^A and x_{ik}^B are the quantities of commodity i which sector k buys on the domestic and on the foreign market respectively. We define x_{ik}^A and x_{ik}^B as two different commodities, even though they will have the same name in the national accounts.²⁾ Let p_i^A be the price of domestically produced commodity i and let p_i^B be the price of imported commodity i .³⁾ The fact that these price indices are different is taken as evidence for the fact that the respective commodities are different. This may in part simply be due to the fact that they are differently weighted averages of the same commodities, though the difference is in most cases more substantial.

We will assume that x_{ik}^A and x_{ik}^B are generally close substitutes, and describe the relationship between them and x_{ik} , the total input of the i 'th commodity into sector k , by the function:

$$x_{ik} = f^{ik}(x_{ik}^A, x_{ik}^B) \quad (2.1)$$

where the inputs x_{ik}^A and x_{ik}^B "produce" x_{ik} . The production function f^{ik} will be assumed to be linearly homogeneous. In the national accounts (2.1) is taken to be linear, reflecting an implicit assumption of free substitutability, and x_{ik} is defined as the Laspeyres aggregate of x_{ik}^A and x_{ik}^B .⁴⁾ In our model, x_{ik} is also defined by (2.1), but the function f^{ik} is unknown, and x_{ik} will generally be unobservable.

Assume that sector k needs a quantity x_{ik} of commodity i , and that the prices p_i^A and p_i^B are given. The producer will then try to minimize the cost of producing x_{ik} , and this minimum cost can be expressed as a function of x_{ik} , p_i^A , and p_i^B :

$$C^{ik}(x_{ik}; p_i^A, p_i^B) = \min_{x_{ik}^A, x_{ik}^B} \{ p_i^A x_{ik}^A + p_i^B x_{ik}^B \mid x_{ik} \geq f^{ik}(x_{ik}^A, x_{ik}^B) \} \quad (2.2)$$

Let us define:

$$r_{ik} = C^{ik}(1, p_i^A, p_i^B) = c^{ik}(p_i^A, p_i^B). \quad (2.3)$$

This is the minimum cost associated with the production of one unit of x_{ik} , and may be interpreted as the price of x_{ik} . It follows from the homogeneity of the production function that $C^{ik}(x_{ik}; p_i^A, p_i^B) = x_{ik} c^{ik}(p_i^A, p_i^B)$. The cost minimizing input of x_{ik}^A and x_{ik}^B as functions of "output" x_{ik} and prices are given by the partial derivatives of (2.2):⁵⁾

$$x_{ik}^A(x_{ik}; p_i^A, p_i^B) = x_{ik} \frac{\partial c^{ik}(p_i^A, p_i^B)}{\partial p_i^A} \quad (2.4)$$

$$x_{ik}^B(x_{ik}; p_i^A, p_i^B) = x_{ik} \frac{\partial c^{ik}(p_i^A, p_i^B)}{\partial p_i^B} \quad (2.5)$$

- 1) The approach outlined in this section can also be applied to the final demand sectors without any significant changes.
- 2) This approach has been extensively and fruitfully utilized by Armington (1969), and Artus and Rhomberg (1973).
- 3) The national accounts give at all but the most detailed level different price indices p_{ik}^A and p_{ik}^B for each receiving sector, and these will be used in the empirical work. See also section 4.
- 4) Or x_{ik} may be defined directly by deflating the related value flow.
- 5) This is Shephard's lemma [Shephard (1953)].

We have interpreted r_{ik} as the unit cost of producing x_{ik} , and the partial derivation of (2.3) w.r.t. p_i^B becomes the cost minimizing import per unit of x_{ik} , i.e. the import share:

$$m_{ik}^B(p_i^A/p_i^B) = \frac{\partial c^{ik}(p_i^A, p_i^B)}{\partial p_i^B}, \quad (2.6)$$

where we have chosen to define the import share as a function of the price ratio p_i^A/p_i^B . This we may do since the derivative of the cost function is homogeneous of degree zero in prices. The domestic share is defined analogously:

$$m_{ik}^A(p_i^A/p_i^B) = \frac{\partial c^{ik}(p_i^A, p_i^B)}{\partial p_i^A}. \quad (2.7)$$

And these shares will satisfy the identity $f^{ik}(m_{ik}^A, m_{ik}^B) = 1$.

Thus far we have taken x_{ik} for given, but x_{ik} is just a further input in the production process of sector k , and will be determined simultaneously by input and output prices, and by the level of activity in sector k .⁶⁾ We will assume that sector k has a separable production structure, and that the upper level function is defined implicitly by:

$$F^k(y_{1k}, \dots, y_{nk}; x_{1k}, \dots, x_{1k}, \dots, x_{nk}; z_{1k}, \dots, z_{mk}) = 0, \quad (2.8)$$

where $y_k = (y_{1k}, \dots, y_{nk})$ and $x_k = (x_{1k}, \dots, x_{nk})$ represent the vector of outputs and commodity inputs, respectively, and $z_k = (z_{1k}, \dots, z_{mk})$ is the vector representing the primary factors. The commodity inputs x_{ik} are given by (2.1) which represent the lower level of the production process.

The producer is assumed to have a given or desired level of activity A_k , and then to maximize profits subject to the condition that A_k be satisfied. A_k may represent the output of one or more commodities, the input of one or more primary factors, or it may represent the presence of limitational factors such as fixed capacity. Let $p^+ = (p_1^+, \dots, p_n^+)$ be the output prices and let $q = (q_1, \dots, q_m)$ be the cost of primary factors, and assume that they are all exogenous. The profit function, for given prices and level of activity, is given by:

$$\pi^k = \Pi^k(A_k; p^+, r_k, q) = \max_{y_k, x_k, z_k} \{ p^+ y_k - r_k x_k - q z_k \mid \text{for given } A_k \text{ and (2.8)} \}, \quad (2.9)$$

where $r_k = (r_{1k}, \dots, r_{nk})$ is given by (2.3).⁷⁾ The demand for the factor x_{ik} is given by the derivative of Π^k w.r.t. the unit cost r_{ik} :

$$x_{ik}(A_k; p^+, r_k, q) = \frac{\partial \Pi^k}{\partial r_{ik}}. \quad (2.10)$$

It is this expression for x_{ik} which enters (2.4) and (2.5) giving the demand for x_{ik}^A and x_{ik}^B as functions of prices and activity level only:

6) Perhaps subject to capacity restrictions or other factor limitations.

7) See McFadden (1978) for a detailed analysis of the properties of (restricted) profit functions.

$$x_{ik}^A = m_{ik}^A \left(\frac{p_i^A}{p_i^B} \right) x_{ik}(A_k; p^+, r_k, q), \quad (2.11)$$

$$x_{ik}^B = m_{ik}^B \left(\frac{p_i^A}{p_i^B} \right) x_{ik}(A_k; p^+, r_k, q). \quad (2.12)$$

This two stage derivation of the factor demand is possible only because of the assumption of homogeneous separability.⁸⁾ It implies a severe restriction on the form of the technology, but it greatly facilitates the empirical work. The main advantage to us is that it makes the ratio of domestic and imported inputs of commodity i to sector k a function of relative prices of the i 'th commodity only:

$$\frac{x_{ik}^A}{x_{ik}^B} = \frac{m_{ik}^A (p_i^A/p_i^B)}{m_{ik}^B (p_i^A/p_i^B)} = y_{ik} \left(\frac{p_i^A}{p_i^B} \right). \quad (2.13)$$

It is this function that we will estimate in section 5.

Example: In the planning models MODIS and MSG one assumes that the upper level production functions (2.8) are "Leontief", i.e. that the y_{ik} and the x_{ik} must be used in fixed proportions. There are only two primary inputs: capital K_k and labor L_k , and these form a separable input group which produces the value added $A_k = f^{n+1,k}(K_k, L_k)$. The input and output coefficients are normalized with respect to the value added:⁹⁾

$$a_{ik}^+ = \frac{y_{ik}}{A_k}, \quad a_{ik}^- = \frac{x_{ik}}{A_k}, \quad a_{n+1,k} = 1.$$

Let q_k and w_k be the price of capital services and labor in section k , and let $r_{n+1,k}(q_k, w_k)$ be the cost per unit of real value added [see (2.3)]. The profit function (2.9) reduces to:

$$\pi^k = A_k \left[\sum_i p_i^+ a_{ik}^+ - \sum_i r_{ik}(p_i^A, p_i^B) a_{ij}^- - r_{n+1,k}(q_k, w_k) \right],$$

while the import of commodity i to sector k is given by:

$$x_{ik}^B = - \frac{\partial \pi^k}{\partial p_i^B} = - \frac{\partial \pi^k}{\partial r_{ik}} \frac{\partial r_{ik}}{\partial p_i^B} = A_k a_{ik}^- m_{ik}^B \left(\frac{p_i^A}{p_i^B} \right).$$

A similar derivation gives the demand for x_{ik}^A , while the ratio between the two is still given by (2.13).

Total demand for import of the i 'th commodity is just the sum of the quantity demanded in the individual sectors:

$$x_i^B = \sum_k x_{ik}^B = \sum_k m_{ik}^B \left(\frac{p_i^A}{p_i^B} \right) x_{ik}(A_k; p^+, r_k, q), \quad (2.14)$$

8) Homogeneity refers to the assumption made about the category functions (2.1).

9) This normalization is characteristic of the two models mentioned, but in other respects the example represents a simplification of their production structure.

This import is seen to be a function of all prices and of the level of activity in each sector. All prices had to be assumed to be exogeneous when we derived the profit function (2.9). And even the simple example above required that p_i^A and p_i^B be exogeneous.

Formally therefore we must require that every sector is a price taker in all markets, i.e. that the elasticity of supply of factors is infinitely elastic, not only on the foreign market, but also on the domestic market. This assumption can only be defended as a first approximation, and is in some instances clearly untenable.

As mentioned above, the primary purpose of this analysis is to generalize the assumption that x_{ik}^A and x_{ik}^B must be used in fixed proportions. We will use the elasticity of substitution σ_{ik} as a measure of the degree to which the two factors are substitutes. This parameter describes completely the second order properties of the function f^{ik} , and is defined in terms of the unit cost function (2.3) as:

$$\sigma_{ik} \left(\frac{p_i^A}{p_i^B} \right) = \frac{c_{12}^{ik} c_1^{ik}}{c_1^{ik} c_2^{ik}} \quad (2.15)$$

The subscripts 1 and 2 represent differentiation w.r.t. the first and the second argument. The elasticity of substitution will be non negative and will in general be a function of relative prices.

Let us define the price elasticity of imports ϵ_{ik}^B as the elasticity of the import share m_{ik}^B w.r.t. the price ratio p_i^A/p_i^B :

$$\epsilon_{ik}^B = \frac{\frac{\partial m_{ik}^B}{\partial (p_i^A/p_i^B)} \frac{p_i^A/p_i^B}{m_{ik}^B}}{\frac{\partial (p_i^A/p_i^B)}{\partial (p_i^A/p_i^B)}} \quad (2.16)$$

It follows from the definition, with the domestic price p_i^A in the numerator, that $\epsilon_{ik}^B \geq 0$. The elasticity ϵ_{ik}^B will be a function of the second derivative of the cost function, and consequently a function of the elasticity of substitution. The first derivative of m_{ik}^B is:

$$\frac{\partial m_{ik}^B}{\partial (p_i^A/p_i^B)} = \frac{\partial}{\partial (p_i^A/p_i^B)} c_2^{ik} \left(\frac{p_i^A}{p_i^B}, 1 \right) = p_i^B c_{12}^{ik} (p_i^A, p_i^B) \quad (2.17)$$

The right hand equality is a consequence of c_2^{ik} being homogeneous of degree minus one in prices.

