92 Sosiale og økonomiske studier Social and Economic Studies

Hilde Christiane Bjørnland

Trends, Cycles and Measures of Persistence in the Norwegian Economy

Hilde Christiane Bjørnland

Trends, Cycles and Measures of Persistence in the Norwegian Economy

Statistisk sentralbyrå • Statistics Norway
Oslo-Kongsvinger 1995

Sosiale og økonomiske studier

Serien *Sosiale og økonomiske studier* omfatter nye forskningsbidrag – monografier og redigerte arbeider – på de områder Statistisk sentralbyrå har forskningsvirksomhet. Analysemetoder og temavalg vil variere, men hovedsakelig vil arbeidene være av anvendt og kvantitativ natur med vekt på utnytting av SSBs data i analyser for samfunnsplanleggingsformål og til allmenn forståelse av sosial og økonomisk utvikling.

Social and Economic Studies

The series *Social and Economic Studies* consists of hitherto unpublished studies in economics, demography and other areas of research in Statistics Norway. Although the studies will vary in analytical methods and in subject matter, they tend to be applied studies based on quantitative analysis of the data sources of Statistics Norway. The research programmes from which the studies originate typically emphasize the development of tools for social and economic planning.

ISBN 82-537-4220-7
ISSN 0806-2056

Emnegruppe

59 Andre samfunnsøkonomiske emner
Ny emnegruppe 1995: 09.90 Metoder, modeller, dokumentasjon

Emneord

Deterministiske trender
Konjunktursvingninger
Spektralanalyse
Stokastiske trender
Strukturelle brudd
Tidsserieanalyse
Varighet av sjokk

Design: Enzo Finger Design
Trykk: Falch Hurtigtrykk

Abstract

Hilde Christiane Bjørnland

Trends, Cycles and Measures of Persistence in the Norwegian Economy

Social and Economic Studies 92 • Statistics Norway 1995

This study analyses empirically the business cycles in Norway using quarterly national accounts from 1967 to 1994. To extract the cyclical component, we need to eliminate the trend component in the data. However, the cycle will not be invariant to whether we describe the trend as stochastic or deterministic.Testing for unit roots allows us to determine whether a series is best described by a stochastic trend or by a deterministic (linear) trend. By using a test for unit roots that allows for a structural break in the slope or the drift of the linear trend alternative, we can reject the unit root hypothesis for e.g. unemployment and investment, and instead describe them by a linear trend with one structural break. The structural break point is unknown a priori, and for unemployment and investment it is estimated to have occurred in the late 1980s. To extract cyclical components in the economic variables, we use a variety of stochastic and deterministic trend alternatives. The detrended data (the business cycles), are thereafter analysed both in the time domain and the frequency domain. In the time domain we concentrate on persistence and correlations, whereas in the frequency domain we establish whether the cycles we have estimated have any important periodic components. We then investigate whether the business cycle components are sensitive to the methods of trend extraction used. We find that for some variables, the measures of business cycles are qualitatively independent of the way we have extracted the trend, although quantitatively, the results may differ somewhat. For instance, analysing the correlations between GDP and other economic time series, we find that some variables are persistently procyclical (e.g. consumption, import, investment and productivity) or persistently countercyclical (e.g. unemployment). However, the magnitude of these correlations varies. For other variables, the business cycles vary considerably with the detrending methods used. For example, traditional exports, real wage and consumer prices show both a procyclical and countercyclical pattern, depending on how we define the trend component in these series. The sensitivity of business cycles to the measurement of the trend, implies that one should be careful not to extract the trend component without first examining the dynamic properties in the data.

Keywords: Unit roots, structural breaks, measures of persistence, trend-cycle decompositions, stylized facts, spectral analysis, time series analysis.

Acknowledgement: Financial support from the Norwegian Research Council is acknowledged.

Sammendrag

Hilde Christiane Bjørnland

Trender, konjunktursvingninger og varighet av sjokk i norsk økonomi

Sosiale og økonomiske studier 92 • Statistisk sentralbyrå 1995

I denne analysen bruker vi kvartalsvise nasjonalregnskapstall til å studere konjunktursvingninger i Norge fra 1967 til 1994. For å analysere konjunktursvingninger, må vi eliminere trenden i tidsseriene. Konjunktursvingningene vil avhenge av om en serie har en deterministisk (lineær) trend eller en stokastisk trend. Vi kan avgjøre om en serie har en lineær eller stokastisk trend ved å teste om en serie er integrert. Vi bruker en test for integrasjon hvor vi inkluderer et strukturelt brudd i stigningen eller nivået på det lineære trendalternativet. Tidspunktet for bruddet blir bestemt i estimeringsprosessen. Analysen viser blant annet at investering og arbeidsledighet kan bli beskrevet ved en lineær trend med et brudd sent på 1980-tallet. Flere metoder brukes deretter for å filtrere ut trenden i tidsseriene, og de filtrerte seriene dvs. konjunktursvingningene, blir analysert både i tidsdomenet og i frekvensdomenet. I tidsdomenet konsentrerer vi oss om varighet av konjunktursvingninger og korrelasjoner mellom to svingninger, mens i frekvensdomenet stadfester vi hvorvidt konjunktursvingningene vi har estimert har viktige periodiske komponenter. Vi analyserer deretter om konjunktursvingningene er sensitive ovenfor de måtene vi har filtrert ut trenden i seriene på. For noen av variablene finner vi at konjunktursvingningene er kvalitativt uavhengige av måten vi har eliminert trenden. Kvantitativt varierer imidlertid resultatene en del. Ved å analysere korrelasjoner mellom BNP og de andre tidsseriene finner vi for eksempel at noen variabler er vedvarende prosykliske (privat konsum, import, investeringer og produktivitet) eller vedvarende kontrasykliske (arbeidsledighet). Størrelsen på disse korrelasjonene vil imidlertid variere. For andre variabler er forløpet for konjunktursvingningene helt avhengig av hvordan vi definerer trenden. Vi finner for eksempel at eksport, pris og reallønninger kan være både prosykliske eller kontrasykliske avhengig av hvordan vi definerer trenden i disse variablene. Skal man analysere konjunktursvingninger bør man derfor være forsiktig å ikke fjerne trenden uten å undersøke de dynamiske egenskapene i tidsseriene først.

Emneord: Deterministiske trender, konjunktursvingninger, spektralanalyse, stokastiske trender, strukturelle brudd, tidsserieanalyse, varighet av sjokk.

Contents

1. Introduction*

'Just as waves following each other on the sea do not repeat each other perfectly, so economic cycles never repeat earlier ones exactly either in duration or in amplitude. Nevertheless, in both cases, it is almost always possible to detect, even in the multitude of individual peculiarities of the phenomena, marks of certain approximate uniformities and regularities.' (Slutsky 1937, p. 105).

Whereas economic researchers have been preoccupied with studies of *economic growth* for more than two centuries, analyses of *business cycles* have only been influential in the history of economic research in the twentieth century. In the ninetieth century, classical economists were mainly preoccupied with the concept of long term equilibrium. Regular cycles were not believed to exist and any short term instability (change in the growth rate) was interpreted in terms of random crisis[1]

The first comprehensive statistical analyses of economic cycles were published by Mitchell (1913, 1927) and Burns and Mitchell (1946). Business cycles were defined as the recurrent sequences of expansions, recessions, contractions, and revivals in the aggregate economic activity that lasted from more than one year, to ten to twelve years. A reference cycle would first be found, and then all other time series would be ordered according to the average lead and lag with regard to this reference cycle. The Burns and Mitchell methodology became advocated by the National Bureau of Economic Research (NBER) and was used in subsequent business cycle chronologies, e.g. Moore (1961) and Friedman and Schwartz (1963).

At the same time as the NBER was engaged with empirical methodology to analyse business cycles, Frisch (1933) had worked on a mathematical model formulating dynamic theories of the business cycle. In his model, Frisch distinguished between impulses and propagation

* The author would like to thank Per Richard Johansen, Knut Moum, Ragnar Nymoen, Terje Skjerpen and Anders Rygh Swensen for useful comments and discussions.

[1] However, some economists like Marx doubted the classical dichotomy of self adjusting markets, and argued for the existence of some sort of cyclical behaviour.

mechanisms in an economy. The impulses in the economy would be exterior stochastic shocks, that would trigger off dampening near regular (transitory) oscillations in the economic variables. The length of the cycle would be determined by the propagation structure of the system, whereas the amplitude, 'the intensity' of the fluctuations, would be determined primarily by the 'exterior cycle'. An economy would be interpreted to be in equilibrium until 'exterior impulses hit the economic mechanism' and set off near regular oscillations. Some of Frisch's basic ideas of the business cycle were taken up by Tinbergen a few years later in one of the first large scale macroeconometric models of the business cycle.[2]

By the end of the 1940s, the Burns and Mitchell methodology had come under severe criticism by Koopmans (1947) one of the leading econometricians at that time, for being measurement without theory. The development of the structural econometric models in the 1950s advocated by the Cowles Commission where Koopmans was a member, shifted the emphasis of macroeconomic research away from the study of business cycles, to the study of macroeconomic policies, required to reduce economic fluctuations. The empirical business cycle programme advocated by the NBER was eventually abolished, and not until the 1970s should empirical business cycle studies again be put on the research agenda on a large scale.[3]

The instability following the oil price shocks of 1973 and 1979 has stimulated a renewed interest in business cycle analysis. Following the recent success of the Real Business Cycle approach to generate artificial data on business cycles e.g. Kydland and Prescott (1982) and Long and Plosser (1983), several empirical studies have set out to present *stylized facts* (or broad regularities), of business cycles. Recent studies include Kydland and Prescott (1990) about the US, Blackburn and Ravn (1992) about the UK, Englund, Persson and Svensson (1992) about Sweden and Fiorito and Kollintzas (1994) about the G7.[4] In presenting their stylized facts, these analyses have followed Lucas' (1977) definition of business cycles as the *co-movement* between the deviations from trend (the business cycle) of gross national/ domestic product – and, the deviations from trend in various aggregate time series.

The instability of the duration and amplitude of the economic fluctuations over the last two centuries, has typically not given us a thorough understanding of what business cycles are like, in terms of their time series properties. As most time series are growing as well as fluctuating, one has to determine how to distinguish between the growth component and the cyclical component in the data. Most empirical analyses of business cycles prior to the early 1980s, saw the decomposition of a time series into a secular (trend) component and a cyclical component as a straightforward exercise. The economic mechanism underlying short- and long-run economic fluctuations would be quite different, and the cyclical and trend component could be studied separately. Output fluctuations were for instance seen as

[2] See Andvig (1981) and Morgan (1990) for a discussion of Frisch's and Tinbergen's contributions to business cycle analysis.
[3] On the lack of interest in business cycle analysis, see for instance Bronfenbrenner (1969).
[4] Koopmans (1947) criticism of empirical business cycle studies was discussed by Kydland and Prescott (1990), who successfully spurred interest in the presentation of stylized facts again.

temporary deviations from a smooth *deterministic trend*, that represented potential GDP. In this framework, the data would be easily detrended prior to analysis of the business cycles. Lesteberg and Wettergreen (1975), Kydland and Prescott (1980) and Blanchard (1981) typically adopted this view of business cycles.

Recent advances in time-series econometrics have seriously questioned this 'traditional view' about business cycles. One of the major debates in macroeconomic literature since the early 1980s has been whether macroeconomic (and financial) time series are better represented as a *random walk* than being stationary around a deterministic trend. Nelson and Kang (1981) showed that if one detrended data that were actually generated by a random walk, one would infer spurious cycles in the data. Nelson and Plosser (1982) brought further criticism to the traditional view, and argued that many macroeconomic time series like GNP, employment, prices, wages, money stock and interest rates were in fact better represented with a single autoregressive unit root (random walk) than as stationary fluctuations around a deterministic trend.