Define the value share for imports $S_{ik}^B = p_i^B x_{ik}^B / (p_i^A x_{ik}^A + p_i^B x_{ik}^B)$. Setting (2.15) and (2.17) into (2.16) gives the price elasticity of imports expressed as a function of the elasticity of substitution:

$$\epsilon_{ik}^B = \sigma_{ik} (1 - S_{ik}^B) \quad (2.18)$$

The price elasticity of total import of commodity i (2.14) will be a weighted average of the elasticities of the individual sectors, with the individual sectors' share of total import as weights:

$$\epsilon_i^B = \frac{\sum_k \frac{\partial x_{ik}^B}{\partial (p_i^A/p_i^B)} \frac{p_i^A/p_i^B}{x_{ik}^B}}{\sum_k \frac{x_{ik}^B}{x_i^B}} \left[\epsilon_{ik}^B + \frac{\partial x_{ik}^B}{\partial (p_i^A/p_i^B)} \frac{p_i^A/p_i^B}{x_{ik}^B} \right] \quad (2.19)$$

The second factor inside the square brackets will be zero if the upper level function is characterized by fixed coefficients.

3. THE MODEL

3.1. Functional form

In the previous section we have shown that as long as we assume that the upper level function F^k [see (2.8)] is homogeneously separable, then the import ratio function (2.13) is independent of the further specification of F^k . But the import ratio γ_{ik} and the import share m_{ik}^B will depend on our specification of the lower level function f^{ik} . We will assume that these functions can be adequately represented by a constant elasticity of substitution (CES) function:

$$x_{ik} = \left[\delta_{ik} \left(\frac{x_{ik}^B}{\delta_{ik}} \right)^{-\rho_{ik}} + (1-\delta_{ik}) \left(\frac{x_{ik}^A}{1-\delta_{ik}} \right)^{-\rho_{ik}} \right]^{-\frac{1}{\rho_{ik}}}, \quad (3.1)$$

where δ_{ik} is the distribution parameter and ρ_{ik} is a substitution parameter related to the elasticity of substitution by $\sigma_{ik} = 1/(1+\rho_{ik})$. CES is a flexible functional form which, with only two inputs, represents a second order approximation to an arbitrary homogeneous function. It also gives a particularly convenient form for the import ratio functions, as will be shown below.

The dual unit cost function (2.3) becomes:

$$r_{ik} = \left[\delta_{ik} (p_i^B)^{1-\sigma_{ik}} + (1-\delta_{ik}) (p_i^A)^{1-\sigma_{ik}} \right]^{\frac{1}{1-\sigma_{ik}}}. \quad (3.2)$$

The import share function and the domestic share function are obtained by taking the derivative of r_{ik} w.r.t. the prices [see (2.6) and (2.7)]:

$$m_{ik}^B \left(\frac{p_i^A}{p_i^B} \right) = \delta_{ik} (p_i^B)^{-\sigma_{ik}} \left[\delta_{ik} (p_i^B)^{1-\sigma_{ik}} + (1-\delta_{ik}) (p_i^A)^{1-\sigma_{ik}} \right]^{\frac{\sigma_{ik}}{1-\sigma_{ik}}}, \quad (3.3)$$

$$m_{ik}^A \left(\frac{p_i^A}{p_i^B} \right) = (1-\delta_{ik}) (p_i^A)^{-\sigma_{ik}} \left[\delta_{ik} (p_i^B)^{1-\sigma_{ik}} + (1-\delta_{ik}) (p_i^A)^{1-\sigma_{ik}} \right]^{\frac{\sigma_{ik}}{1-\sigma_{ik}}}.$$

The import ratio becomes simply:

$$\gamma_{ik} = \frac{m_{ik}^A}{m_{ik}^B} = \frac{1-\delta_{ik}}{\delta_{ik}} \left(\frac{p_i^A}{p_i^B} \right)^{-\sigma_{ik}}, \quad (3.4)$$

an expression which is log linear in the unknown parameters. Let $c_{ik} = \ln [(1-\delta_{ik})/\delta_{ik}]$. The ratio of domestic to imported deliveries can be written

$$\ln \frac{x_{ik}^A}{x_{ik}^B} = c_{ik} - \sigma_{ik} \ln \left(\frac{p_i^A}{p_i^B} \right). \quad (3.5)$$

One could of course have started directly with the functional form (3.5) and then proceeded to estimate it. The above derivation, however, sets the relationship in a broader context which facilitates its interpretation and points out more clearly its limitations.

3.2. Dynamic and stochastic specification¹⁾

The model presented above is static, and may be looked upon as a description of the long run equilibrium.

In the empirical work we will assume that effective prices, i.e. the prices determining the input ratio x^A/x^B , are formed by adaptive expectation, and that the current effective price²⁾ \bar{p}_t is a weighted average of current and past prices:

$$\bar{p}_t = \sum_{\tau=0}^L \alpha_{\tau} p_{t-\tau}, \quad \alpha_{\tau} = \frac{a_{\tau}}{\sum_{\tau=0}^L a_{\tau}} \quad (3.6)$$

The parameter L represents the longest lag, $p_t = \ln(p_t^A/p_t^B)$, and the a_{τ} parameters are explained below. The logarithm of the import ratio $x_t = \ln(x_t^A/x_t^B)$ can then be written [see (3.5)]:

$$x_t = c + \sum_{\tau=0}^L a_{\tau} p_{t-\tau} + \lambda t + u_t \quad (3.7)$$

where t represents a trend included to represent (non-neutral) technological change, changing commodity mix, etc., and where the error terms are assured to follow a first order autoregressive process

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad (3.8)$$

the ε_t being serially independently distributed. The coefficient a_0 represents the short run elasticity of substitution, while $\sum_{\tau} a_{\tau}$ becomes the long run elasticity σ . We will apriori choose a relatively simple lag structure a_{τ} , $\tau = 0, \dots, L$ of the form:

- 1) $L = 2$,
- 2) $a_2 = .5a_1$,

i.e. the longest lag is two periods and p_{t-2} has half the weight of p_{t-1} . The effective price can be written

$$\bar{p}_t = \frac{1}{a_0 + a_1} \left[a_0 p_t + a_1 \left(\frac{2}{3} p_{t-1} + \frac{1}{3} p_{t-2} \right) \right]. \quad (3.9)$$

This lag structure is economical with the use of parameters, and sufficiently flexible to allow for an increased effect of prices in the second year. Additionally it allows for the following interesting special cases:

- $a_0 = 0$ - no influence of current period prices,
- $a_1 = 0$ - no influence from past prices,
- $a_0 = a_1$ - linearly distributed lag.

1) This subsection treats only the flow of a single commodity i to a single sector k. The subscripts i and k will therefore be dropped.
2) The argument of part a) of this section now applies with respect to these effective prices.

Defining the variable

$$p_t^L = \frac{2}{3} p_{t-1} + \frac{1}{3} p_{t-2} \quad (3.10)$$

the model (3.7) becomes:

$$x_t = c + a_0 p_t + a_1 p_t^L + \lambda \cdot t + u_t \quad (3.11)$$

where the error terms are serially correlated. Equation (3.11) with $\lambda = 0$, is the basic model of this analysis. Combining (3.8) and (3.11) with $\lambda = 0$ gives the restricted transformed equation (RTE)³⁾:

$$x_t = k_0 + a_0 p_t + a_1 p_t^L - \rho a_0 p_{t-1} - \rho a_1 p_{t-1}^L + \rho x_{t-1} + \epsilon_t \quad (3.12)$$

Equation (3.12) is characterized by two nonlinear restrictions. Writing it in unrestricted transformed equation (UTE) form it becomes

$$x_t = k_0 + a_0 p_t + a_1 p_t^L + k_1 p_{t-1} + k_2 p_{t-1}^L + k_3 x_{t-1} + \epsilon_t \quad (3.13)$$

The restrictions:

$$k_1 = -k_3 a_0$$

$$k_2 = -k_3 a_1$$

become a test of the specification (3.12) with first order autoregressive error terms. Failure to accept (3.12) would indicate that our basic model is misspecified.⁴⁾

Figure 3.1 presents the various formulations being tested in this paper. This scheme is essentially that of Sargan (1964) and Hendry (1974), augmented to include various lag specifications on the prices. Eq. 1 is the basic model while eq. 3 represents the same model with uncorrelated error terms. The set of equations 11, 12, 13, 14 and 31, 32, 33, 34 represents various hypotheses about the lag structure of the prices. Eq. 0 is the UTE and provides us with a test of the basic model.

We have, in addition, included two alternative formulations, eq. 2 and eq. 21, which include the lagged endogenous variable. The presence of a significant lagged endogenous variable in these equations may indicate, in addition to serial correlation, the presence of a partial adjustment mechanism.⁵⁾ In addition, 6 of these equations, marked with a T in fig. 1, were also estimated with a time trend.

Some of the formulations in figure 3.1 are nonlinear in the parameters. We will therefore use the likelihood ratio to test the significance of the various formulations. Assume that the i 'th equation represents a parametric restriction on the coefficients of the j 'th equation. Let L_i be the value of the likelihood function of the i 'th equation, let S_j be the sum of squared residuals, and let k_j be the number of parameters estimated. On the assumption that the hypothesis embodied in the i 'th equation is true, then

$$\lambda = -2 \ln \frac{L_i}{L_j} = T \ln \frac{S_i}{S_j} \sim \chi^2(k_j - k_i), \quad (3.14)$$

i.e. λ will have an asymptotic chi-square distribution with $k_j - k_i$ degrees of freedom.

In the empirical section (sec. 5) we have started the analysis by testing the autoregressive formulation of eq. 1 against the UTE, represented by eq. 0. But regardless of whether the basic model (i.e. eq. 1) was rejected or not, we chose to proceed down the test tree, conditional upon the hypothesis of eq. 1. We always used a 5 per cent confidence level at each step, unless otherwise mentioned. If the procedure accepted two or more of the "parallel" hypothesis 11, 12, and 13,⁶⁾ or 31, 32, and 33, we chose among them on the basis of the lowest SSR (sum of squared residuals), which is equivalent to choosing the one with the highest value of the likelihood function. This procedure led unambiguously to a "best" formulation, conditional upon which no more restrictive hypothesis could be accepted. Only rarely did we consider the hypothesis embodied in eqs. 2 and 21, or consider the role of the time trend.

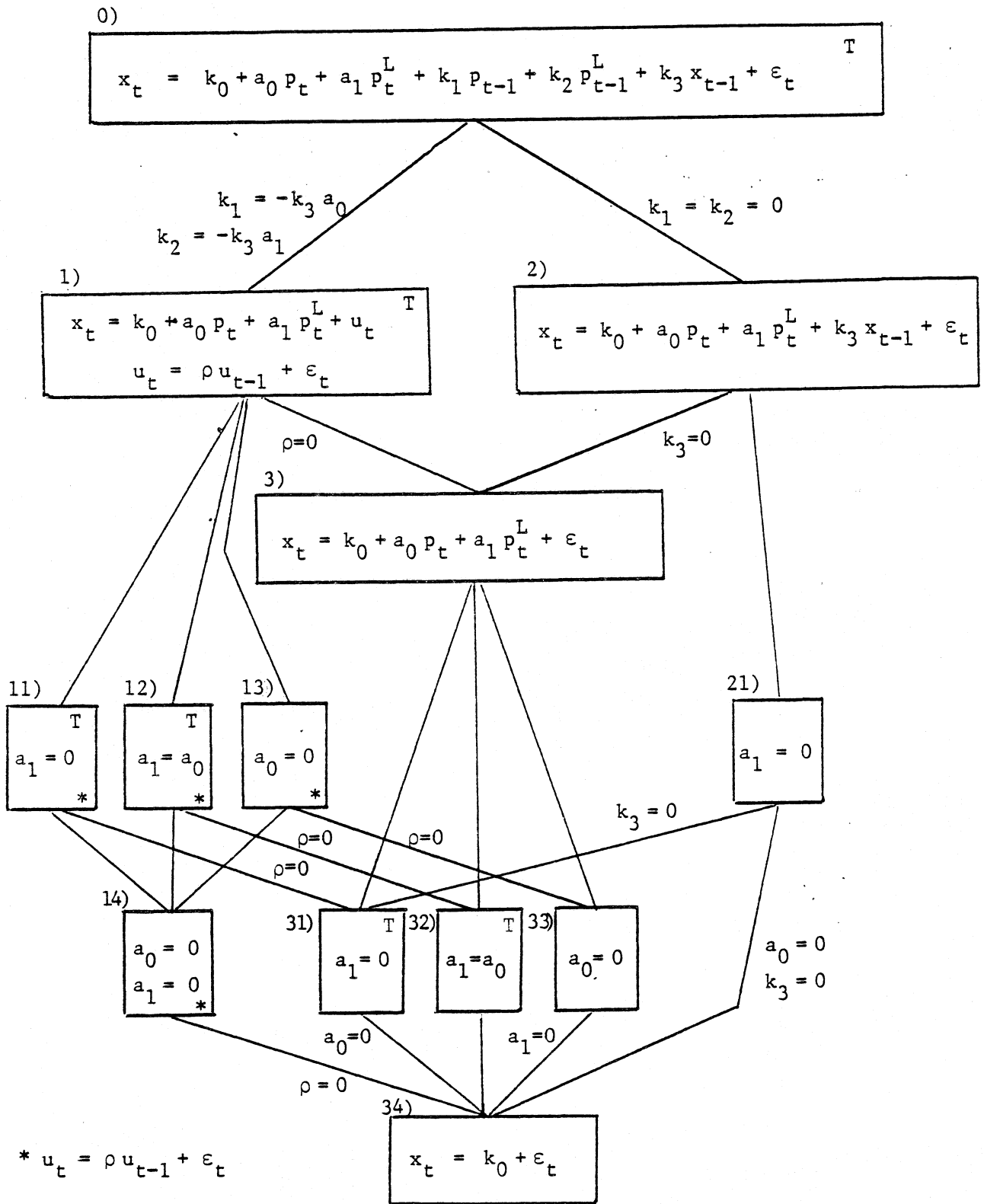
3) See Hendry (1974).

4) The presence of serial correlation in the UTE, as indicated by the DW or the Durbin h statistic, might indicate that even this model is misspecified.

5) We may later return to an analysis of such a model.

6) Eq. 3 should perhaps be considered "parallel" to 11, 12, and 13.

Fig. 3.1
Outline of estimated equations



4. THE DATA

The data for this analysis are all taken from the "old" national accounts, covering the years 1949 to 1969.¹⁾ These accounts were presented on a sector by sector basis, the commodities being classified according to their principal sector of production. The national accounts have been aggregated to 29 commodities, 29 production sectors and 15 final demand sectors.²⁾ This level of aggregation corresponds to that of the MSG model.

Table 4.1 presents a list of the commodities (and production sector classification) and some summary data for 1961. The first two columns give the numerical codes and the names of the commodities. The third column gives the value of total import, the fourth gives the value of "supply for domestic use" defined as Norwegian production (presented in column six) less exports plus imports. The fifth column gives the value shares for imports.

The national accounts are available in four value sets: producers' and purchasers' values measured in both current and constant (1961) prices.³⁾ We have chosen to measure the value of inputs in current purchasers' prices and the volume of input in constant producers' prices, interpreting the change in the volume of trade margins as price changes.⁴⁾

We have thus far limited the analysis to the study of price substitution in the production sectors.⁵⁾ Table 4.2 presents the 1961 input matrix for the production sectors, the typical element of the matrix being $p_{ik}^A x_{ik}^A + p_{ik}^B x_{ik}^B$, where we have taken explicit account of the fact that the price of the i 'th commodity (imported and domestically produced) differ among recipient sectors.⁶⁾

-
- 1) The conversion to new SNA (system of national accounts) in 1969 limits the length of the available time series.
 - 2) Four investment sectors, nine private consumption sectors, exports and inventory investment.
 - 3) The constant price data for 1949 to 1961 were measured in 1955 prices, while data for the period 1961 to 1969 were measured in 1961 prices.
 - 4) Given the prevalent use of fixed weights in computing trade margins, this distinction may be of little consequence.
 - 5) Excluding sector 34: public administration.
 - 6) In 1961 all the price indices are unity.

Table 4.1. Commodity flows, 1961. Mill. Nkr. (Purchasers' prices)

Commodity/sector		Commodity			Sector
Code	Name	Import	Supply for domestic use ¹⁾	Import share	Domestic production
01	Agriculture	1 327.0	5 989.9	0.221	4 825.6
02	Forestry	206.8	1 288.6	0.160	1 117.0
03	Fishing	12.2	872.0	0.013	1 119.6
04	Mining (incl. crude oil)	476.5	705.6	0.675	439.7
05	Food processing	459.7	5 825.5	0.079	6 430.2
06	Beverages, tobacco and chocolate ...	236.6	2 078.9	0.114	1 851.0
07	Textiles and wearing apparel	1 468.8	4 315.1	0.340	2 996.7
08	Wood and wood products	207.5	2 036.2	0.102	1 932.9
09	Paper and paper products	111.0	1 497.5	0.074	2 590.1
11	Chemicals (incl. petroleum refining)	2 017.3	3 819.9	0.528	2 502.0
12	Mineral products	237.7	989.7	0.240	803.5
13	Basic metals	1 367.5	2 372.8	0.576	2 674.0
15	Machinery	3 360.8	6 159.2	0.546	3 218.9
16	Electrical machinery and products ..	695.4	1 729.9	0.402	1 124.0
17	Building and repair of vessels	2 354.8	3 644.6	0.646	1 453.4
18	Other manufacturing (printing, rubber products, glass, etc.)	617.9	2 509.7	0.246	1 960.7
19	Electricity, gas, and water supply .	131.3	1 576.4	0.083	1 453.7
20	Construction	-	6 512.9	-	6 512.9
21	Trade	79.2	9 039.2	0.009	9 228.0
22	Restaurants and hotels	415.9	-	415.9
23	Real estate services	-	2 108.0	-	2 108.0
24	Finance and insurance	42.8	1 068.7	0.040	1 025.9
25	Communication	7.6	782.8	0.010	775.2
27	Domestic transports	2.0	2 599.7	0.001	2 944.2
28	Health services	-	926.4	-	926.4
29	Education and research	-	883.5	-	883.5
30	Other services	18.4	1 539.4	0.012	1 521.2
31	Shipping	-	44.0	-	6 313.0
34	Public administration	-	2 069.3	-	2 069.3

1) "Supply for domestic use" equals "domestic production" less export, pluss import.

Table 4.2. 1961 Commodity input matrix for the production sectors¹⁾

Commodity	Receiving sector														
	01	02	03	04	05	06	07	08	09	11	12	13	15	16	
01	1 386.0	1.5	0.	0.	2 117.7	94.8	126.3	1.1	0.	99.1	0.	0.	1.2	0.2	
02	0.	0.	0.	0.	1.6	0.	0.3	321.3	578.3	1.5	0.1	0.1	0.2	0.	
03	38.2	0.	17.8	0.	587.3	0.	8.3	0.	0.	1.7	0.	0.	0.	0.	
04	5.1	0.	0.3	0.9	7.5	0.2	0.3	0.3	16.2	290.5	49.2	209.9	0.1	0.4	
05	568.1	0.	20.5	0.	1 371.2	35.2	0.3	1.7	2.9	18.8	0.8	0.3	2.1	2.1	
06	1.5	0.	0.	0.	3.4	72.2	0.	0.	1.8	2.8	0.	0.	0.5	0.	
07	2.9	0.	4.4	0.	2.6	0.	764.1	32.4	13.7	2.8	0.6	0.5	14.1	2.3	
08	4.4	0.	2.7	0.8	18.9	0.3	2.3	345.4	40.6	5.3	5.2	11.9	19.6	29.5	
09	1.5	0.	0.	0.3	69.9	17.5	16.4	5.4	705.9	83.6	15.5	0.	12.6	7.3	
11	235.7	0.8	57.7	7.5	182.6	9.3	76.4	33.0	95.0	385.9	27.9	306.9	46.7	19.4	
12	3.3	0.	0.	1.4	10.8	6.3	2.0	7.1	6.0	11.6	63.4	12.6	7.9	9.3	
13	0.2	0.	0.	0.2	0.6	0.	1.2	10.9	8.8	9.9	13.1	669.5	492.7	104.3	
15	7.2	0.	2.7	17.3	66.7	9.4	22.4	52.5	20.2	27.5	4.8	16.1	393.5	30.9	
16	0.	0.	1.0	0.	0.	0.	0.1	0.	0.5	0.	0.2	2.8	40.0	171.6	
17	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.8	0.	52.9	36.2	2.1	
18	0.	0.	0.	6.1	11.2	8.2	20.3	10.5	20.0	25.4	4.7	13.2	24.9	15.7	
19	36.2	0.	0.	14.9	40.0	3.2	12.3	19.2	60.0	96.2	17.3	188.7	23.8	4.7	
20	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
21	1.0	0.	1.0	6.5	11.8	0.	0.	0.	16.2	1.0	0.	3.0	3.0	0.	
22	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
23	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
24	6.2	0.	12.1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
25	7.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
27	0.	0.	1.5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
28	11.6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
29	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
30	12.5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
31	0.	0.	8.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
34	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	