The failure of Nelson and Plosser (1982) to reject the hypothesis of a unit root in many macroeconomic time series had severe implications for further macroeconomic research. Instead of regarding output fluctuations as trend reverting, the existence of a unit root in the time series implies that a large fraction of stochastic shocks to output fluctuations would not die out. Each shock would have a permanent effect on the series, so for a pure unit root, all fluctuations would represent permanent changes in the trend rather than stationary fluctuations around a deterministic trend. In this sense, series with a unit root were said to contain a *stochastic trend* which would be sensitive to economic shocks and would evolve as a stochastic process. The series itself is nonstationary, but its first differences will be stationary. The presence of unit roots implied that the traditional trend and business cycle decomposition would be incorrect. Business cycles and the secular component could no longer be seen as separate and independent components, as the fluctuations in a series with a unit root would itself represent an accumulation of permanent shocks.

Nelson and Plosser's findings have spurred interest in questions about the long run effects (persistence) of macroeconomic shocks. Whereas shocks to a series that is stationary around a deterministic trend are only transitory, shocks to a random walk will have permanent effects and will therefore persist forever. Several studies like Watson (1986), Campbell and Mankiw (1987a, 1987b, 1989) and Cochrane (1988) have set out to measure the long-term effects of a shock/innovation in the level of the macroeconomic variables. A high degree of persistence was also taken to imply a relatively important permanent component in the time series. The relative magnitude of the permanent component in economic variables has remained controversial.

More recently, Perron (1989) and Rappoport and Reichlin (1989), have argued that the persistent effects of the shocks/innovations may have been severely exaggerated as economists have failed to take into account the fact that there may have been an important structural change in the trend. This structural change in the trend may be due to an episode like the oil price shock in 1973 which reduced the growth rate of industrial production for several years in most OECD countries. Hence, instead of arguing that time series are accumulations of a series of permanent stochastic shocks as for a random walk, time series may still display transitory fluctuations around a determinist trend, when one allow for a structural break in the trend.

These findings have motivated tests of the unit root hypothesis against the trend-stationary alternative where the deterministic trend is allowed to have a structural break (a single jump in the level of the trend) and/or a structural shift (change in the growth rate). Whereas the tests proposed by Perron (1989) require prior knowledge of the breakpoint, Banerjee, Lumsdaine and Stock (1992), Christiano (1992) and Zivot and Andrews (1992) have suggested tests of unit roots against a trend break/shift alternative that treat the break/shift point as unknown a priori. These tests find less support against the unit root hypothesis in favour of the break/shift in trend hypothesis than Perron (1989).

The introduction of the unit root hypothesis into economic literature was also thought to have serious implications for central macroeconomic questions. If aggregate demand disturbances like monetary shocks were assumed to be transitory, and the magnitude of the long-term effects of shocks to the economy was small, then the disturbances would most likely be in the form of transitory monetary disturbances. Traditional monetary models or Keynesian models that produce only temporary deviations from trend, would find their source of fluctuations from these types of aggregate demand disturbances. If, on the other hand, the magnitude of the long term effects was large, then most macroeconomic disturbances would be non-monetary. A large permanent component would instead imply that the source of economic fluctuations was due to (real) supply side disturbances such as technology shocks, a view emphasized by the Real Business Cycle approach advocated by Prescott (1986). However, more recently West (1988) has shown that several theoretical models other than the real business cycle approach can be compatible with a high degree of persistence in output.

Based on an altogether different argument, Quah (1990,1992) has argued that univariate characterisation of aggregate time series may be uninformative for economic theory. To be able to capture the rich dynamics of the multivariate world, multivariate models like Shapiro and Watson (1988) and Blanchard and Quah (1989) should be specified that consider among other things, long run and possible cointegration restrictions among variables. On the other hand, by using univariate models one can draw inferences from the time series models without explicitly having to test restrictions derived from e.g. economic theory that have been imposed on the data. Nevertheless, economic researchers have had to rely on either economic or statistical theories when documenting the stylized facts of business cycles, as is emphasized by Blanchard and Fischer (1989).

This study analyses empirically the economic fluctuations in quarterly national accounts in Norway using univariate statistical techniques. Given the uncertainty with regard to the appropriate trend representation in the series, the first chapters of this study are devoted to formal tests of unit roots and measures of persistence that can shed some light on the underlying dynamic processes in the data. The final chapters of this study calculate and present the stylized facts of business cycles in Norway, using both the stochastic and the deterministic trend alternatives. The results using the different trend alternatives are then evaluated in light of the statistical properties documented in the first chapters of this study. The stylized facts are presented both in the time domain and the frequency domain. In the time domain we focus on autocovariances and correlations, whereas in the frequency domain we interpret the power spectrum (or the power spectral density function). Here, an economic series that experiences fluctuations associated with business cycles will display a peak at the relevant business cycle frequencies in the spectrum. While these two methods are uniquely determined by each other, one may be more useful than the other in different circumstances. For instance, the autocovariances will give information on serial dependence in the variables, whereas the power spectrum tells us whether there are any important periodic components in the data.

Chapter 2 first sets out to explain the difference between a model that is stationary around a deterministic trend and a model with a stochastic trend (a unit root) where only the first differences are stationary. Some different unit root test procedures are then described. We first test for unit roots against the trend-stationary alternative, (the traditional augmented Dickey-Fuller test) and thereafter test for unit roots against the trend break/shift alternative using the procedure developed by Banerjee, Lumsdaine and Stock (1992). The different unit root tests are applied to thirteen Norwegian quarterly macroeconomic time series. For investment, government consumption, real wage and unemployment rate, we find strong evidence *against* the unit root hypothesis in favour of the trend break/shift alternative.

In chapter 3 we measure persistence in the series based on the infinite moving average of the first differences of the series and a nonparametric variance ratio. Although we are not able to distinguish between a trend-stationary and difference-stationary process based on this information, some findings nevertheless stand out. Generally, the Norwegian time series display little persistence, especially compared to other international series. Those series that show high persistence (investment, government consumption, unemployment, prices, M2 and oil prices), show a considerable fall in persistence when adjusted for a break in the trend or when allowed to be represented as integrated of second order, that is, they have to be differenced twice to be stationary. In this sense, the findings from chapter 2 are supported.

Chapter 4 describes and evaluates the different detrending methods that will be used in this study. Among the deterministic trends, we use polynomial functions of time and deterministic trends with structural breaks. Among the stochastic trends, we use the Hodrick-Prescott filter, which has been extensively used in the Real Business Cycle literature to evaluate the cyclical properties of artificial data against observed empirical fluctuations, and the Beveridge-Nelson decomposition, which is based on the Nelson and Plosser (1982) notion of stochastic trends. The final method is a frequency domain filter, where the trend is neither measured as deterministic nor as stochastic, but the business cycle is assumed to have a periodicity of about two to eight years. Each method is illustrated and examined through a set of figures of the trend and the cycle generated. In addition we examine the cyclical properties through the spectrum, where we concentrate on the contributions made by the various periodic components in the series.

In chapter 5 we analyse the stylized facts of the economic fluctuations in Norway based on the data we have detrended in chapter 4. The results are first presented in chapter 5.1 as a set of summary statistics in the time domain for the whole sample period 1967-1993, reporting volatility, persistence and comovements with GDP. For some variables (e.g. consumption, import, investment and productivity), the stylized facts are suggestive, indicating that the business cycles in these variables are positively correlated with the business cycle in GDP. The business cycle in unemployment on the other hand, is highly negative correlated with the cycle in GDP. For other variables, the results vary considerably with the decomposition method used. Especially for government expenditure, interest rates and oil prices, it is hard to establish what the stylized facts are at all. In chapter 5.2, we investigate the dynamic stability of the sample movements, where we concentrate on the comovement with GDP. Chapter 5.3 analyses the relations between two variables in the frequency domain, that is, we investigate the coherence.

As symmetries in economic fluctuations across countries are particularly important when these countries are to coordinate their economic policies, we further investigate business cycles across several countries in the time domain in chapter 5.4. This is done to see if the business cycles in Norway have been in phase with international business cycles. Chapter 6 concludes.

This study is similar in methodology to Blackburn and Ravn (1991) about the UK and Canova (1993) about the US, in that we explicitly consider several detrending methods when analysing the stylized facts of business cycles. However, this study differs significantly, in that we have applied a comprehensive analysis of the underlying dynamic properties of the time series, which we use extensively to discriminate between the results obtained using the different detrending methods. To our knowledge, some of the detrending methods presented here have not been applied to Norwegian quarterly national accounts before. However, for an application of the Hodrick-Prescott filter to Norwegian quarterly national accounts, see Swensen (1995). The present study is also the first paper to consider unit root tests on Norwegian quarterly national accounts, when the alternative is a trend with a break that is unknown prior to the testing procedure.

2. Deterministic trends and random walks

In empirical studies of time series, the researcher is confronted with the statistical problem of how one should represent an economic series. Most economic variables like output and prices, are characterized as nonstationary as they grow over time, hence they contain a secular (trend) component.[5] Most economic time series will in addition display cycles, that are most commonly referred to as business cycles. Although these cycles are not periodic, they are thought to be fairly regular. In addition to cyclical movements, the variables may exhibit seasonal movements which compared to business cycles are thought of as quite periodic, occurring at a frequency corresponding to say a year. For some variables the seasonal component will be the most important component and in some financial markets, like stock and exchange rate tradings, there may be several 'seasonal cycles' during a day. Finally, a time series may have an irregular component, (white noise). Whereas it will be hard to distinguish these components from each other if they interact multiplicatively, by taking the logarithm of the economic time series, the series can be represented as the sum of these components which can be modelled statistically:

$$\textit{Log (observed value)} = \textit{Trend} + \textit{Cycle} + \textit{Season} + \textit{White Noise}$$

Based on economic or statistical reasoning, several approaches have been put forward in the literature for numerically measuring the cyclical and growth component of the non stationary time series.

2.1 Measuring trends – some background

The traditional idea is that the economy is growing along a *smooth* growth path, but is being disturbed by cyclical fluctuations that have only transitory (non lasting) effects. A conventional way to study the economic movements is to think of the economy as being hit

[5] We use the term stationarity when a time series is weakly or covariance stationary. That is, when its mean and variance are independent of time, and the covariance between values of the time series at two time points will depend only on the distance between these time points and not on time itself. A stronger condition is imposed if a series is strictly stationary. If a time series is both weakly stationary and normally distributed, then it is also stationary in the strict sense, (see e.g. Harvey 1993, pp. 10-11).

by two types of shocks; shocks with permanent effects and shocks with transitory effects. Shocks with permanent effects will determine the trend, represented as the "natural" growth path in the economic series. These "permanent shocks" will typically be real factors like technology changes, capital accumulation and population growth. Shocks with temporary effects will on the other hand determine the cyclical movements. These fluctuations will be transitory, i.e. fade away over time. "Transitory shocks" will typically be monetary adjustments or government spendings.

In models that define the growth rate as a smooth deterministic trend, any (unanticipated) stochastic shocks will be interpreted as having only transitory effects, that eventually fade away. Fluctuations in output will reflect temporary deviations from trend only. Until recently, this way of accounting for fluctuations in output has been the main tradition in macroeconomics. The implications for these models are that they do not allow for unanticipated stochastic shocks to have permanent effects on the time series, since all cyclical movements are transitory by definition. Hence, any unanticipated permanent stochastic shocks would wrongly be attributed to the cycle. Nelson and Kang (1981) showed that to treat the secular component as deterministic when it is instead stochastic, may give spurious cyclical characteristics. Empirical investigation of the cycle would tend to over-estimate the persistence and variation of the cycle, whereas the importance of the real factors that influence the secular component would be under-estimated. Further, as these models imply that the long run path of the time series is deterministic, it is also perfectly predictable, (as an extrapolation of a linear regression on time). If instead the trend is represented as a stochastic function, then any forecast of the trend will diverge from the actual series over time and this deviation will grow without bound.