	Receiving sector													
	17	18	19	20	21	22	23	24	25	27	28	29	30	31
01	0.2	5.0	0.	0.	0.	0.	0.	0.	0.	0.8	0.	0.	0.	0.
02	2.9	1.9	0.	24.7	0.2	0.	0.	0.	0.	0.	0.	0.	0.	0.
03	0.	0.1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
04	0.7	0.5	6.8	39.1	0.	0.	0.	0.	0.	8.3	0.1	0.	0.	0.
05	1.9	0.6	0.	0.	14.0	0.	0.	0.	0.	0.	0.	0.	0.	0.
06	0.1	0.2	0.	0.	0.5	0.	0.	0.	0.	0.	2.0	0.	0.1	0.
07	4.6	34.9	0.	47.2	9.8	0.	0.	0.	0.	0.9	0.4	0.	5.8	4.6
08	28.1	5.8	0.	662.8	19.5	0.	0.	0.	0.	0.	0.	0.	0.	2.7
09	0.3	143.8	0.	41.9	137.6	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	29.1	70.0	5.3	226.0	71.5	0.	0.	0.	0.	139.0	4.1	0.	11.2	86.4
12	6.6	1.0	0.	519.1	3.4	0.	0.	0.	0.	0.	0.	0.	0.	0.7
13	218.0	25.8	0.	379.1	0.	0.	0.	0.	0.	0.	4.1	0.	0.	0.
15	111.0	6.0	0.4	571.0	13.7	0.	0.	0.	0.	6.7	0.	0.	0.	4.9
16	20.2	0.5	0.	293.6	0.	0.	0.	0.	0.	0.8	0.	0.	0.	1.5
17	160.7	0.	0.	36.6	0.	0.	0.	0.	0.	0.	0.	0.	0.	9.3
18	7.5	343.2	0.	33.0	23.4	0.	0.	0.	0.6	0.	0.	0.	0.	0.4
19	7.8	9.0	340.6	0.	0.	0.	0.	0.	0.	14.1	5.2	0.	0.	0.
20	9.3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	22.7	0.	0.	1.0	0.	11.0	0.	0.	11.8	0.
22	0.	0.	0.	0.	18.5	0.	0.	9.	0.	0.	0.	0.	0.	0.
23	0.	0.	0.	1.0	200.0	50.0	0.	19.0	2.0	44.0	10.0	5.0	50.0	4.0
24	0.	0.	0.	2.2	32.0	0.	30.0	42.8	0.	20.7	0.	5.0	0.	54.8
25	0.	0.	0.	1.6	0.	0.	1.3	24.0	7.6	0.	0.	0.	0.	0.
27	0.	0.	0.	0.	1213.7	6.8	0.	0.	31.4	89.7	0.	0.	0.	123.6
28	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
29	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30	0.	68.3	0.	26.9	0.	3.5	4.9	0.	0.	0.	0.	0.	14.0	0.
31	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	12.0
34	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

1) Sum of domestically produced and imported inputs. Measured in (1961) producers' prices (it should for consistency with table 4.1 have been measured in purchasers' prices).

In principle we would have to estimate $28^2 = 784$ import ratio functions, but a majority of these flows are zero, while in other instances either x_{ik}^A or x_{ik}^B may be zero for some or all the years. We decided to estimate only those γ_{ik} for which we have complete time series for both x_{ik}^A and x_{ik}^B . This criteria excluded deliveries of commodities 20, 22, 23, 28, 29, 31 and 34 because they are not imported to any production sector in any year, and it excluded commodities 03, 18, 21, 24, 25 and 30 because all flows of those commodities to the production sectors (i.e. x_{ik}^A or x_{ik}^B) were zero in some year. Along the same lines, we excluded the production sectors 22 and 29 because they do not receive any imported inputs and sectors 2, 19, 23, 24, 25 and 28 because either x_{ik}^A or x_{ik}^B was zero in some year. This leaves us with 16 commodities and 20 production sectors to be analyzed. But many of these flows are also zero, or either x_{ik}^A or x_{ik}^B is zero in some year.

Table 4.3 presents an import value shares matrix for all those flows for which complete time series for both x_{ik}^A and x_{ik}^B are available. It will be seen that we are left with 86 import ratios which can be estimated on the basis of complete observations: for the commodities 06 and 27 we have only one recipient sector with complete data, while commodity 11, chemicals is delivered to all sectors included in table 4.3¹⁾ A look at table 4.3 will also reveal substantial differences among sectors in the magnitude of the import shares for the same commodity.

1) In section 5 below we present select estimates from 31 of these flows.

Table 4.3. 1961 Import share matrix¹⁾ for production sectors for flows with complete data

Commodity	Receiving sector									
	1	3	4	5	6	7	8	9	11	12
1	-	-	-	0.116	0.932	0.739	-	-	-	-
2	-	-	-	-	-	-	0.060	0.311	-	-
4	0.078	-	-	-	-	-	-	0.506	0.941	0.245
5	0.008	-	-	0.078	0.531	-	-	-	-	-
6	-	-	-	-	0.322	-	-	-	-	-
7	-	-	-	-	-	0.491	0.290	0.992	-	-
8	-	-	-	-	-	-	0.241	-	-	-
9	-	-	-	-	-	-	-	0.066	-	-
11	0.252	0.679	0.693	0.446	0.634	0.630	0.406	0.554	0.525	0.616
12	-	-	-	-	0.539	-	-	-	0.431	0.191
13	-	-	-	0.166	-	-	-	0.318	-	0.274
15	-	-	-	0.046	-	0.758	0.219	0.594	-	-
16	-	-	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-	-	-
19	0.030	-	-	-	-	-	-	-	0.071	0.104
27	-	-	-	-	-	-	-	-	-	-

	Receiving sector									
	13	14	15	16	17	18	19	20	21	22
1	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-
4	0.616	-	-	-	-	-	-	0.891	-	-
5	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-
7	-	0.560	-	-	0.512	0.502	0.449	-	-	-
8	-	0.224	-	-	-	0.042	-	-	-	-
9	-	-	-	-	0.027	0.212	-	-	-	-
11	0.778	0.439	0.603	0.357	0.605	0.189	0.643	0.731	0.357	0.483
12	0.515	0.531	0.559	-	-	0.188	-	-	-	-
13	0.760	0.631	0.614	0.684	0.550	0.480	-	-	-	-
15	0.453	0.525	0.326	0.409	0.333	0.227	-	-	-	-
16	-	0.540	0.766	0.549	-	0.269	-	-	-	-
17	0.820	-	-	0.405	-	-	-	-	-	-
19	0.220	0.025	-	-	-	-	-	-	-	-
27	-	-	-	-	-	-	-	0.022	-	-

elasticity has the wrong sign. Strengthening our rejection criteria, we can accept eq. 13 at the 2% significance level ($\chi_1^2 = 4.5$) compared with eq. 1. Eq. 13 assigns a large role to lagged prices:

$$\text{eq. 1: } x_t = 1.00 + 1.52 p_t - 3.82 p_t^L \quad \rho = .79 \quad R^2 = .886 \\ \quad \quad \quad (.45) \quad (.74) \quad (.72) \quad \quad \quad (.14) \quad \quad \quad DW = 1.36$$

$$\text{eq.13: } x_t = 1.32 - 3.68 p_{t-1} \quad \rho = .78 \quad R^2 = .853 \\ \quad \quad \quad (.45) \quad (.79) \quad \quad \quad (.15) \quad \quad \quad DW = 1.63$$

The import ratio γ_{jk} associated with this flow has fallen at an average rate of 14 per cent per year over the period 1949-1969.

Commodity 07 - Textiles

Receiv. sector	Input value	Import share
07	764.1	.491
08	32.4	.290
09	13.7	.993
15	14.1	.560
18	34.9	.513
20	47.2	.502
21	9.8	.449

The demand for commodity 07 is dominated by the production sector 07, which uses 80 per cent of the textiles used as inputs in the production sectors (most textiles go directly to final demand). The hypothesis of eq. 1 is accepted readily in all seven cases, but in only two of them (07 and 09) do we get significant price effects. In the case of the remaining five sectors, a constant alone does "best" for sector 18, while a purely autoregressive model does best for the rest.

Table 5.1. Textiles - Select regression results

S e c t o r 07 - Manufacture of textiles

$$\text{eq. 1 } x_t = .00 - 1.42 p_t + .07 p_t^L \quad \rho = .71 \quad R^2 = .948 \\ \quad \quad \quad (.07) \quad (.24) \quad (.28) \quad \quad \quad (.17) \quad \quad \quad DW = 1.61$$

$$\text{eq.11 } x_t = .01 - 1.39 p_t \quad \rho = .72 \quad R^2 = .948 \\ \quad \quad \quad (.07) \quad (.19) \quad \quad \quad (.16) \quad \quad \quad DW = 1.65$$

S e c t o r 08 - Manufacture of wood and wood products

$$\text{eq. 1 } x_t = -.25 + 1.06 p_t - 1.05 p_t^L \quad \rho = .61 \quad R^2 = .499 \\ \quad \quad \quad (.36) \quad (.93) \quad (1.19) \quad \quad \quad (.19) \quad \quad \quad DW = 1.84$$

$$\text{eq.14 } x_t = -.20 \quad \rho = .63 \quad R^2 = .435 \\ \quad \quad \quad (.35) \quad \quad \quad (.18) \quad \quad \quad DW = 1.66$$

S e c t o r 09 - Manufacture of paper and paper products

$$\text{eq. 1 } x_t = -1.15 - 2.05 p_t + .83 p_t^L \quad \rho = .74 \quad R^2 = .747 \\ \quad \quad \quad (.52) \quad (.32) \quad (.53) \quad \quad \quad (.16) \quad \quad \quad DW = 2.13$$

$$\text{eq.11 } x_t = -1.23 - 2.33 p_t \quad \rho = .79 \quad R^2 = .708 \\ \quad \quad \quad (.67) \quad (.28) \quad \quad \quad (.14) \quad \quad \quad DW = 2.10$$

Table 5.1 (cont.). Textiles - Select regression results

Sector 15 - Manufacture of non-electrical machinery

$$\text{eq. 1 } x_t = \begin{matrix} -.31 \\ (.37) \end{matrix} + \begin{matrix} 1.04 \\ (1.32) \end{matrix} p_t + \begin{matrix} .37 \\ (1.63) \end{matrix} p_t^L \quad \rho = \begin{matrix} .52 \\ (.20) \end{matrix} \quad R^2 = \begin{matrix} .349 \\ DW = 1.98 \end{matrix}$$

$$\text{eq.14 } x_t = \begin{matrix} -.19 \\ (.36) \end{matrix} \quad \rho = \begin{matrix} .56 \\ (.20) \end{matrix} \quad R^2 = \begin{matrix} .316 \\ DW = 1.89 \end{matrix}$$

Sector 18 - Other manufacture

$$\text{eq. 1 } x_t = \begin{matrix} -.33 \\ (.14) \end{matrix} - \begin{matrix} .22 \\ (1.68) \end{matrix} p_t + \begin{matrix} .23 \\ (1.53) \end{matrix} p_t^L \quad \rho = \begin{matrix} .21 \\ (.23) \end{matrix} \quad R^2 = \begin{matrix} .042 \\ DW = 2.11 \end{matrix}$$

$$\text{eq.34 } x_t = \begin{matrix} -.34 \\ (.10) \end{matrix} \quad R^2 = \begin{matrix} .000 \\ DW = 1.65 \end{matrix}$$