The introduction of unit roots into the literature, changed the traditional idea that economic fluctuations were stationary around a deterministic trend. Nelson and Plosser (1982) in their influential paper, failed to reject the null hypothesis that many macroeconomic variables were in fact better represented with an autoregressive unit root rather than stationary processes around a deterministic trend. The existence of a unit root in the time series implies that stochastic shocks to the time series will not eventually die out as in a trend reverting model, but will have permanent effects on the growth rate in the series. Depending on the model specified, stochastic shocks will affect the long run growth rate and the transitory component to varying degrees, and for a pure random walk, all fluctuations in the series will represent permanent changes to the secular component (growth rate).[6] A series with a unit root is said to contain a stochastic trend. In the Nelson and Plosser terminology, processes that are stationary after removal of a deterministic trend are defined as *trend-stationary* (TS). A nonstationary series that can be made stationary by first differencing is termed *difference-stationary* (DS). Generally, a nonstationary series can contain both a deterministic and a stochastic trend.

Below we discuss the implications of a random shock (an uncorrelated random innovation) to the TS and DS model. According to the traditional view of macroeconomic fluctuations

[6] As will be seen later, time series models can be specified that contains both a stochastic trend and a stationary component that are perfectly correlated, (Beveridge and Nelson 1981).

(advocated by e.g. Kydland and Prescott 1980 and Blanchard 1981), a time series y_t can be written as the sum of the trend component being here a polynomial of first degree in time, and a stochastic (cyclical) function c_t:

$$
\begin{aligned}
y_t &= \alpha_0 + \alpha_1 t + c_t \\
\phi(L)c_t &= \theta(L)\varepsilon_t
\end{aligned}
\tag{2.1}
$$

where we assume that $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator L of order p and q respectively so $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p = \Sigma\phi_i L^i$ and $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q = \Sigma\theta_i L^i$ where the lag operator L is such that $L^i\varepsilon_t = \varepsilon_{t-i}$. ε_t is a sequence of uncorrelated random innovations. For both the TS and DS model, we assume that $\theta(L)$ has roots strictly outside the unit circle, i.e. satisfying the condition of invertibility. The difference between the TS and the DS model can be seen by analysing the implications of a unit autoregressive root in the polynomial $\phi(L)$.

In the TS model, $\phi(L)$ has roots that are strictly outside the unit circle. That is, c_t is a stationary ARMA(p,q) process, and y_t is stationary around the deterministic trend $(\alpha_0+\alpha_1 t)$. For simplicity, assume that c_t follows an AR(1) process, which by backward substitution yields an infinite moving average representation of c_t:

$$
\begin{aligned}
c_t &= \phi c_{t-1} + \varepsilon_t \\
c_t &= \sum_{i=0}^{\infty} \phi^i \varepsilon_{t-i}
\end{aligned}
\tag{2.2}
$$

For stationarity, $|\phi|<1$ is required. Hence, when the process in (2.2) is stationary, the effect of a shock today dies out over time, and an innovation in the process will not change one's forecast of the process in the long run. This implies that stochastic shocks today will only generate transitory movements in c_t, and have no persistent or permanent effect in the future. As will be defined more formally in chapter 3, a model like (2.2) is said to generate zero persistence.

In the DS model, $\phi(L)$ will have one unit root at zero frequency, and all other roots will be strictly outside the unit circle, $\phi(L)=(1-L)\,\phi^*(L)$, where $\phi^*(L)$ has all its roots strictly outside the unit circle. c_t will now be represented by its first difference $\phi(L)c_t \equiv \phi^*(L)\Delta c_t$, which will be stationary.[7] For simplicity, assume $\phi^*(L) = 1$ so (2.2) reduces to a random walk which can be solved to yield:

[7] In the notation that follows we will use (1-L) and Δ interchangeably.

$$(1-L)c_t = \varepsilon_t$$

(2.3)

$$c_t = \sum_{i=0}^{t-1} \varepsilon_{t-i}$$

with c_0 taken to be zero. From (2.3) it can be seen that each shock ε_t will contribute its full value to c_t rather than its discounted value $\phi^i\varepsilon_{t-i}$ as in (2.2). Shock to a random walk will therefore not die out but will persist forever. In a model like (2.3), all fluctuations in c_t will be made up by permanent changes in c_t, and there will be no transitory movements.

A DS model that yields a comparison to the TS model in (2.1) can be defined by specifying y_t with a unit root, so the first differences of y_t will be stationary around a fixed mean. Subtract y_{t-1} from both sides of (2.1) and recall that when c_t contained one unit root at zero frequency we defined $\phi(L)=(1\text{-}L)\ \phi^*(L)$, where $\phi^*(L)$ has all its roots outside the unit circle:

$$(1-L)y_t = \alpha_1 + A(L)\varepsilon_t$$

(2.4)

$$A(L) = \phi^*(L)^{-1}\theta(L)$$

where A(L) is stationary by definition, as $\phi^*(L)$ and $\theta(L)$ satisfies the stationarity and invertibility conditions. More generally, Beveridge and Nelson (1981) showed that any difference-stationary process as (2.4) can be decomposed into a random walk (permanent) component and a stationary (transitory) component. Define $A^*(L)=(1\text{-}L)^{-1}[A(L)\text{-}A(1)]$, then (2.4) can be written as:

$$(1-L)y_t = \alpha_1 + [A(1) + (1-L)A^*(L)]\varepsilon_t$$

(2.5)

$$y_t = y_0 + \alpha_1 t + A(1)\sum_{s=1}^{t}\varepsilon_s + A^*(L)\varepsilon_t$$

The link between a TS model like (2.1) and a DS model as in (2.5), can now be seen clearly. Both the TS and the DS processes can be written as linear processes of time, with a deterministic trend component, $(\alpha_0 + \alpha_1 t)$ in the TS case (cf. (2.1)) and a deterministic trend component like $(y_0 + \alpha_1 t)$ in the DS case (cf. (2.5)). However, whereas the intercept in the TS model (α_0) is a fixed parameter, the intercept in the DS model (y_0) is a function of historical events. Further, whereas the deviations from trend in the TS model are stationary ARMA representations $[\phi\ (L)c_t=\theta(L)\varepsilon_t]$, the deviations from the deterministic trend in the DS model are accumulations of stationary innovations: That is, in (2.5) the cyclical component c_t consists of both a stochastic (permanent) component which has serially uncorrelated increments $A(1)\Sigma^t_{s=1}\varepsilon_s$, and a transitory component given by $A^*(L)\varepsilon_t$, which is stationary by definition of A(L). For a TS model, a current negative(positive) shock has no effect in the future, the growth rates of y_t will rise(fall) above the average growth rate for a few periods, until the trend line again is re-established. In the DS model, y_t is nonstationary, so there is no trend reversion in response to stochastic shocks.

A convenient way to distinguish between a (trend) stationary series and a nonstationary series, is to define the variables in terms of integration. A stationary series is said to be integrated of zero order I(0), whereas a nonstationary series that is only stationary after taking first differences, is known to be integrated of first order I(1). From (2.5) it can be observed that if $A(1)=0$, then y_t will be I(0). That is, y_t follows a covariance stationary ARMA process, (around a deterministic trend), whereas if $A(1)\neq 0$, y_t is I(1), where covariance stationary is first obtained when y_t is first differenced. Testing for unit roots then implies to test whether y_t is difference-stationary against the alternative that y_t is trend-stationary.

The Nelson and Plosser (1982) study concluded that macroeconomic variables like GDP, employment, prices and money in US were better characterized as having a unit root, rather than being trend-stationary. These findings were carried out using the then recent developments in econometric procedures, especially the Dickey and Fuller (1979) tests for autoregressive unit roots against the trend-stationary alternative. Following Nelson and Plosser's findings, the nature of macroeconomic fluctuations have been subject to an intense debate in the economic literature. More recently, unit root tests have been proposed that allow for infrequent *structural breaks* in the trend under the alternative hypothesis. In chapter 2.2, we discuss some test procedures for testing for unit roots especially against the alternative of stationary fluctuations around a trend with a break. The tests discussed are then applied to the Norwegian quarterly data in chapter 2.3 and 2.4.

2.2 Tests for the unit root and trend-break hypotheses

A common practice for determining the underlying process of a series has been to test the hypothesis that a process is a random walk against the alternative that the series is trend-stationary. Consider a regression model that is based on the AR(1) process as in (2.2) $y_t= \phi y_{t-1}+\varepsilon_t$, which can be rearranged to yield: $\Delta y_t=\mu y_{t-1}+\varepsilon_t$, where $\mu=\phi-1$. The true hypothesis is a test for unit root, that is a test for $\mu=0$, versus the alternative of stationarity where $\mu < 0$. This can be carried out by calculating a "t-statistic" on the OLS estimate on μ. However, the asymptotic distribution of the t-statistic under the null hypothesis of a unit root is non-Gaussian and will be downward biased. Without correcting for this bias, one can wrongly conclude that the series is stationary when it in fact is a random walk. The relevant critical values for the t-statistics were calculated by Monte Carlo simulations by Dickey and Fuller, and are reported in e.g. Fuller (1976).

The properties of the asymptotic distribution of the t-statistic will change when one adds a constant term and a time trend to the estimated regression model. However, the applicability of the models will also depend on what is known as the data generating process (DGP). Below we will add a constant term and a time trend in the estimated regression, $\Delta y_t = \alpha_0 + \alpha_1 t + \mu y_{t-1} + \varepsilon_t$, and assume the true process is a random walk with drift, $y_t = \alpha_0 + y_{t-1} + \varepsilon_t$. Critical values are again reported in Fuller (1976).[8]

One can further allow for more serial correlation in the residuals in the process, so y_t follows a higher order AR process, rather than an AR(1) process as above. A test for unit roots when the y-series follows an AR(p+1) process and allowing for both a constant and a time trend in the regression model can then be carried out by testing $\mu=0$, versus the alternative that $\mu<0$ in:

$$
(2.6) \qquad
\begin{aligned}
y_t &= \alpha_0 + \alpha_1 t + \sum_{j=1}^{p+1} \phi_j y_{t-j} + \varepsilon_t \\
\Delta y_t &= \alpha_0 + \alpha_1 t + \mu y_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta y_{t-j} + \varepsilon_t
\end{aligned}
$$

where $\mu = \Sigma_j^{p+1} \phi_j - 1$ and $\gamma_j = -\Sigma^p_{k=j+1}\phi_k$, $j = 1,2, \ldots p$. This generalisation is known as the augmented Dickey-Fuller test. It can be shown that the asymptotic augmented Dickey-Fuller distribution for μ in (2.6) is the same as the asymptotic augmented Dickey-Fuller distribution for μ in the AR(1) case including a constant and a trend, and the critical values can be obtained by Fuller (1976). Below we denote the general augmented Dickey-Fuller (ADF) t-test for the unit root hypothesis of $\mu=0$ against the trend-stationary alternative for t_{ADF}.

Nelson and Plosser (1982) and many other subsequent studies, have failed to reject the null-hypothesis of a single autoregressive unit root in several international macroeconomic variables. However, due to among others Schwert (1989), Christiano and Eichenbaum (1990) and Rudebusch (1992), it is now well known that the standard unit root tests have low power to discriminate between a DS and TS process, especially when the trend-stationary process has roots that are fairly close to unity. Further, as pointed out by Perron (1989), the standard tests for a unit root against the trend-stationary alternative fail to reject the unit root hypothesis if the true data generating process is a stationary process around a trend with one structural break. By using tests that allow for structural breaks, Perron (1989) and Rappoport and Reichlin (1989) found much less evidence of unit roots than had been previously reported. These findings have important implications. A series that is stationary around a trend with one structural break, will imply that only one stochastic shock has permanent effect on the series, rather than a whole series of shocks as in the DS model. Misspecifying a 'breaking trend' model as an integrated process would mean that one would attribute more persistence to innovations in the economic variables than might be the true case. Hence, incorrect conclusions may be drawn on the response of the economic variables to different shocks.

[8] Adding a constant term but no time trend in the regression when the null hypothesis is a random walk with drift, will make the estimated coefficients asymptotically Gaussian. The asymptotic distribution will be exactly the same as for the coefficients in a deterministic time trend regression like (2.1). The reason is that the time trend will asymptotically dominate the other components (for a textbook discussion of this issue, see e.g. Hamilton 1994, pp. 495-497).

Below we expand the ADF tests against the trend-stationary alternative by allowing the series to have a structural break in the trend. However, Perron's approach has been criticised by among other Christiano (1992) as being biased in favour of the structural break alternative, as they treated the break point as known a priori. Instead, we follow Banerjee, Lumsdaine and Stock (1992) and treat the possible break point as unknown in the time series.

Banerjee, Lumsdaine and Stock (1992) suggested three classes of test statistics: Recursive, rolling and sequential tests. Both the recursive and rolling test statistics can be carried out using (2.6). The idea is to test whether only parts of the series contain a unit root, and the other parts can be represented as a stationary process around a deterministic trend. Under the null hypothesis, $\alpha_1=0$ and $\mu=0$. Recursive and rolling test statistics have been frequently used in econometric analysis and are important tools for analysing stability of coefficient estimates over time, (for a list of relevant references, see Banerjee, Lumsdaine and Stock, 1992).[9] The *recursive* test extend the ADF test for unit root against the trend-stationary alternative ($\mu<0$) by recursively computing the ADF-statistics for μ. The test statistics are then computed using the subsamples $t=1,2...,k$ where $k= k_0,...,T$, k_0 is the initialization sample and the recursive tests are carried out until $t=T$, (the full sample). For the recursive test-statistics we report (1) $t^{rec}_{ADF\text{-}min}\equiv \min_{ko\leq k\leq T} t_{ADF}(k)$; The minimal t_{ADF} value over all recursively computed t_{ADF} statistics and (2) $t^{rec}_{ADF\text{-}max}\equiv \max_{ko\leq k\leq T} t_{ADF}(k)$; The maximal t_{ADF} value over all recursively computed t_{ADF} statistics. For the *rolling* test statistics we extend the ADF test by computing the test statistic from subsamples that are a constant fraction of the full sample, $t=k-(k_0)+1,...k$, $k=k_0,...,T$. For the rolling test statistics we compute; (1) $t^{rol}_{ADF\text{-}min}\equiv \min_{ko\leq k\leq T} t_{ADF}(k)$; The minimal t_{ADF} value over all the rollingely computed t_{ADF} statistics and (2) $t^{rol}_{ADF\text{-}max}\equiv \max_{ko\leq k\leq T} t_{ADF}(k)$; The maximal t_{ADF} value over all the rollingely computed t_{ADF} statistics.

The testing strategy in the *sequential test*, is to test the null hypothesis of a unit root against the alternative hypothesis that the series is stationary around a deterministic time trend with a one time change occurring at an unknown point in time. The change in the trend is either modelled as a single *shift* in the trend (change in the growth rate in the trend) (case A below) or as a single *break* in the trend (change in the level of the trend) (case B below).[10] The test is computed sequentially using the full sample. Consider a random walk with drift model for the unit root null hypothesis:

[9] Another example of a rolling statistic is presented in chapter 5.2, where we calculate several correlation coefficients between two series using a fixed part of the sample which we shift forwards a period each time. These «rolling» correlations are used to analyse the stability of the stylized facts of business cycles over time.

[10] Zivot and Andrews (1992) have also developed an asymptotic distribution theory for a 'breaking trend alternative' that is quite similar to that of Banerjee, Lumsdaine and Stock (1992), but they also allow for a «third alternative», both a shift and a break in the trend at the same time period. To keep the exposition simple, we decided to follow Banerjee, Lumsdaine and Stock (1992) and considered only either a shift or a break in the trend. However, for some variables we will test for both a break and a shift at the same time in the trend.

(2.7) $$y_t = \alpha_0 + y_{t-1} + \varepsilon_t$$

The two trend stationary alternative hypotheses (with either a break or a shift in the trend) can be summarized using the following equation:

(2.8) $$y_t = \alpha_0 + \alpha_1 t + \alpha_2 DU_t(k) + \varepsilon_t$$

where $DU_t(k)$ is a dummy variable that captures the possible change in the trend at period k. In case (A) (change in the slope (or shift) in the trend), $DU_t(k)$ is specified as:

(2.8a) (A) $DU_t(k) = t - k$ if $t > k$, 0 otherwise.

whereas in case (B) (change in the level (or break) in the trend), $DU_t(k)$ is specified as:

(2.8b) (B) $DU_t(k) = 1$ if $t > k$, 0 otherwise.

To test for a unit root versus the alternative of a change in the trend can now be estimated by using the following regression:

(2.9) $$\begin{aligned} y_t &= \alpha_0 + \alpha_1 t + \alpha_2 DU_t(k) + \sum_{j=1}^{p+1} \phi_j y_{t-j} + \varepsilon_t \\ \Delta y_t &= \alpha_0 + \alpha_1 t + \alpha_2 DU_t(k) + \mu y_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta y_{t-j} + \varepsilon_t \end{aligned}$$

where $\mu = \Sigma_j^{p+1} \phi_j - 1$ and $\gamma_j = -\Sigma^p_{k=j+1}\phi_k$, $j = 1,2, ...p.$ as specified in (2.6), and $DU_t(k)$ is defined as above. We compute three test-statistics for cases A and B. For case A, we define; (1) $F^A_{DU\text{-}k^*} \equiv \max_{ko \le k \le T\text{-}ko} F(k)_{DU}$; The maximum F-value for testing the null hypothesis, $\alpha_2=0$ over all sequentially computed F-statistics, (2) $t^A_{ADF\text{-}k^*}$; The $t(k)_{ADF}$ value corresponding to the k-value (k*) chosen by the maximum F-statistic $F^A_{DU\text{-}k^*}$ and (3) $t^A_{ADF\text{-}min} \equiv \min_{ko \le k \le T\text{-}ko} t(k)_{ADF}$; The minimal t_{ADF} value over all sequentially computed t_{ADF} statistics. Similar statistics are computed for case (B), where the statistics are now indexed by B instead of A. Finally, in the cases where we cannot reject the null hypothesis of a unit root, we define a case B', where we restrict case B so that $\mu=0$ and $\alpha_1=0$, and test whether there has been a single shift in the *mean growth* rate. In case B' we compute (4) $t^{B'}_{DU\text{-}k^*}$, which is the minimum absolute t-statistics on the coefficient on $DU_t(k)$.

The finite sample critical values and the empirical size and nominal power of the test statistics described above, are established by Monte Carlo simulation in Banerjee, Lumsdaine and Stock (1992). Except for the rolling max statistics, $t^{rol}_{ADF\text{-}max}$, all tests reported there have sizes near their levels when the true model is a Gaussian AR(1). When analysing the power against changing AR coefficients in the middle of the sample, (the largest root is one in half of the sample and less than one in the other half), all tests reported there performed well, especially, the recursive test $t^{rec}_{ADF\text{-}min}$, for a changing coefficients in the first half of the sample, (which is not surprising as it includes the initial observations). However, for the nominal power against a trend shift, the traditional t_{ADF} test and the maximum recursive $t^{rec}_{ADF\text{-}max}$ and rolling $t^{rol}_{ADF\text{-}max}$ tests all fail to reject the unit root null against the trend shift alternative. The minimum recursive $t^{rec}_{ADF\text{-}min}$ and rolling $t^{rol}_{ADF\text{-}min}$ tests statistics have somewhat higher power. The sequential F-statistics, $F^{A,B}_{DU\text{-}k*}$ perform very well, and have high power against both the trend shift and the trend break alternative. The sequential t-statistics, $t^{A,B}_{ADF\text{-}k*}$, $t^{A,B}_{ADF\text{-}min}$ have also relative high power against both alternatives. Below we test for whether the unit root can be rejected in favour of the trend break/shift hypotheses as a suitable model for the underlying process of aggregate dynamic behaviour in the Norwegian economy, bearing in mind the properties of the test statistics reported above. Finally, we conclude by performing sensitivity analysis, to investigate the robustness of the tests.

2.3 Empirical evidence

All empirical analyses in this chapter use seasonally adjusted quarterly Norwegian national accounts from 1967Q1 to 1994Q1, except for the interest rate where the sample is from 1971Q4 to 1994Q1. A definition of the variables with their respective abbreviations are presented in table 2.1. Appendix A gives a further description of the variables used in this analysis. All tests statistics are computed using RATS.

In table 2.2a we report the full-sample ADF test, together with the rolling and recursive 'subsample' ADF tests corresponding to model (2.6). Table 2.2b reports their critical values. In table 2.3a we report the sequential tests corresponding to the estimated regression in (2.9) and table 2.3b reports their critical values. All calculations are based on fourth order autoregression (p=4) using seasonally adjusted data. To investigate whether the results are sensitive to the choice of p, we recalculate the sequential test statistics in the next section using p=8. Finally, we will perform the tests using data that are not seasonally adjusted. We follow Banerjee, Lumsdaine and Stock (1992) and set the trimming parameter for the recursive statistics k_0 = 26 (which corresponds to 25 pct. of the sample), for the rolling test statistics, k_0= 35 (1/3 of the sample) and for the sequential tests, k_0= 16 (15 pct. of the sample).

Using the standard full sample unit root test t_{ADF}, we cannot reject the unit root null for any of the variables at the 10 pct. level, although for import, the unit root hypothesis can be rejected at the 20 pct. level, (see table 2.2a). However, we can reject the hypothesis that all variables are integrated of second order I(2), against the hypothesis that they are I(1), (these results are not reported here). Except for the unemployment rate and real wage, the recursive and rolling test statistics give little evidence against the unit root/ no break hypothesis, and the unit root null hypothesis can not be rejected at the 10 pct. level. However, for the unemployment rate, the minimum recursive test $t^{rec}_{ADF\text{-}min}$ rejects the unit root null hypothesis at the 5 pct. level and for real wage, the unit root hypothesis is rejected at the 2.5 pct. level by the minimum rolling test, $t^{rol}_{ADF\text{-}min}$. As both the rolling and recursive tests statistics have relative little power against a trend shift alternative, we now turn instead to the sequential test statistics.

Table 2.1. Variables and definitions[1]:

Series:	Definition:	Series:	Definition:
GDP	Gross Domestic Product	U	Unemployment rate
C	Private Consumption	RWG	Real Wage
G	Government Consumption	CPI	Consumer Price Index
I	Investment	M2	Money Supply, M2
X	Traditional Export	R	3-months interest rate
M	Traditional Import	OP	Oil price in Norwegian currency
PR	Productivity		

[1] All variables are measured in logs, except for the unemployment rate and the interest rate that are measured in levels.