Sector 20 - Building and construction

$$\text{eq. 1 } x_t = \begin{matrix} -.54 \\ (.09) \end{matrix} + \begin{matrix} .09 \\ (.36) \end{matrix} p_t + \begin{matrix} .25 \\ (.40) \end{matrix} p_t^L \quad \rho = \begin{matrix} .54 \\ (.20) \end{matrix} \quad R^2 = \begin{matrix} .454 \\ DW = 1.65 \end{matrix}$$

$$\text{eq.14 } x_t = \begin{matrix} -.52 \\ (.11) \end{matrix} \quad \rho = \begin{matrix} .63 \\ (.18) \end{matrix} \quad R^2 = \begin{matrix} .398 \\ DW = 1.66 \end{matrix}$$

Sector 21 - Wholesale and retail trade

$$\text{eq. 1 } x_t = \begin{matrix} 1.08 \\ (.48) \end{matrix} - \begin{matrix} 2.16 \\ (1.21) \end{matrix} p_t + \begin{matrix} .25 \\ (1.63) \end{matrix} p_t^L \quad \rho = \begin{matrix} .60 \\ (.19) \end{matrix} \quad R^2 = \begin{matrix} .499 \\ DW = 1.75 \end{matrix}$$

$$\text{eq.14 } x_t = \begin{matrix} 1.00 \\ (.48) \end{matrix} \quad \rho = \begin{matrix} .59 \\ (.19) \end{matrix} \quad R^2 = \begin{matrix} .384 \\ DW = 1.84 \end{matrix}$$

Commodity 08 - Wood and wood products

Receiv. sector	Input value	Import share
08	345.4	.242
15	19.6	.225
20	662.8	.043

Sector 08 - Manufacture of wood and wood products

The assumption of autoregressive residuals is clearly accepted, and both a_0 and a_1 are significant in eq. 1 and approximately of the same magnitude ($a_0 = -2.02$ and $a_1 = -1.68$). The χ^2 test scheme indicates not significant serial correlation in eq. 12 and suggests eq. 32 as the preferred formulation:

$$\text{eq. 1 } x_t = \begin{matrix} 1.26 \\ (.05) \end{matrix} - \begin{matrix} 2.02 \\ (.65) \end{matrix} p_t - \begin{matrix} 1.68 \\ (.59) \end{matrix} p_t^L \quad \rho = \begin{matrix} .278 \\ (.226) \end{matrix} \quad R^2 = \begin{matrix} .632 \\ DW = 2.33 \end{matrix}$$

$$\text{eq.32 } x_t = \begin{matrix} 1.20 \\ (.04) \end{matrix} - \begin{matrix} 2.52 \\ (.57) \end{matrix} L(p_t) \quad R^2 = \begin{matrix} .55 \\ DW = 2.12 \end{matrix}$$

The time trend is in no case significant.

S e c t o r 15 - Manufacturing of non-electrical machinery

The DW in eq. 0 indicates correlated errors even in this general formulation, but we can accept the AR formulation, conditional upon the model of eq. 0. The coefficient a_1 is insignificant, while the autocorrelation coefficient is significant in eq. 1. The test scheme leads to either eqs. 11 or 14:

$$\begin{array}{lll} \text{eq. 1 } x_t = 2.01 - 1.59 p_t + .05 p_t^L & \rho = .53 & R^2 = .511 \\ \quad \quad \quad (.28) \quad (.11) \quad (1.17) & \quad \quad (.20) & \quad \quad DW = 1.68 \\ \\ \text{eq.11 } x_t = 2.01 - 1.58 p_t & \rho = .52 & R^2 = .51 \\ \quad \quad \quad (.27) \quad (.96) & \quad \quad (.20) & \quad \quad DW = 1.68 \\ \\ \text{eq.14 } x_t = 1.88 & \rho = .68 & R^2 = .452 \\ \quad \quad \quad (.39) & \quad \quad (.17) & \quad \quad DW = 1.78 \end{array}$$

However, eq. 14 indicates that we cannot reject this purely AR formulation. The trend is never significant, but comparison with eq. 0T indicates that we can accept eq. 11 but reject eq. 14.

S e c t o r 20 - Building and construction

The residuals are strongly correlated, but the χ^2 -test of eq. 1 indicates acceptance of the AR specification, though eq. 2 and eq. 21 indicate significant role for the lagged endogenous variable. The coefficients a_0 and a_1 are of the same order of magnitude, though not significant, in eq. 1. Based on the SSR criterion we choose eq. 12 over eq. 11 and eq. 13:

$$\begin{array}{lll} \text{eq. 1 } x_t = 4.10 - 1.88 p_t - 2.33 p_t^L & \rho = .62 & R^2 = .772 \\ \quad \quad \quad (.41) \quad (1.11) \quad (1.26) & \quad \quad (.18) & \quad \quad DW = 2.03 \\ \\ \text{eq.12 } x_t = 4.10 - 4.15 L(p_t) & \rho = .62 & R^2 = .77 \\ \quad \quad \quad (.39) \quad (1.61) & \quad \quad (.19) & \quad \quad DW = 2.06 \\ \\ \text{eq.14 } x_t = 3.22 & \rho = .89 & R^2 = .744 \\ \quad \quad \quad (1.13) & \quad \quad (.11) & \quad \quad DW = 1.56 \end{array}$$

The purely AR formulation of eq. 14 does well, despite the significant coefficient on prices in eq. 12 and cannot, by the χ^2 -test, be rejected either conditional on eq. 0 or on eq. 12. The trend is in no case significant.

Commodity 09 - Paper and paper products

Receiv. sector	Input value	Import share
09	705.9	.067
18	143.8	.028
20	41.9	.212

S e c t o r 09 - Manufacture of paper and paper products:

The AR formulation of eq. 1 is not rejected, and eq. 1 is in fact the preferred equation:

$$\text{eq. 1 } x_t = 2.43 - 1.52 p_t - 2.89 p_t^L \quad \rho = .76 \quad R^2 = 0.90 \\ \quad \quad \quad (.19) \quad (.34) \quad (.54) \quad \quad \quad (.15) \quad \quad \quad DW = 1.26$$

One must reject the hypothesis that $a_0 = a_1$. The trend is not significant in 0T or 1T, and a test of eq. 1 against eq. 0T does not lead to rejection.

S e c t o r 18 - Other manufacturing:

The AR structure of eq. 1 is accepted. The lagged price variable in this equation is insignificant, so that our χ^2 -test scheme chooses eq. 11 unequivocally: the test of eq. 11 against eq. 1 having $\chi^2 = .25$.

$$\text{eq. 1 } x_t = \begin{matrix} 3.36 \\ (.10) \end{matrix} - \begin{matrix} 1.29 \\ (.27) \end{matrix} p_t + \begin{matrix} .14 \\ (.31) \end{matrix} p_t^L \quad \rho = \begin{matrix} .45 \\ (.21) \end{matrix} \quad R^2 = .855 \quad DW = 2.16$$

$$\text{eq.11 } x_t = \begin{matrix} 3.35 \\ (.18) \end{matrix} - \begin{matrix} 1.19 \\ (.18) \end{matrix} p_t \quad \rho = \begin{matrix} .45 \\ (.21) \end{matrix} \quad R^2 = .853 \quad DW = 2.21$$

The trend is not significant.

S e c t o r 20 - Building and construction

The AR formulation of eq. 1 (and eq. 1T) is clearly rejected ($\chi^2 = 36.00$): our basic model seems to be misspecified. The regressions 2 and 3 suggest a significant role for lagged prices, but they have a significantly wrong sign. Only in the UTE eqs. 0 and 0T is the current price significantly negative. The test scheme would choose eq. 13 with $a_1 = 1.91$ (.78). Insisting on a negative price coefficient, we choose eq. 11:

$$\text{eq. 1 } x_t = \begin{matrix} .93 \\ (.15) \end{matrix} - \begin{matrix} .21 \\ (.59) \end{matrix} p_t + \begin{matrix} 1.83 \\ (.81) \end{matrix} p_t^L \quad \rho = \begin{matrix} .57 \\ (.19) \end{matrix} \quad R^2 = .648 \quad DW = 1.83$$

$$\text{eq.11 } x_t = \begin{matrix} 1.08 \\ (.34) \end{matrix} - \begin{matrix} .60 \\ (.58) \end{matrix} p_t \quad \rho = \begin{matrix} .80 \\ (.14) \end{matrix} \quad R^2 = .58 \quad DW = 1.59$$

though it appears to be the AR structure of the error terms which "explains" most of the change ($R^2 = .56$ for eq. 14), and eq. 14 cannot be rejected conditional upon eq. 11. Inclusion of a trend does not alter the above picture.

Commodity 11 - Chemicals

Receiv. sector	Input value	Import share
01	235.7	.253
05	182.6	.447
11	385.9	.526
13	306.9	.778
20	226.0	.190
27	139.0	.357

Chemicals represent the most widely used commodity having positive flows of both domestically produced and imported products to all production sectors included in this analysis in all years from 1949 to 1969. We have chosen to analyse the six largest recipients of chemicals, i.e. those receiving an input of chemicals of over 100 mill. N.kr in 1961.

The autoregressive formulation of eq. 1 was accepted in all cases except for sector 13. For sector 27, our selection criteria picks eq. 11, an equation with a significant positive price coefficient. As an alternative we have also included eq. 13, where the price term has the right sign, but which is rejected when compared with eq. 1.

Table 5.2. Chemicals - Selected regression results

Sector 01 - Agriculture			
eq. 1	$x_t = 1.87 - 2.64 p_t - .82 p_t^L$ (.97) (.78) (.72)	$\rho = .92$ (.09)	$R^2 = .416$ DW = 1.68
eq.11	$x_t = .98 - 2.19 p_t$ (.54) (.74)	$\rho = .86$ (.12)	$R^2 = .367$ DW = 1.57
Sector 05 - Food processing			
eq. 1	$x_t = .27 - .78 p_t - .67 p_t^L$ (.04) (.27) (.27)	$\rho = .17$ (.23)	$R^2 = .918$ DW = 1.60
eq.32	$x_t = .26 - 1.44 L(p_t)$ (.03) (.11)		$R^2 = .915$ DW = 1.35
Sector 11 - Manufacture of chemicals			
eq. 1	$x_t = -.03 - 1.42 p_t - .16 p_t^L$ (.21) (.47) (.57)	$\rho = .79$ (.15)	$R^2 = .875$ DW = 1.13
eq.11	$x_t = -.04 - 1.43 p_t$ (.21) (.46)	$\rho = .80$ (.14)	$R^2 = .874$ DW = 1.12
Sector 13 - Manufacture of basic metals			
eq. 1	$x_t = -1.41 + 2.46 p_t - 5.43 p_t^L$ (.11) (1.90) (2.14)	$\rho = -.35$ (.22)	$R^2 = .330$ DW = 2.08
eq.33	$x_t = -1.36 - 2.67 p_t^L$ (.14) (1.03)		$R^2 = .294$ DW = 2.29
Sector 20 - Building and construction			
eq. 1	$x_t = 1.19 - .23 p_t - .07 p_t^L$ (.04) (.25) (.28)	$\rho = -.03$ (.24)	$R^2 = .260$ DW = 1.12
eq.31	$x_t = 1.19 - .29 p_t$ (.04) (.12)		$R^2 = .255$ DW = 1.17
Sector 27 - Domestic transport			
eq. 1	$x_t = -.38 + 3.07 p_t - 1.65 p_t^L$ (1.63) (1.27) (1.73)	$\rho = .93$ (.09)	$R^2 = .878$ DW = 1.37
eq.11	$x_t = -.91 + 3.38 p_t$ (1.07) (1.26)	$\rho = .90$ (.10)	$R^2 = .870$ DW = 1.35
eq.13	$x_t = 1.58 - 2.50 p_t^L$ (3.08) (1.90)	$\rho = .96$ (.07)	$R^2 = .833$ DW = 1.44

Thus far in this section we have concentrated entirely on the second order parameter of the production function f_{ik} [see (2.1)] identifying the elasticity of substitution σ_{ik} [see (3.5)] with the long run elasticity $a_{ik0} + a_{ik1}$ of this section.³⁾ We can estimate the price elasticities of the import demand functions using (2.18), while the estimates of the distribution parameters $\hat{\delta}_{ik}$ can be computed from $\hat{c}_{ik} = \ln(1 - \hat{\delta}_{ik}) / \hat{\delta}_{ik}$. The estimates $\hat{\sigma}_{ik}$ and $\hat{\delta}_{ik}$ completely specify the function f_{ik} , providing us with an estimate \hat{f}_{ik} of this function and, if desired, an estimate \hat{x}_{ik} of the unobservable x_{ik} .