Table 2.2a. Augmented Dickey-Fuller, recursive and rolling unit roots tests[1]

Series	ADF	Recursive ADF		Rolling ADF	
	t_{ADF}	$t^{rec}_{ADF\text{-}min}$	$t^{rec}_{ADF\text{-}max}$	$t^{rol}_{ADF\text{-}min}$	$t^{rol}_{ADF\text{-}max}$
GDP	-1.36	-2.88	-0.52	-3.87	-0.37
C	-2.20	-3.20	-0.47	-3.84	-0.35
G	-0.23	-2.38	1.79	-3.33	1.33
I	-0.76	-3.74	-0.13	-3.64	0.89
X	-2.50	-3.28	-0.61	-4.12	-1.21
M	-3.04	-3.20	-0.21	-3.20	-0.61
PR	-1.57	-3.26	-0.56	-4.31	-0.32
U	-2.30	-4.34[b]	0.19	-4.28	-0.77
RWG	-1.64	-2.98	-0.79	-5.44[a]	1.17
CPI	0.42	-3.24	0.42	-3.35	0.19
M2	1.57	-2.34	1.72	-3.32	0.17
R	-0.91	-3.66	-0.91	-4.66	-1.08
OP	-0.98	-3.08	0.79	-3.53	0.78

[1] For a definition of the variables, see table 2.1.

[a] Rejection of the unit root hypothesis at the 2.5 pct. level

[b] Rejection of the unit root hypothesis at the 5 pct. level

Table 2.2b. Critical values for ADF, recursive and rolling unit roots tests

Percentage	ADF	Recursive ADF		Rolling ADF	
	t_{ADF}	$t^{rec}_{ADF\text{-}min}$	$t^{rec}_{ADF\text{-}max}$	$t^{rol}_{ADF\text{-}min}$	$t^{rol}_{ADF\text{-}max}$
2.5	-3.73	-4.62	-2.21	-5.29	-1.66
5.0	-3.45	-4.33	-1.99	-5.01	-1.49
10.0	-3.15	-4.00	-1.73	-4.71	-1.31

The critical values for the full sample Augmented Dickey-Fuller statistic were taken from Table 8.5.2 in Fuller (1976) table 8.5.2 p. 373. The critical values for the rolling and recursive statistics were taken from Banerjee, Lumsdaine and Stock (1992) table 1 p. 277, where the values are computed by Monte Carlo simulations for T=100 and p=0, (none of the tests have distributions that depend on p).

Despite the fact that the sequential tests have higher power against the break/shift alternatives than the recursive and rolling tests, only for government consumption and unemployment are there substantial evidence against the null hypothesis of a unit root. Nevertheless, there is some evidence for some other variables that there has been a trend break (or shift) during the estimation period, although in most cases it is not significant enough to reject the unit root hypothesis. For most of the variables, the maximum F-statistics, F_{DU-k*} and the minimum t-statistics $t_{ADF-min}$, suggest the same break or shift point.

From table 2.3a, we conclude that for the *trend-shift alternative,* the F-statistic, F^{A}_{DU-k*}, is significant at the 2.5 pct. level for prices, at the 5 pct. level for government consumption and at the 10 pct. level for oil prices, M2 and interest rates. Based on the t-statistics, t^{A}_{ADF-k*} and $t^{A}_{ADF-min}$, the unit root null hypothesis can only be rejected against the stationary trend-shift alternative at the 10 pct. level for government consumption. For the *trend-break alternative,* the F-statistic, F^{B}_{DU-k*}, is significant at the 10 pct. level for export, at the 5 pct. level for M2 and at the 2.5 pct. level for investment and unemployment. However, the unit root/no- break null hypothesis can only be rejected at the 5 pct. level against the stationary trend break alternative for the unemployment rate based on t^{B}_{ADF-k*} and $t^{B}_{ADF-min}$.

Hence, there seems to be clear evidence against the unit root hypothesis only for the unemployment rate and government expenditures, although there are some evidence of changing coefficients of some form for many variables. Further, the rejection of the unit root hypothesis for real wage based on the rolling unit root test is not confirmed here by the sequential unit root tests. For the series where we can not reject the unit root hypothesis, we finally test whether we can reject the notion of a constant drift rate in favour of a shift in the drift rate in the series, (case B'). From table 2.3a, it can be seen that the null-hypothesis of constant drift is rejected in favour of a shift in the drift for investments and M2 at the 2.5 pct. level, and for prices at the 10 pct. level.

In the next chapter we analyse whether the result presented above is sensitive to the choice of p, (the number of AR lags used in the regression) and the seasonal adjustment procedure applied. By the end of the chapter, we also investigate whether some variables that showed evidence of both a change in the slope and the level of the trend, can be specified with both a shift and a break in the trend at the same time. The test procedure used there is that of Zivot and Andrews (1992).

Table 2.3a. Sequential unit roots tests[1,2]

Series:	(A) Shift in trend (change in slope) k^*	$F^A_{DU\text{-}k*}$	$t^A_{ADF\text{-}k*}$	$t^A_{ADF\text{-}min}$	(B) Break in trend (change in level) k^*	$F^B_{DU\text{-}k*}$	$t^B_{ADF\text{-}k*}$	$t^B_{ADF\text{-}min}$	(B') Shift in intercept if I(1) k^*	$t^{B'}_{DU\text{-}k*}$
GDP	1986Q1	6.09	-2.84	-2.84	1988Q1 (88Q2)	11.59	-3.38	-3.45	1986Q3	-2.46
C	1986Q1	9.07	-3.78	-3.78	1988Q2	14.04	-4.34	-4.34	1986Q3	-2.20
G	1982Q1	18.44[b]	-4.25[c]	-4.25[c]	1975Q2	9.09	-2.31	-2.31	-	-
I	1986Q3	11.19	-3.31	-3.31	1988Q2	25.90[a]	-4.02	-4.02	1988Q1	-3.71[a]
X	1982Q4	9.30	-3.89	-3.89	1974Q2 (74Q3)	16.33[c]	-3.97	-4.22	1974Q2	-1.60
M	1986Q1	2.04	-3.31	-3.31	1988Q2	8.11	-4.23	-4.23	1986Q3	-1.15
PR	1984Q4	6.00	-2.93	-2.93	1986Q2 (72Q2)	5.03	-2.60	-2.70	1975Q3	-2.70
U	1986Q2	6.23	-3.40	-3.40	1988Q2	19.36[b]	-4.87[b]	-4.87[b]	-	-
RWG	1977Q4	8.45	-3.30	-3.30	1973Q3	11.7	-3.66	-3.66	1978Q1	-1.90
CPI	1987Q2	19.25[a]	-3.65	-3.65	1988Q2	6.01	-0.83	-0.83	1988Q2	-3.07[a]
M2	1987Q4	16.29[c]	-3.10	-3.10	1988Q2 (89Q3)	18.78[b]	-0.81	-1.00	1988Q2	-5.61[a]
R	1986Q4	15.51[c]	-3.92	-3.92	1979Q1	9.99	-2.76	-2.76	1982Q3	-2.06
OP	1981Q3	14.08[c]	-3.84	-3.84	1985Q3	9.86	-2.96	-3.11	1981Q3	-2.17

[1] For a definition of the variables, see table 2.1.

[2] k* indicates the break date suggested by $F^{A,B}_{DU\text{-}k*}$. The break date suggested by $t^{A,B}_{ADF\text{-}min}$ is given in parenthesis below if different from k*.

[a] Rejection of the unit root hypothesis at the 2.5 pct. level

[b] Rejection of the unit root hypothesis at the 5 pct. level

[c] Rejection of the unit root hypothesis at the 10 pct. level

Table 2.3b. Critical values for sequential unit roots tests

Percentage	(1) Shift in trend $F^A_{DU\text{-}k*}$	$t^A_{ADF\text{-}k*}$	$t^A_{ADF\text{-}min}$	(2) Break in trend $F^B_{DU\text{-}k*}$	$t^B_{ADF\text{-}k*}$	$t^B_{ADF\text{-}min}$	(3) I(1) Break $t^{B'}_{DU\text{-}k*}$
2.5	19.15	-4.76	-4.76	20.83	-5.07	-5.07	3.40
5.0	16.30	-4.47	-4.48	18.62	-4.80	-4.80	3.13
10.0	13.64	-4.19	-4.20	16.20	-4.52	-4.54	2.84

The critical values for the Sequential test statistics were taken from Banerjee, Lumsdaine and Stock (1992) table 2 p. 278, where the values are computed by Monte Carlo simulations for T=100 and p=0, (none of the tests have distributions that depends on p).

2.4 Sensitivity results

In this chapter we perform sensitivity analysis of the results above. We first investigate the choice of p, (the number of AR lags). In the above analysis, we chose p=4 as a base value when analysing quarterly data. However, among others, Schwert (1989) has shown that the results of the unit root tests will be sensitive to the serial dependence in the error term, and higher order AR lags may be more appropriate for capturing the serial correlation in the data. Also, an extra number of regressors will not affect the size of the unit root tests, although its power may decrease. To investigate whether the above results have been sensitive to the choice of p, we recalculate the ADF- tests and the sequential test ($F_{DU\text{-}k^*}$ and $t_{ADF\text{-}k^*}$), using p=8.[11] The results are shown in table B.1 in appendix B.

The conclusions from the ADF-tests for one unit root are essentially unchanged with p=8, although some of the coefficients have varied. However, we can no longer reject the hypothesis that prices and M2 are I(2) at the 2.5 pct. level and that investments is I(2) at the 10 pct. level (these results are not reported). Based on p=8, the result of the sequential tests changes for some variables, and in some cases, the break/shift points (k*) are shifted one or two quarters forwards or backwards, (for real wages it is shifted one year backwards). Most important, we can now reject the unit root hypothesis for investment and real wage in favour of the *trend shift* alternative at the 5 pct. and 2.5 pct. level respectively. The trend shifts occurred in 1986Q3 for investment (as was suggested in table 2.3) and in 1976Q4 for real wage. We can still reject the unit root hypothesis for the unemployment rate against the *trend break* alternative, although now only at the 10 pct. level. The results for government consumption is unchanged, although the trend shift now occurs two quarters later, namely in 1982Q3. Some of the statistics reported above have become less significant, and $F^B_{DU\text{-}k^*}$ for exports and $F^A_{DU\text{-}k^*}$ for CPI, oil prices and interest rates are no longer significant at the 10 pct. level.

Recent studies, (see Jaeger and Kunst 1990, Ghysels 1990 and Ghysels and Perron 1993 among many others), have examined the power of unit roots tests when the data have been seasonally adjusted. It is well known that summation procedures that attenuate the high frequencies may infer some spuriousity at the low frequencies. They concluded that seasonal adjustment procedures often create an upward bias of persistence and reduces the power of the tests of the unit root null hypothesis, creating a bias towards non-rejection of the unit root hypothesis. As all data in the analysis above have been seasonally adjusted using the common moving average X11-ARIMA procedure, (see appendix A), we recalculate the findings using p=8, when the data are not seasonally adjusted. The results are presented in table B.2 in appendix B.

The conclusions based on the ADF tests reported for p=8 are basically unchanged from when the same data were seasonally adjusted.[12] The results for the sequential tests for p=8, for the unadjusted variables are also essentially unchanged from when the data were seasonally adjusted, although the break/shift points (k*) for some variables are shifted one or two quarters forwards or backwards. For real wage, the trend shift point is shifted one year forward again, (to 1977Q4).

In the above analysis we tested for either a break or a shift in the trend. As mentioned above, Zivot and Andrews (1992) allowed for a third possibility, namely both a change in

[11] For a choice of p less than 4, the Ljung-Box Q -statistics show significant evidence of serial correlation in the residuals.

[12] We have also calculated the ADF tests for p=4 when the data are not seasonally adjusted, but the results are unchanged from the ADF tests for p=4 using seasonally adjusted data, except for import where we find that we can reject the unit root-null against the trend-stationary hypothesis at the 10 pct. level. The test statistics are not reported here.

the level (break) in the trend and a change in the slope (shift) in the trend in the same time period. Given that for some variables we have seen evidence of both a change in the slope and the level of the trend, we test for both a break and a shift in the trend at the same time for private consumption, government consumption, investment, export, GDP, real wage and unemployment. Only for investment could we reject the unit root hypothesis in favour of the trend break and shift alternative, when the change in the slope and the level of the trend was estimated in 1985Q4. However, the trend shift and break alternative is only significant at the 10 pct. level, whereas the trend shift alternative found in table B.1 for investment is significant at the 5 pct. level.[13]

To sum up, for none of the variables can we reject the unit root hypothesis in favour of a deterministic linear trend altogether, although for import, we are close to rejection of the unit root hypothesis. Prices and M2 may either be represented as I(2) or as I(1) with a reduction in the drift. When adjusting for a break or a shift in the trend, for unemployment rate, government consumption, investment and real wage, we can reject the hypothesis of a unit root.

In some international analysis of output using *yearly data* from about 1870 to 1985 on several countries,[14] Raj (1992) and Serletis (1994) have used the test procedure proposed by Zivot and Andrews (1992) to test for breaking trend functions. Both could reject the hypothesis of a unit root against the breaking-trend alternative for most countries, but not for Norway, Sweden, Australia and Italy according to Raj (1992) and Norway and Sweden according to Serletis (1994).

In figure 2.1-2.4, we have graphed unemployment, investment, real wage and government consumption with their estimated trend functions respectively. The unemployment rate may be represented as stationary fluctuations around a trend with break, when the break occurred in 1988Q2. Government expenditures, investment and real wages may be represented with a trend with shift, when the shift occurred in 1982Q1 (or 1982Q3) for government consumption, 1986Q3 for investment and in 1976Q4 for real wage.

For government consumption and real wage, the growth rate was higher in the period before the shift than after the shift. For investment, the trend is negative after 1986, indicating that investment has fallen regularly on a stable trend since 1986. (Using the alternative representation for investment defined by Zivot and Andrews (1992), where both the trend shift and break alternative was estimated to 1985Q4 above, gives a negative growth rate after 1985).

[13] The minimum t statistics for both a change in the slope and the level in the trend at same time $t^{A\&B}_{ADF-min}$, equals -4.85 for investments whereas the critical value that rejects the null hypothesis of non-stationarity at the 10 pct. level calculated by Zivot and Andrew (1992, table 4 p. 257) is -4.82. For real wage, $t^{A\&B}_{ADF-min}$ = -4.80, hence it is just below the 10 pct. critical value, and we can not reject the unit root hypothesis.

[14] The sample consists of Australia, Canada, Denmark, France, Germany, Italy, Norway, Sweden, UK and US.

Figure 2.1. Unemployment with estimated trend

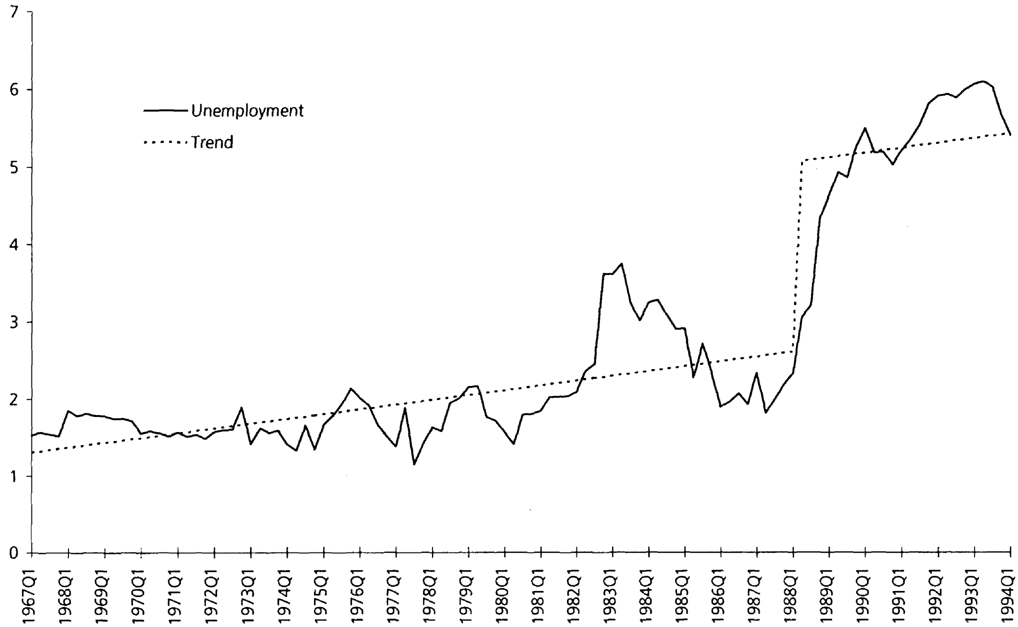


Figure 2.2. Investment with estimated trend

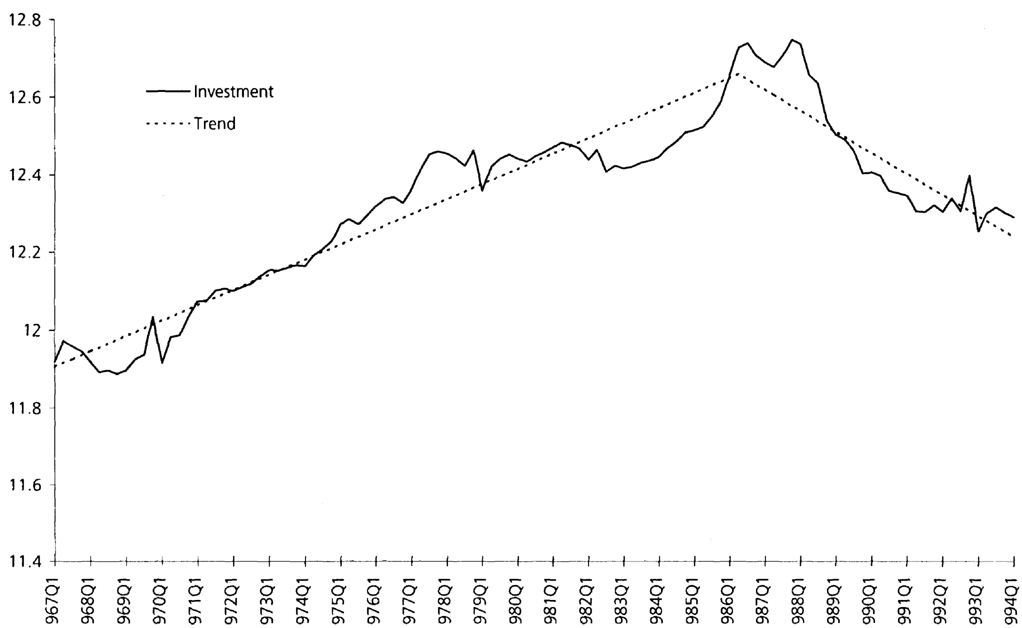


Figure 2.3. Real wage with estimated trend

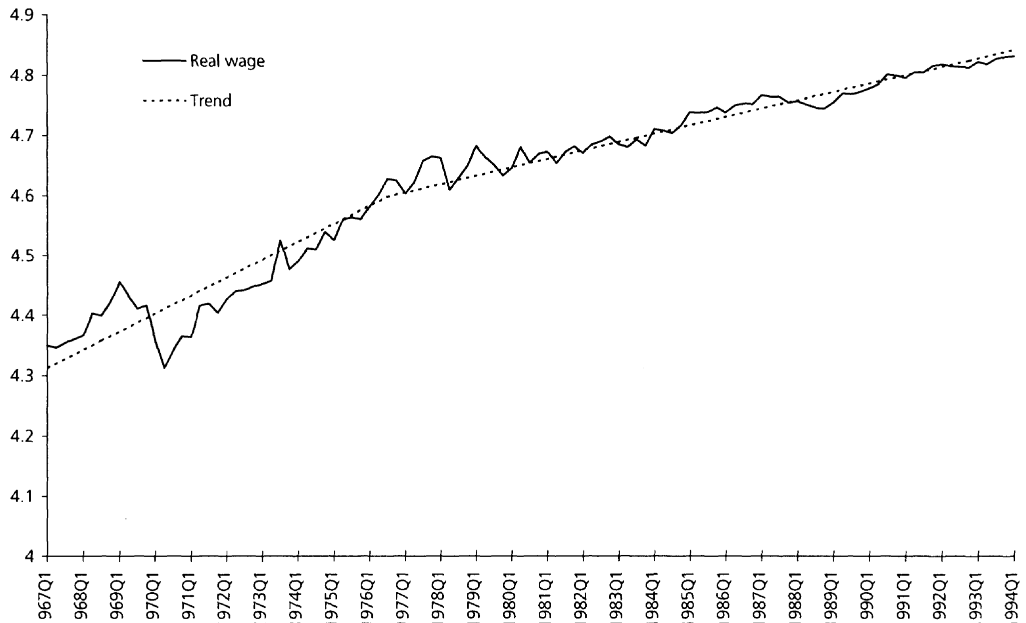
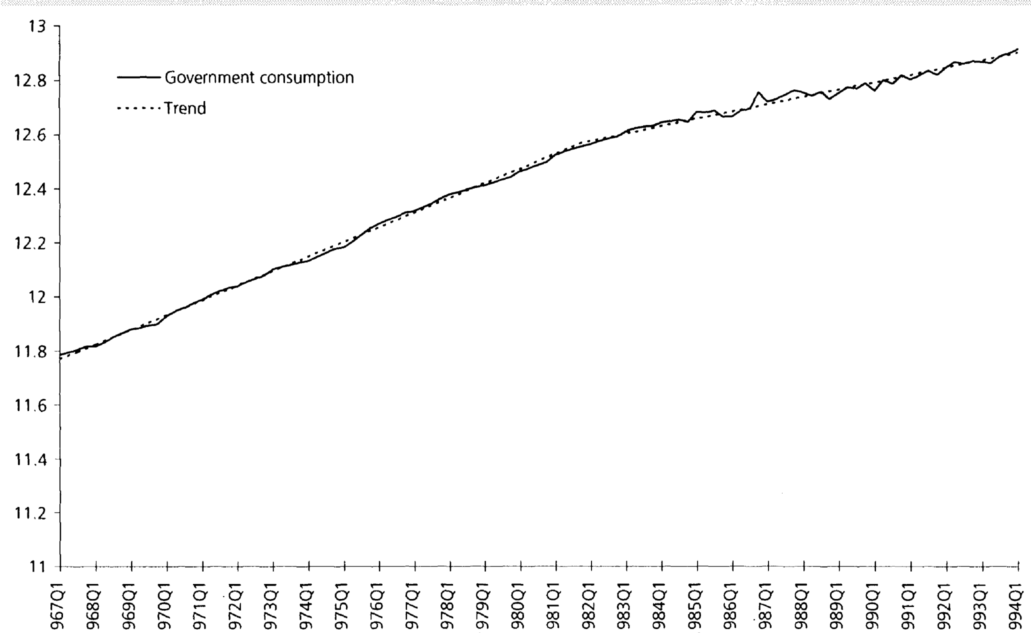


Figure 2.4. Government consumption with estimated trend



Several periods may have been important in explaining these breaks/shifts. The lowering of the growth rate for investment and the upward shift in the unemployment rate, both occurred in a period of financial crisis and recession in the late 1980s. The preceding years had been characterized by a hugh consumption and investment boom, that was primarily set off by the financial deregulation in the middle 1980s. The shift in the trend for real wages may be understood on the background that in 1976, working hours pr. week were reduced from 42.5 hours to 40 hours. Norway used also income policies and direct price controls on several occasions in 1975 and from 1978 -1981. The lowering of growth rates in government consumption coincides with the change in government policies from 1982, when the conservative party took over after several years of labour government.