3) We have earlier ignored the commodity and sector subscripts on the a coefficients.

6. SOME MULTIVARIATE ESTIMATES

Thus far we have estimated each import ratio equation γ_{ik} individually. But it seems reasonable to expect the disturbances from the various equations referring to the demand for the same commodity, i.e. γ_{ik} , $k=1, \dots, n$, to be correlated. Many of the same omitted factors may influence both the demand of input i from the various sectors, and, perhaps more importantly, the supply of the i 'th commodity. In addition, we are interested in testing various hypotheses about the elasticity of substitution.

We will therefore analyze the following multivariate model for the i 'th commodity:

$$x_t = c + \sum_{\tau=0}^L \hat{a}_\tau p_{t-\tau} + u_t \quad (6.1)$$

where c is an n -dimensional vector of constants, and

$x_t = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))'$ and $u_t = (u_{i1}(t), u_{i2}(t), \dots, u_{in}(t))'$ are the (column) vectors of import ratios and residuals of the i 'th commodity in period t ¹⁾. \hat{a}_τ is an $n \times n$ diagonal matrix of parameters and $p_{t-\tau}$ is an n dimensional vector of relative prices in year $t-\tau$. The exact form of the lag structure will depend on the model formulation. In our basic model [see (3.11)] equation (6.1) becomes:

$$x_t = c + \hat{a}_0 p_t + \hat{a}_1 p_t^L + u_t \quad (6.2)$$

The errors will be assumed to be first order serially correlated, i.e.

$$u_t = \hat{\rho} u_{t-1} + \varepsilon_t \quad (6.3)$$

where ρ is the n -vector of first order correlation coefficients and ε_t is a serially independently distributed n -vector with mean zero and covariance matrix Σ^2 .

We have chosen to estimate three models for each set of γ_{ik} , $k=1, \dots, n$, using this multivariate method:

- i) eq. 1 - the basic model of this analysis
- ii) eq.12 - the linearly distributed lag model
- iii) eq. P - the set of preferred models as derived in sec. 5.

An example of these estimates (for commodity 09 - paper and paper products) is presented in columns 3, 5, and 7 in table 6.1. The first column gives the names of the estimated coefficients, while the second column gives the estimates obtained by single equation non-linear least squares.³⁾ A comparison of the single equation estimates and those of eq. 1 reveal the expected gain in efficiency.

The main purpose of introducing the multivariate model is to test the equality of the elasticity of substitution across the production sectors. We have estimated the following three restricted equation systems corresponding to i-iii above:

- iv) eq. IS - eq. 1 with the restriction that $a_{k0} = a_0$, and $a_{k1} = a_1$, $k=1, \dots, n$.
- v) eq.12S - eq.12 with $a_{k0} = a_0$, $k=1, \dots, n$.
- vi) eq. PS - eq. P with $a_{k0} = a_0$, $a_{k1} = a_1$, $k=1, \dots, n$,
unless a_{k0} or a_{k1} already are zero in the preferred equation.

1) The commodity index i is fixed in the analysis of this section, and has been ignored. The index $n = n_i$ is the number of production sectors included in the estimation of the i 'th commodity.

2) Good arguments could also be made for the residuals ε_{ik} , $i = 1, \dots, m$, i.e. for the various commodities used by the same sector, to be correlated.

3) The equivalent estimates in sec. 5 were obtained by the Cochrane-Orcutt method. The parameters of the multivariate model are estimated using the RTE formulation (3.12). The constant term of the present multivariate estimates and the single equation estimates of section 5 are therefore related by $k_k = c_k (1-\rho_k)$.

The last comment suggests some of the ambiguity in defining equality of the elasticity of substitution, when the equations are not all of the same type. We have three measures of the substitution parameter: the short run elasticity a_{k0} , the delayed elasticity a_{k1} , and the long run elasticity $a_{k0}+a_{k1}$.⁴⁾ The most reasonable definition of equality would perhaps refer to the long run parameters. We have chosen a simpler definition as evidenced by vi) above. The estimates of equation systems 1S, 12S, and PS are presented in columns 4, 6, and 8 of table 6.1.

Table 6.1. Commodity 09 - Paper and paper products. Coefficient estimates, multivariate analysis

Coefficient	Single eq. estimates	eq. 1	eq. 1S	eq. 12	eq. 12S	pref.	pref.S
Sector 09 ¹⁾							
k_1	.59 (.39)	.52 (.33)	.24 (.38)	.27 (.42)	.24 (.34)	.66 (.31)	.56 (.38)
a_{10}	-1.52 (.35)	-1.51 (.30)	-.81 (.16)	-2.47 (.70)	-1.05 (.14)	-1.56 (.30)	-1.17 (.13)
a_{11}	-2.89 (.56)	-2.94 (.50)	.31 (.18)			-3.17 (.48)	-2.76 (.46)
ρ_1	.76 (.16)	.79 (.14)	.87 (.16)	.87 (.17)	.87 (.14)	.73 (.13)	.77 (.16)
Sector 18							
k_2	1.84 (.61)	2.15 (.47)	2.39 (.49)	2.86 (.53)	2.84 (.51)	2.15 (.45)	2.11 (.45)
a_{20}	-1.29 (.28)	-1.26 (.22)		-1.07 (.15)		-1.18 (.14)	
a_{21}	.14 (.33)	.07 (.25)					
ρ_2	.45 (.19)	.36 (.14)	.27 (.15)	.12 (.16)	.12 (.16)	.36 (.14)	.37 (.14)
Sector 20							
k_3	.40 (.22)	.35 (.18)	.21 (.12)	.26 (.14)	.20 (.12)	.20 (.13)	.20 (.13)
a_{30}	-.21 (.61)	-.12 (.45)		.31 (.90)		-.59 (.47)	
a_{31}	1.85 (.88)	1.68 (.72)					
ρ_3	.57 (.24)	.63 (.19)	.82 (.12)	.72 (.15)	.82 (.12)	.82 (.13)	.84 (.13)
ln L		13.2342	1.9714	3.6069	1.5152	11.1476	9.7309
No.of coeff.	12	12	8	9	7	10	8

1) The indices 1, 2, and 3 represent the sectors 08, 18, 20 respectively.

4) In the linearly distributed lag formulation a_{k0} is the long run parameter.

We have estimated the system (6.1) and (6.3) using multivariate maximum likelihood. The value of the log likelihood function is:

$$\ln L = -\frac{nT}{2} (1 + \ln 2\pi) - \frac{T}{2} \ln |\hat{\Sigma}|, \quad (6.4)$$

where $|\hat{\Sigma}|$ is the generalized residual variance.⁵⁾ Let H_i be the hypothesis that the parametric restrictions embodied in the formulation of the i 'th equation system are true, and let H_j be another less restrictive hypothesis. Then a test of H_i conditional upon H_j is provided by the likelihood ratio test:

$$-2 \ln \frac{L_i}{L_j} = T (\ln |\hat{\Sigma}_i| - \ln |\hat{\Sigma}_j|) \underset{A}{\sim} \chi^2(k_j - k_i) \quad (6.5)$$

which has an asymptotic chi-square distribution with $k_j - k_i$ degrees of freedom.

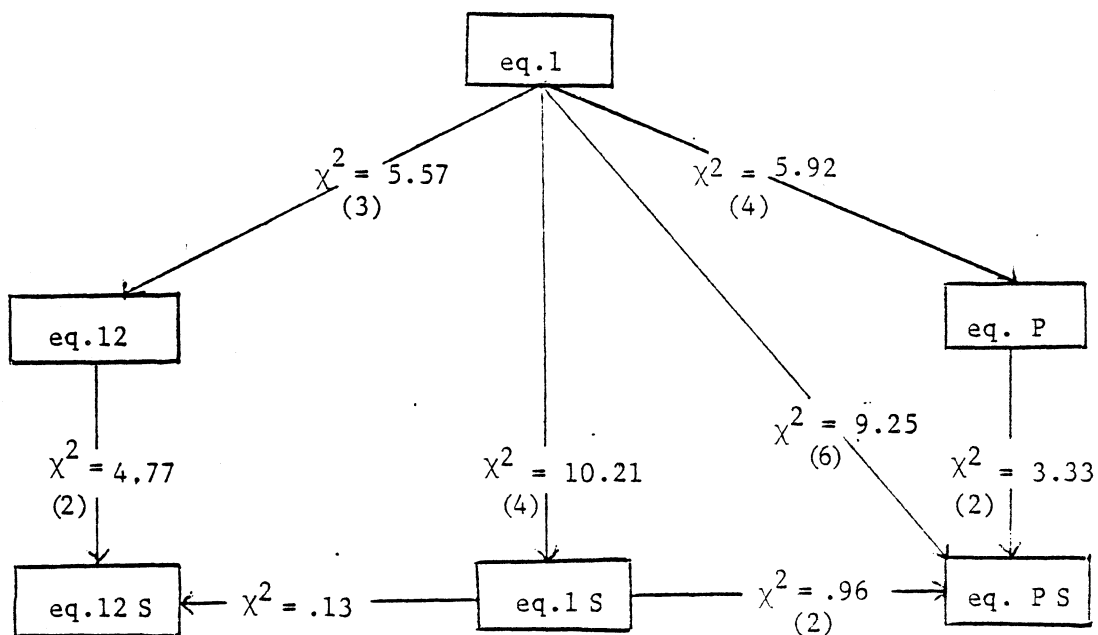
We have carried out these tests only for commodities 08 and 09, and the results are presented in fig. 6.1. For commodity 08, the preferred equations are: sector 08 - eq. 32, sector 15 - eq. 11, and sector 20 - eq. 12. For commodity 09, the preferred equations are: sector 09 - eq. 1, sector 18 - eq. 11, and sector 20 - eq. 11. In both cases equality is rejected in the basic model, and accepted in eq. 12 and the preferred model. It would seem that equality in the basic model is too restrictive because it imposes the same lag structure on all sectors, an hypothesis which we may already have rejected when choosing a preferred formulation.⁶⁾ But there seems to be evidence for the equality of the substitution parameters, once one has determined the type of lag structure.

5) $\hat{\Sigma}$ is the maximum likelihood estimate of the covariance matrix Σ .

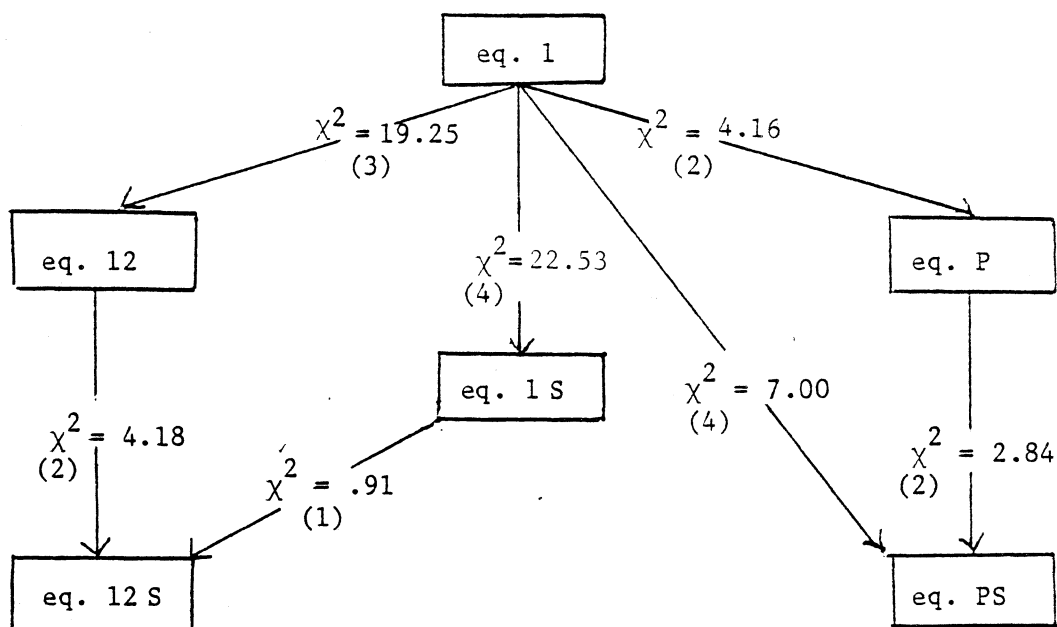
6) See section 5.

Table 6.2 - Parameter Tests in the Multivariate Model¹⁾

08 Wood and wood products



09 Paper and paper products



1) Numbers in parentheses are degrees of freedom. See text for further explanation.

7. CONCLUDING REMARKS

This analysis has been a test both of the methodology and of the data. One could easily fault the data when the theory seems to fail, or one could lay the blame on the model when the data seem unreasonable. There are, particularly for some of the commodities, many reasons why the model may be inadequate. But the estimates presented above do suggest that there often is significant price responsiveness in the demand for imports of a given commodity by a given sector. It may therefore be advantageous to introduce the import share functions m_{ik}^B [see (2.6)] as explicit functions of relative prices into the import share matrix M^B [see (1.5)]. This is particularly the case, since, as mentioned in the introduction, it seems to be the only way to combine price substitution with the use of a detailed matrix of import shares.

We had hoped in this analysis to use the added information contained in the data by sector to obtain more efficient estimates of the substitution parameter, given that this was the same for all sectors. Tentative conclusions thus far do not seem to support the last assumption. It would still seem reasonable, however, to use such a common value for the elasticity of substitution for those sectors where for one reason or another we do not get good estimates (in some cases the import share may better be left as an exogeneous variable).

It may be worth while to look again at (1.1) and (1.2). In a planning model we are primarily interested in explaining the total import x_i^B of the different commodities. This suggests a comparison of the following three models:

- i) - estimate (1.1) directly
- ii) - the approach of this paper [i.e. (1.2)], using (1.3) to obtain x_i^B
- iii) - model ii) with the added restriction that the substitution parameter be the same for all sectors

We have not yet estimated i) using the present body of data, but we will do so and we then intend to compare the predictive ability of the three models over the sample period. This will not give us a statistical test, but may give a better idea of the gains that can be expected from an implementation of the current approach.

APPENDIX A - Tables

This appendix consists of 6 tables giving detailed estimates of the equations outlined in fig. 3.1 for the commodities "wood and wood products" (08) and "paper and paper products" (09). For each commodity, there is one table for each of the three production sectors receiving that commodity and having complete time series on import and domestic supplies.¹⁾ The first column gives the number of the equation estimated (see fig. 3.1): a star indicates that the equation is estimated by the Cochrane-Orcutt procedure, and a T indicates the inclusion of a time trend, i.e. eq. 1T is eq. 1 with a time trend added. For each equation the first row presents the coefficient estimates and the summary statistics, and the second row gives the standard errors of the coefficients. The following abbreviations are used in the column heading:

- CONST - the constant term.
- P - the coefficient of p_t , the current price variable. In equations estimated by polynomially distributed lag (12, 32, 12T, 32 T) the coefficient shown is the sum of the individual lag coefficients.
- PLAG - the coefficient of $p_t^L = (.67 p_{t-1} + .33 p_{t-2})$.
- TREND - the coefficient of the time trend, the trend being - 10 in 1949 and + 10 in 1969.
- XLAG,RHO the coefficient of the lagged endogenous variable x_{t-1} in unrestricted equations, and the first order correlation coefficient for equations estimated by the Cochrane-Orcutt-method (and market with a star in the first column).
- RSQ - square of the multiple correlation coefficient.
- DW - Durbin-Watson statistic.
- SER - standard error of the regression, corrected for degrees of freedom.
- SSR - sum of squared residuals (multiplied by 10).

1) See sec. 4 for more detailed explanation of the selection of sectors included in the analysis.

Table A-1

EQ.NR.	CONST.	P	PLAG	TREND	XLAG,RHO	RSQ	DW	SER	SSR*10
0	0,911 0,200	-2,093 0,904	1,720 4,735	- -	0,277 0,187	0,651 -	2,188 -	0,149 -	2,668 -
1 *	1,262 0,053	-2,017 0,650	-1,680 0,592	- -	0,278 -	0,632 -	2,332 -	0,137 -	2,811 -
2	0,900 0,186	-2,008 0,751	-0,622 0,568	- -	0,283 0,167	0,637 -	2,278 -	0,141 -	2,775 -
3	1,208 0,043	-1,648 0,764	-0,998 0,555	- -	- -	0,562 -	2,102 -	0,149 -	3,347 -
11 *	1,225 0,049	-2,481 0,687	- -	- -	0,093 -	0,480 -	1,995 -	0,158 -	3,971 -
12 *	1,267 0,053	-3,830 0,810	- -	- -	0,302 -	0,630 -	2,359 -	0,133 -	2,827 -
13 *	1,165 0,044	- -	-1,801 0,549	- -	0,093 -	0,436 -	2,410 -	0,164 -	4,310 -
14 *	1,133 0,069	- -	- -	- -	0,320 -	0,113 -	1,494 -	0,200 -	6,772 -
21	0,829 0,175	-2,529 0,585	- -	- -	0,354 0,154	0,606 -	2,179 -	0,142 -	3,013 -
31	1,220 0,045	-2,462 0,657	- -	- -	- -	0,467 -	1,996 -	0,159 -	4,068 -
32	1,198 0,038	-2,518 0,566	- -	- -	- -	0,553 -	2,121 -	0,146 -	3,414 -
33	1,158 0,040	- -	-1,708 0,496	- -	- -	0,426 -	2,252 -	0,166 -	4,385 -
34	1,126 0,050	- -	- -	- -	- -	- -	1,115 -	0,212 -	7,638 -
0T	0,848 0,208	-2,277 0,917	2,502 4,770	0,013 0,012	0,324 0,191	0,683 -	2,322 -	0,148 -	2,423 -
1T*	1,268 0,055	-2,263 0,707	-2,241 0,856	0,011 0,013	0,294 -	0,649 -	2,447 -	0,138 -	2,679 -
11T*	1,226 0,051	-2,071 0,813	- -	-0,009 0,010	0,127 -	0,510 -	1,943 -	0,158 -	3,746 -
12T*	1,270 0,053	-4,538 1,145	- -	0,011 0,012	0,297 -	0,649 -	2,449 -	0,134 -	2,679 -
31T	1,216 0,046	-2,070 0,822	- -	-0,007 0,009	- -	0,490 -	1,935 -	0,161 -	3,898 -
32T	1,200 0,039	-2,873 0,919	- -	0,005 0,011	- -	0,560 -	2,150 -	0,150 -	3,359 -

Table A-2

EQ. NR.	CONST.	P	PLAG	TREND	XLAG, RHO	RSG	DW	SER	SSR*10
0	0,715 0,484	-1,170 1,291	-15,529 9,570	- -	0,643 0,229	0,634 -	1,224 -	0,519 -	32,342 -
1 *	2,011 0,276	-1,595 1,113	0,047 1,169	- -	0,525 -	0,511 -	1,682 -	0,537 -	43,198 -
2	0,938 0,514	-1,017 1,393	0,231 1,305	- -	0,525 0,243	0,492 -	1,711 -	0,566 -	44,920 -
3	2,005 0,160	-2,338 1,397	0,579 1,446	- -	- -	0,322 -	1,026 -	0,632 -	59,934 -
11 *	2,010 0,266	-1,575 0,957	- -	- -	0,525 -	0,511 -	1,683 -	0,520 -	43,203 -
12 *	1,975 0,277	-1,586 1,197	- -	- -	0,541 -	0,488 -	1,709 -	0,532 -	45,226 -
13 *	1,897 0,373	- -	-0,228 1,227	- -	0,653 -	0,452 -	1,774 -	0,550 -	48,419 -
14 *	1,882 0,393	- -	- -	- -	0,680 -	0,452 -	1,779 -	0,534 -	48,443 -
21	0,919 0,486	-0,813 0,762	- -	- -	0,531 0,233	0,491 -	1,707 -	0,548 -	45,020 -
31	1,984 0,147	-1,855 0,684	- -	- -	- -	0,315 -	0,977 -	0,615 -	60,575 -
32	1,945 0,150	-1,813 0,743	- -	- -	- -	0,271 -	0,932 -	0,635 -	64,449 -
33	1,909 0,157	- -	-1,512 0,767	- -	- -	0,196 -	0,927 -	0,667 -	71,129 -
34	1,923 0,170	- -	- -	- -	- -	- -	0,649 -	0,721 -	88,416 -
0T	0,796 0,461	-0,381 1,325	-15,863 9,061	-0,088 0,057	0,667 0,218	0,700 -	1,353 -	0,491 -	26,565 -
1T*	2,353 0,360	-0,490 1,263	2,411 1,900	-0,174 0,112	0,563 -	0,582 -	1,838 -	0,514 -	36,966 -
11T*	2,110 0,294	-0,876 1,259	- -	-0,057 0,066	0,530 -	0,534 -	1,751 -	0,524 -	41,166 -
12T*	2,157 0,334	0,461 2,365	- -	-0,104 0,104	0,544 -	0,521 -	1,814 -	0,532 -	42,381 -
31T	2,011 0,157	-1,151 1,412	- -	-0,033 0,058	- -	0,330 -	0,959 -	0,629 -	59,275 -
32T	2,017 0,175	-0,418 1,863	- -	-0,059 0,072	- -	0,302 -	0,941 -	0,641 -	61,695 -