Finally, three points of caution must be noted. In the above framework we have assumed that there has been one large break/shift in the series. Most of the tests reported above have higher power against larger breaks/shifts than against smaller breaks/shifts. Obviously, they may not detect a situation where the break is small, but nevertheless may have had permanent effect on the series. Secondly, we have only testet for one break in the series. For Norway, the most important breaks seemed to occur in the late 1980s, coinciding with the period of financial deregulation. However, the oil price shock of 1973 may also have had an effect on the series, and for e.g. export, there is an indication of a break both in the early seventies and eighties. Thirdly, whereas the sequential F-test has high power against the alternative especially when the break/shift occurred at the end of the sample, the sequential ADF-tests have higher power the earlier the break/shift occurred. For most of the variables analysed in this sample, the F-tests indicate that the break occurred in the latter 40 pct. of the sample, and the sequential ADF-tests may have too little power to detect the breaks there.

All this points in the direction that there may be more series that can be represented with a change in the trend than what we have suggested above. In the above framework we tested for breaks/shifts in trends when we could not reject that the underlying process was I(1). Given the low power of the ADF unit roots test, in some of the cases it would be better to test for breaks/shifts in the trend without pretesting for unit root. However, for the purpose of this paper, the analyses above suffices.

Another way of tackling the issue of distinguishing between a unit root and a trend-stationary process, has been to analyse *how* much long-term forecasts respond to initial shocks, that is *how* persistent the effect of shocks to these macroeconomic time series are. If the long term response is zero, the series will be characterized as trend stationary, whereas if the long term response is one-to-one, the series will be characterized as a pure random walk. Any number in between zero and one implies that there will be some trend reversion, whereas a number above one indicates that a series will continue to diverge from its previously forecasted value. Following the influential work of Nelson and Plosser (1982), much research in the macroeconomic literature has been devoted to establish measures of the magnitude of the persistence of shocks to macroeconomic variables, e.g. Campbell and Mankiw (1987a, 1987b, 1989). Another way of interpreting persistence is due to Cochrane (1988) and Watson (1986). They showed that when the models could be measured as a combination of a stationary component and a random walk, the random walk would carry the permanent part, and an interpretation of a measures of persistence would be to measure how big is the *random walk* in the series. Below we set out the various measures of persistence, also when we again consider the alternative to be a stationary process with a breaking trend.

3. Measures of persistence in the series

In order to analyse persistence it is useful to start from the moving average representation of a model similar to (2.4). When the first differences of y_t are stationary, they can be given by a infinite moving average *difference-stationary Wold representation:*

$$\Delta y_t = \alpha_1 + A(L)\varepsilon_t$$

(3.1)
$$A(L) = \sum_{i=0}^{\infty} A_i L^i, \quad A_0 = 1$$

where the ε_t's are uncorrelated random innovations with variance σ^2, and $\Sigma_i i \, |A_i| \langle \infty$. The impact of an innovation (or shock) in period t on the *growth rate* in period t+k, is A_k, since it corresponds to that part of the growth rate of y_t that can not be predicted from univariate information at time t-1. Further, the impact of a shock on the *level* of y in period t+k, (y_{t+k}) would be $1 + A_1 + .. + A_k$. The ultimate impact of a shock, will be the infinite sum of these coefficients. Campbell and Mankiw (1987a) suggested that the sum of the moving average coefficients, A(1), would be a good measure of persistence as it measures the ultimate effect to an immediate effect of a shock to the variables. A(1)=1 for a random walk, and shocks will persist forever. If A(1)>1, the ultimate effect is larger than the immediate effect, and shocks tend to explode. If A(1)<1, the ultimate effect is less than the immediate effect, and shocks will tend to die out. In the limiting case for a stationary series around a deterministic trend, A(1) will equal zero. By estimating finite stationary ARMA models to the first differences of the series as in (2.4), where $A(L) = \phi^*(L)^{-1}\,\theta(L)$, the coefficients of A(1) can be calculated directly by estimating $A(1) = \phi^*(1)^{-1}/\theta(1)$ through the parameters in the $\phi^*(1)$ and $\theta(1)$ polynomial.

Cochrane (1988) proposed another measure of persistence that could be estimated non-parametrically. The idea is that if y_t follows a random walk $y_t = y_{t-1} + \varepsilon_t$, with ε_t again defined as uncorrelated random innovations with variance σ^2, the variance of the k-differences of y_t grows linearly with the variance of the innovation ε_t so: $var(y_t - y_{t-k}) = k\sigma^2$. If instead y_t is defined as a stationary process (i.e. after removing a linear trend),

$y_t = \phi y_{t-1} + \varepsilon_t$, the variance of its k-differences approaches a constant twice the unconditional variance of the series: var $(y_t - y_{t-k}) \rightarrow 2\text{var}(y_t) = 2\sigma^2/(1-\phi^2)$. Cochrane (1988) suggested that 1/k times the ratio of the variance of the k period differences to the variance of the one period differences would be a good measure of persistence, which could also be written as the sum of autocorrelations of Δy_t:

$$V^k = \frac{1}{k}\frac{\text{var}(y_t - y_{t-k})}{\text{var}(y_t - y_{t-1})} \tag{3.2}$$

$$= 1 + 2\sum_{j=1}^{k-1}(1 - \frac{j}{k})\rho_j$$

where $\rho_j \equiv C_j/C_0 \equiv \text{cov}(\Delta y_t, \Delta y_{t-j})/\text{var}(\Delta y_t)$ is the jth autocorrelation of the process, C_j denotes the jth autocovariance of the process and C_0 denotes the variance of the process. In the limit, V^k provides a natural measure of persistence, as it can be expressed as the two side infinite sum of autocorrelations, $V \equiv \lim_{k\rightarrow\infty} V^k = \Sigma^{\infty}_{j=-\infty} \rho_j$, where now V equals 1 if y_t is a random walk, and converges to zero if y_t is stationary.

A(1) and V can be related by linking autocorrelations to moving average coefficients, (see Harvey 1993, pp. 28-29 for a proof), so $C(z) = A(z)A(z^{-1})\text{var}(\varepsilon)$, where $C(z) = \Sigma_{-\infty}^{\infty} C_j z^j$ is the autocovariance-generating function and var(ε) is the variance of a univariate innovations to a differenced process as in (3.1). Hence, $V = A(1)^2\ \text{var}(\varepsilon)/\text{var}(\Delta y)$. V is the lower bound of A(1), and only for two processes, the stationary process and a random walk, will A(1) and V be the same.

A(1) and V can also be interpreted in terms of the relative importance of the permanent component (random walk) in any decomposition of a series into a permanent component with serially uncorrelated disturbances, and a transitory component. Further, the variance of the permanent component in any such decomposition can be identified from the spectral density of the increments in the original series. More generally, based on the Beveridge and Nelson (1981) decomposition, we showed in (2.5) that by defining $A^*(L) = (1-L)^{-1}[A(L)-A(1)]$, any difference-stationary Wold representation as in (3.1) can be decomposed into a random walk (permanent) component and a stationary (transitory) component:

$$y_t = y_0 + \alpha_1 t + A(1)\sum_{s=1}^{t}\varepsilon_s + A^*(L)\varepsilon_t \tag{3.3}$$

From (3.3) it is easy to see that the trend is now made up of a deterministic component, $(y_0 + \alpha_1 t)$ but also a stochastic (permanent) component $A(1)\Sigma^t_{s=1}\varepsilon_s$. The transitory component is given by $A^*(L)\varepsilon_t$. A high degree of persistence implies that the movements in the stochastic trend itself account for a large proportion of the movements in the variable, as the current observation of the variable is made up by the summation of the historical disturbances. The higher the degree of persistence, the smaller will the business cycle be. From the definition of spectral density, we have that $S_{\Delta y}(e^{-i0}) = (1+ 2\Sigma_1^{\infty}\rho_j)$ var (Δy_t) (see e.g. Hamilton 1994, pp. 152-155). Hence, the innovation variance of the stochastic trend g_t defined as $g_t \equiv A(1)\Sigma_{s=1}^{t}\varepsilon_s$, will equal ($2\pi$ times) the normalised spectral density of Δy_t at frequency zero:

$$(3.4) \qquad \text{var}(\Delta g_t) = A(1)^2 \text{var}(\varepsilon_t) = V \text{var}(\Delta y_t) = S_{\Delta y_t}(e^{-i0})$$

V can now be interpreted as the innovation variance of the random walk component divided by the variance of the total change in the economic variable. A(1), can be defined as the standard deviation of the innovation in the random walk component divided by the standard deviation of the univariate innovation to the economic variable. Hence, A(1) (and V) can be used to capture the random walk component, or the unit root. Cochrane (1988) also showed that any I(1) process that can be represented as the Wold moving average representation in (3.1) (which is a decomposition into a random walk and a stationary component), will have the innovation variance of the random walk component given by (3.4), independent of whether the permanent and transitory components are correlated or not.

In the next chapter, we turn to the issue of measuring persistence, as defined above. It is now well known, that in finite samples, the variance ratio or any estimate of the spectral density at the origin may have poor properties to distinguish between a TS and a DS process. Essentially, as Cochrane (1991) points out, any TS process can be approximated arbitrarily well by a unit root process, in the sense that the autocovariance structure will be arbitrarily close. No aspects of the autocorrelation $(1\text{-}L)y_t$ other than their infinite sum or no aspects of the periodigram ordinates other than at frequency zero provide us with information about whether a series is trend-stationary or difference-stationary. A test for a pure random walk against the alternative trend-stationary can nevertheless be distinguished by adding some restrictions. Cochrane (1988) assumes that the slope of the spectral density is small in a region near zero, so that evidence from ordinates other than zero can provide evidence about its value at the frequency zero.

On the other hand, Quah (1992) has argued that the measures of persistence presented above, will not identify the *magnitude* of the permanent component. Regardless of the magnitude found of V at frequency zero, Quah (1992) shows that the permanent component in every integrated time series can be taken to be arbitrarily smooth, (so increments to the permanent component have arbitrarily small variance) and that the transitory component will dominate the time series. This decomposition is possible even when the permanent component and transitory components are uncorrelated at all leads and lags. Only for a random walk, can the interpretation of the spectral density at frequency zero indicate the size of the permanent component. In light of these criticisms, the results below should be interpreted as an indicator of the underlying dynamics in the series, rather than providing a precise magnitude of the permanent component and a further attempt to categorize the series into trend-stationary or difference-stationary.

3.1 Estimation

A(1) is found directly by estimating ARMA models for the first differences of y, thereby establishing the coefficients of the θ's and the ϕ's. However, dynamic misspecifying the Wold representation (3.1), may lead to incorrect estimates of A(1) and some care should be taken with regard to the ARMA specifications of the first differences of y_t. Cochrane (1988) argues that fitting low order ARMA models to the first differences of output, gives too much weight to the short run dynamics and too little weight to the long run dynamics. In this sense they fail to capture the behaviour of output as they systematically over-estimate the permanent component in the observed series. Cochrane (1988) shows that output in fact does return to a trend after a shock, but that this trend reversion occur several years after the initial shock, implying that the short run properties of output will be consistent with a model with persistent shocks. Fitting a time series model to the short run properties will therefore incorrectly infer a great deal of long run persistence. Intuitively, this seems reasonable as by modelling short run dynamics, we may ignore high order statistically (insignificant) autocorrelations. Also, if there are a set of positive autocorrelations at the short lags and a small random walk component at the long lags, the maximum likelihood will match the short-run behaviour but misrepresent the long run behaviour, (see Cochrane 1988, for a further discussion of this issue and the discussion in appendix E with regard to estimating ARMA models).