EQ.NR.	CONST.	P	PLAG	TREND	XLAG,RHO	RSQ	DW	SER	SSR*10
0	1,149 0,680	-1,231 1,585	-8,483 10,358	-	0,707 0,165	0,810	2,010	0,564	38,207
1 *	4,097 0,413	-1,881 1,105	-2,328 1,263	-	0,623	0,772	2,027	0,554	46,028
2	1,105 0,643	-0,742 1,228	-0,630 1,029	-	0,708 0,159	0,794	1,878	0,544	41,506
3	3,888 0,228	-2,395 1,758	-1,091 1,538	-	-	0,502	0,727	0,818	100,304
11 *	3,371 1,129	-0,239 1,110	-	-	0,874	0,744	1,593	0,567	51,495
12 *	4,098 0,391	-4,154 1,612	-	-	0,616	0,771	2,057	0,538	46,240
13 *	2,955 1,550	-	0,406 1,673	-	0,906	0,745	1,569	0,567	51,449
14 *	3,220 1,125	-	-	-	0,887	0,744	1,562	0,551	51,595
21	1,104 0,629	-1,331 0,749	-	-	0,718 0,155	0,788	2,069	0,533	42,618
31	3,954 0,205	-3,464 0,892	-	-	-	0,485	0,895	0,805	103,671
32	3,842 0,193	-3,393 0,854	-	-	-	0,497	0,660	0,796	101,431
33	3,725 0,199	-	-2,887 0,813	-	-	0,441	0,630	0,839	112,724
34	3,656 0,257	-	-	-	-	-	0,264	1,089	201,494
0T	1,174 0,789	-1,198 1,718	-8,662 11,103	-0,009 0,128	0,704 0,178	0,810	1,998	0,589	38,189
1T*	4,315 0,548	-0,565 1,456	-0,200 2,138	-0,172 0,129	0,695	0,793	1,778	0,545	41,623
11T*	4,318 0,535	-0,478 1,132	-	-0,182 0,085	0,698	0,793	1,748	0,527	41,653
12T*	4,309 0,530	-1,061 2,800	-	-0,161 0,114	0,694	0,793	1,783	0,528	41,754
31T	3,917 0,207	-1,425 2,103	-	-0,092 0,086	-	0,522	0,653	0,801	96,326
32T	3,866 0,200	-1,262 3,175	-	-0,094 0,134	-	0,512	0,592	0,809	98,241

EQ.NR.	CONST.	P	PLAG	TREND	XLAG,RHO	RSQ	DW	SER	SSR*10
0	0,280 0,460	-1,479 0,340	1,151 2,522	- -	0,879 0,188	0,920 -	1,419 -	0,186 -	4,173 -
1 *	2,426 0,190	-1,516 0,336	-2,889 0,544	- -	0,756 -	0,902 -	1,262 -	0,185 -	5,135 -
2	0,818 0,524	-1,366 0,406	-0,550 0,754	- -	0,647 0,213	0,857 -	1,574 -	0,231 -	7,448 -
3	2,399 0,069	-1,475 0,503	-2,389 0,559	- -	- -	0,763 -	0,492 -	0,287 -	12,348 -
11 *	1,273 1,036	-0,485 0,417	- -	- -	0,937 -	0,742 -	1,599 -	0,290 -	13,459 -
12 *	2,369 0,184	-3,419 0,789	- -	- -	0,702 -	0,847 -	1,787 -	0,224 -	7,999 -
13 *	1,967 0,501	- -	-1,390 0,684	- -	0,873 -	0,772 -	1,748 -	0,272 -	11,866 -
14 *	0,940 1,178	- -	- -	- -	0,944 -	0,720 -	1,513 -	0,293 -	14,581 -
21	0,514 0,313	-1,424 0,391	- -	- -	0,771 0,125	0,852 -	1,914 -	0,227 -	7,731 -
31	2,417 0,100	-2,453 0,646	- -	- -	- -	0,474 -	1,243 -	0,414 -	27,405 -
32	2,410 0,069	-3,798 0,552	- -	- -	- -	0,747 -	0,727 -	0,287 -	13,187 -
33	2,355 0,082	- -	-3,136 0,604	- -	- -	0,627 -	0,884 -	0,349 -	19,442 -
34	2,335 0,131	- -	- -	- -	- -	- -	0,262 -	0,554 -	52,143 -
0T	0,686 0,493	-1,215 0,355	-0,231 2,495	-0,030 0,018	0,730 0,197	0,936 -	1,444 -	0,174 -	3,331 -
1T*	2,679 0,257	-1,362 0,337	-2,520 0,578	-0,060 0,039	0,773 -	0,916 -	1,577 -	0,177 -	4,404 -
11T*	2,517 0,142	-0,580 0,444	- -	-0,094 0,025	0,532 -	0,814 -	1,518 -	0,254 -	9,680 -
12T*	2,544 0,157	-2,707 0,792	- -	-0,063 0,029	0,644 -	0,883 -	1,933 -	0,202 -	6,122 -
31T	2,472 0,072	-0,935 0,587	- -	-0,070 0,017	- -	0,753 -	1,045 -	0,293 -	12,369 -
32T	2,445 0,064	-2,315 0,864	- -	-0,043 0,020	- -	0,805 -	0,804 -	0,260 -	10,176 -

EQ. NR.	CONST.	P	PLAG	TREND	XLAG, RHO	RSW	DW	SER	SSR*10
0	1,665 0,588	-1,003 0,312	1,253 2,358	-	0,476 0,178	0,886 -	2,598 -	0,219 -	5,768 -
1 *	3,356 0,102	-1,287 0,271	0,142 0,308	-	0,451 -	0,855 -	2,156 -	0,221 -	7,353 -
2	2,129 0,614	-1,135 0,342	0,409 0,400	-	0,363 0,189	0,835 -	2,016 -	0,244 -	8,366 -
3	3,300 0,074	-1,112 0,371	0,068 0,389	-	- -	0,792 -	1,084 -	0,265 -	10,564 -
11 *	3,349 0,098	-1,193 0,180	- -	-	0,450 -	0,853 -	2,208 -	0,216 -	7,457 -
12 *	3,268 0,091	-1,175 0,203	- -	-	0,323 -	0,792 -	2,316 -	0,257 -	10,544 -
13 *	3,188 0,089	- -	-1,011 0,205	-	0,145 -	0,677 -	2,136 -	0,320 -	16,407 -
14 *	2,931 0,495	- -	- -	-	0,832 -	0,575 -	2,581 -	0,356 -	21,599 -
21	2,377 0,565	-0,839 0,183	- -	-	0,277 0,170	0,823 -	2,011 -	0,245 -	8,992 -
31	3,294 0,063	-1,052 0,135	- -	-	- -	0,791 -	1,108 -	0,257 -	10,585 -
32	3,240 0,066	-1,072 0,152	- -	-	- -	0,757 -	1,423 -	0,273 -	12,326 -
33	3,183 0,077	- -	-1,012 0,179	-	- -	0,667 -	1,745 -	0,325 -	16,883 -
34	3,166 0,129	- -	- -	-	- -	- -	0,414 -	0,546 -	50,762 -
0T	1,703 0,642	-0,960 0,393	1,086 2,599	-0,012 0,059	0,469 0,188	0,887 -	2,596 -	0,229 -	5,748 -
1T*	3,465 0,121	-0,925 0,344	0,860 0,533	-0,098 0,061	0,490 -	0,878 -	2,528 -	0,211 -	6,218 -
11T*	3,362 0,104	-1,062 0,351	- -	-0,016 0,035	0,454 -	0,855 -	2,333 -	0,221 -	7,357 -
12T*	3,288 0,103	-0,821 0,754	- -	-0,030 0,064	0,302 -	0,795 -	2,411 -	0,263 -	10,400 -
31T	3,298 0,064	-0,770 0,432	- -	-0,026 0,037	- -	0,798 -	1,226 -	0,262 -	10,262 -
32T	3,270 0,078	-0,572 0,683	- -	-0,043 0,057	- -	0,766 -	1,541 -	0,281 -	11,879 -

EQ.NR.	CONST.	P	PLAG	TREND	XLAG,RHO	RSQ	DW	SER	SSR*10
0	0,649 0,111	-0,666 0,282	7,861 2,184	- -	0,367 0,119	0,952 -	2,211 -	0,109 -	1,428 -
1 *	0,932 0,151	-0,213 0,591	1,830 0,811	- -	0,571 -	0,648 -	1,628 -	0,265 -	10,546 -
2	0,449 0,114	-0,395 0,440	2,168 0,477	- -	0,558 0,118	0,831 -	2,620 -	0,190 -	5,043 -
3	0,936 0,074	-0,552 0,683	2,915 0,699	- -	- -	0,563 -	1,261 -	0,295 -	13,085 -
11 *	1,079 0,340	-0,595 0,585	- -	- -	0,802 -	0,581 -	1,592 -	0,280 -	12,541 -
12 *	0,925 0,257	0,305 1,124	- -	- -	0,728 -	0,557 -	1,463 -	0,288 -	13,261 -
13 *	0,916 0,132	- -	1,908 0,778	- -	0,544 -	0,645 -	1,764 -	0,258 -	10,631 -
14 *	0,955 0,265	- -	- -	- -	0,754 -	0,556 -	1,459 -	0,280 -	13,297 -
21	0,236 0,158	0,572 0,587	- -	- -	0,737 0,170	0,582 -	1,709 -	0,289 -	12,506 -
31	0,851 0,101	0,832 0,849	- -	- -	- -	0,057 -	0,597 -	0,420 -	28,239 -
32	0,864 0,084	2,308 0,848	- -	- -	- -	0,316 -	0,901 -	0,358 -	20,469 -
33	0,918 0,070	- -	2,640 0,604	- -	- -	0,544 -	1,126 -	0,292 -	13,655 -
34	0,871 0,099	- -	- -	- -	- -	- -	0,433 -	0,420 -	29,932 -
0T	0,734 0,121	-0,735 0,275	6,938 2,190	0,010 0,007	0,252 0,140	0,960 -	2,133 -	0,105 -	1,203 -
1T*	0,853 0,100	-0,466 0,582	1,815 0,728	0,037 0,019	0,344 -	0,712 -	1,874 -	0,248 -	8,626 -
11T*	0,777 0,199	-0,565 0,609	- -	0,059 0,033	0,620 -	0,628 -	1,473 -	0,272 -	11,125 -
12T*	0,762 0,155	0,489 1,074	- -	0,048 0,028	0,528 -	0,612 -	1,397 -	0,278 -	11,605 -
31T	0,796 0,085	-0,085 0,759	- -	0,051 0,017	- -	0,409 -	0,688 -	0,343 -	17,695 -
32T	0,811 0,080	1,295 0,893	- -	0,037 0,017	- -	0,481 -	0,957 -	0,322 -	15,532 -

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