A nonparametric estimator of V^k in (3.2) can be found by replacing the population autocorrelations (ρ_j) with the sample autocorrelations, (r_j). As k increases with the sample size, the estimator (written as $\hat{V}$) consistently estimates V. With sample autocorrelations in place of population correlations in (3.2), $\hat{V}^k$ is asymptotically equal to the Bartlett estimator of the spectral density at frequency zero with its standard error given by, (Priestley 1982, p. 463):

$$\text{(3.5)} \qquad \text{S.E.}[\hat{V}^k] = (\hat{V}^k)\sqrt{\frac{4}{3T/k}}$$

In small samples, $\hat{V}^k$ can be biased and the asymptotic standard errors may incorrectly estimate the actual standard errors. For a random walk with drift, the mean value of $\hat{V}^k$ is approximately (T-k+1)/T rather than 1. To correct for this downward bias for a random walk, we follow Cochrane (1988) and Campbell and Mankiw (1987a), and multiply r_j with T/(T-k+1). Note nevertheless, simulations in Cochrane (1988) showed that when the series have a small random walk component or are trend-stationary, there may be an upward bias in $\hat{V}$ as an estimate of the random walk component.

In finite samples, the appropriate k has to be chosen. Although a high k is preferable, choosing a too high k may give excess trend reversion, as when k get closer to the sample size the estimator will approach zero. If on the other hand a too low k is chosen, too few autocorrelations will be included, and patterns of trend reversion found in the higher autocorrelations will not be detected. In a Monte Carlo study of the behaviour of $\hat{V}^k$, Campbell and Mankiw (1987b) found that in a sample of 130, k must be at least 30, and preferable 40 and 50 if they should be able to discriminate between a random walk process and a stationary AR(2) process. However, the appropriate choice of the k that will distinguish trend-stationary and difference-stationary properties will vary between the variables. Perron (1993) and the references he sites, argue that the exact mean square error of the estimated V^k is minimised using a large value of k when V is small and a small value of k, when V is large. For this reason we consider a set of values of k corresponding to k=10, 20, 40 and 60.

Perron (1993), emphasized that when a series is stationary around a trend with a shift (2.8a) the sample autocorrelations will not consistently estimate the population autocorrelations, (although for a break in the trend (2.8b), the population autocorrelations are correctly estimated). To correct for this bias, we follow Perron (1993) and estimate a set of values of V^k for k=10, and 40, where we use Δx_t instead of Δy_t in the formula for V^k in (3.2) above, where Δx_t is defined as the residual in the following expression:

$$(3.6) \qquad \Delta y_t = \alpha_1 + \alpha_2 DU_t + \Delta x_t$$

where $DU_t = 1(t>k^*)$. (Note that (3.6) is written in this form so that under the hypothesis of a shifting trend, (3.6) corresponds to: $y_t = \alpha_0 + \alpha_1 t + \alpha_2 DB_t + \text{noise}$, where now $DB_t = (t-k^*)1(t>k^*)$ which corresponds to case A in chapter 2.2). k^* is the estimated date found in chapter 2. We pick all dates for k^* from case A in table 2.3a where we use four AR lags (p=4) in the estimation procedure, except for real wages where the dates of the most significant shift in the slope of the trend is found in table B.1 where we use eight AR lags (p=8) in the estimation procedure. For the other variables, the estimated dates do not change much whether we use p=4 or p=8. The test is denoted $\hat{V}^k_{DU}$.

3.2 Empirical evidence

In table 3.1, we estimate persistence defined by $\hat{V}^k$, $\hat{V}^k_{DU}$ and A(1) truncated for two different ARIMA models used later in the Beveridge-Nelson decomposition in chapter 4.6 (see also appendix E). We first report the results for $\hat{V}^k$ and the corresponding standard errors calculated by (3.5) for k= 10, 20, 40 and 60. Based on these results, GDP, consumption, export, import, productivity and real wage show little evidence of persistence, all having most of its values well below unity. Unemployment has values of $\hat{V}^k$ that fluctuates around one, whereas the interest rate has values of $\hat{V}^k$ that reaches above one from k=40. Oil prices, government consumption, investment, CPI and M2 all show considerable evidence of persistence with $\hat{V}^k$ increasing rapidly with k.

Although the estimates of A(1) show values that differ slightly from some of the values of $\hat{V}^k$, they do not turn around the main findings supported by the V-ratio, and government consumption, investment, oil prices, and especially M2 and CPI have values of A(1) above one. Unemployment indicates values both above and below one. Note that the low-order ARIMA models produce a higher value for A(1) than the high order ARIMA models do, except for productivity, real wages and investment where they show little difference. This supports what we noted above, that fitting low order ARIMA models may give too much weight to the short run dynamics and too little weight to the long run dynamics for some variables.

Most of the variables that have values of $\hat{V}^k$ in excess of one, (interest rates, government consumption, investment, oil prices, CPI and M2) are also those variables that in chapter 2 showed either evidence (based on the t or F statistics) of being represented by a linear trend with a shift (investment, government consumption, interest rates and oil prices) or an I(2) or I(1) process with changing drift, (CPI and M2). When correcting for the bias caused if the variables where represented with a deterministic trend with shift, persistence measured by $\hat{V}^k_{DU}$ is falling below one for government consumption, interest rates and investment for all values for k, and for oil prices for k>10. Although for real wage we could also reject the unit root hypothesis in favour of a deterministic trend with shift in chapter 2, the values for $\hat{V}^k$ are below one already before we correct for this shift. Adjusting for the shift in the trend, the values for $\hat{V}^k_{DU}$ falls even further. However, in contrast to the other variables that experience a shift in the trend in the middle 1980s, real wage has a significant shift point early in the sample, (1976). For most of the sample, the variable is thereafter stationary around a deterministic trend, which is appropriate captured by the autocorrelation structure of $\hat{V}^k$. Unemployment was found to be trend-stationary around a breaking trend (case B in chapter 2.2). Allowing for a shift in the trend here reduces nevertheless $\hat{V}^k_{DU}$ somewhat, but only for k>10 is $\hat{V}^k_{DU}$ below unity. Only CPI and M2 have values of $\hat{V}^k_{DU}$ above one for all k, although M2 now has values close to one.

Another interesting feature is the fact that, when there are no adjustment for a shift in the trend, the lowest value for $\hat{V}^k$ (for k=40 or k=60) among all the variables, is found for import. Given that in chapter 2 import was the variable that showed most evidence of being stationary around a deterministic trend, the results from chapter 2 seem to be somewhat confirmed here. Further, for the import series, the pattern for $\hat{V}^k$ and $\hat{V}^k_{DU}$ as k increases are almost identical.

To sum up, productivity, real wages, GDP and all its components show little evidence of persistence, as all have values of $\hat{V}^k$ or $\hat{V}^k_{DU}$ that are well below unity. However, based on $\hat{V}^k$ and $\hat{V}^k_{DU}$ it is difficult to establish which of these series are better represented as trend-stationary or difference-stationary processes. Nevertheless, two findings can be summarized from table 3.1 and the discussion above. First, the variable that shows some evidence against the unit root hypothesis in favour of the linear trend hypothesis based on the ADF unit root tests in chapter 2, (import), is the same variable that has the lowest value for $\hat{V}^k$ for k=40 or k=60 of all variables in table 3.1. Second, those values that have the highest persistence measured by $\hat{V}^k$ in table 3.1, (unemployment, government consumption, interest rates, investment, oil prices, CPI and M2), are the same variables that supported the trend shift/break alternative or where better represented as I(2) or I(1) with a drift in chapter 2. Correcting for the bias of having a shift in the trend, for the variables that

Table 3.1. Measures of persistence[1]

Series	$\hat{V}^k$ k=10	$\hat{V}^k$ k=20	$\hat{V}^k$ k=40	$\hat{V}^k$ k=60	$\hat{V}^k_{DU}$ k=10	$\hat{V}^k_{DU}$ k=40	A(1) High-order	A(1) Low-order
GDP	0.42	0.45	0.50	0.67	0.30	0.19	0.66	0.72
	(0.15)	(0.22)	(0.35)	(0.58)	(0.15)	(0.14)		
C	0.77	0.73	0.71	1.05	0.63	0.31	0.89	0.99
	(0.27)	(0.36)	(0.50)	(0.90)	(0.22)	(0.22)		
G	0.76	1.26	2.31	3.19	0.25	0.12	1.20	NA
	(0.27)	(0.62)	(1.62)	(2.73)	(0.08)	(0.08)		
I	1.62	1.93	2.39	3.44	0.92	0.47	1.20	1.17
	(0.57)	(0.95)	(1.67)	(1.88)	(0.32)	(0.33)		
X	0.62	0.49	0.54	0.54	0.59	0.80	0.60	NA
	(0.22)	(0.24)	(0.38)	(0.46)	(0.21)	(0.56)		
M	1.03	0.75	0.38	0.47	1.01	0.37	0.66	NA
	(0.36)	(0.37)	(0.26)	(0.40)	(0.35)	(0.26)		
PR	0.30	0.34	0.54	0.80	0.25	0.37	0.57	0.46
	(0.10)	(0.17)	(0.38)	(0.68)	(0.09)	(0.26)		
U	1.43	1.18	0.97	1.22	1.23	0.61	0.87	1.26
	(0.50)	(0.58)	(0.68)	(1.05)	(0.43)	(0.43)		
RWG	0.51	0.49	0.46	0.56	0.44	0.26	0.94	0.83
	(0.18)	(0.24)	(0.32)	(0.48)	(0.16)	(0.18)		
CPI	4.84	7.30	12.02	14.87	3.09	3.59	3.62	4.43
	(1.69)	(3.61)	(8.41)	(12.74)	(1.08)	(2.51)		
M2	3.37	5.36	8.44	11.70	1.33	1.03	3.81	4.07
	(1.18)	(2.65)	(5.90)	(10.02)	(0.47)	(0.72)		
R	0.55	0.61	1.01	1.44	0.36	0.14	0.51	NA
	(0.19)	(0.30)	(0.70)	(1.24)	(0.13)	(0.10)		
OP	1.35	1.55	2.09	2.71	1.07	0.56	1.31	NA
	(0.47)	(0.76)	(1.46)	(2.32)	(0.38)	(0.40)		

[1] For a definition of the variables, see table 2.1.

rejected the unit root in favour of a shift in the trend, persistence measured by $\hat{V}^k_{DU}$ falls considerably. Persistence falls also somewhat for unemployment when correcting for a shift in the trend, although unemployment was found to be best represented with a break in the trend (case B) in chapter 2. In the end, only CPI and M2 show clear evidence of persistence as $\hat{V}^k_{DU}$ exceed one for all k.

It is interesting to examine estimates of persistence and trends with breaks/shifts for other countries. Campbell and Mankiw (1989) estimate V^k using quarterly data for GDP (or GNP) in the G7 countries. All countries except UK show considerable persistence. Corrected for small sample bias, estimated V^k (k=60) for UK equals 0.85 whereas in Japan, estimated V^k =13.71 for k=60. However, although UK output is less persistent than the other G7 countries, it is no clear evidence that output is stationary around a deterministic trend and Dickey-Fuller tests at the 10 pct. level fail to reject the null hypothesis of a unit root. Further, Banerjee, Lumsdaine and Stock (1992) found that those countries that experienced the highest variance ratio, where those countries where the hypothesis of a deterministic trend with break/shift was accepted in favour of the unit root-hypothesis, (Japan, France and Germany). For the countries with lowest variance ratio, (UK and US), there were no evidence against the unit root null.

Finally, in an international study of persistence using *yearly* data from 1871-1985, Cogley (1990) found point estimates of V^k ($k=20$) close to 1.4 for GDP in Norway. USA had the smallest point estimate in the sample, with V^k about 0.5. The largest point estimate was found for Italy and France, where V^k varies between 1.8 and 2.0.

4. Trend-cycle decompositions

Below, we will present six univariate decompositions that yield stylized facts of business cycles. In the first two, the series are modelled as stationary cycles around a deterministic trend, where the trend is either a polynomial function of time (of first or second degree), or a deterministic trend with break. The three next decompositions are stochastic in nature. The first, the Hodrick-Prescott filter is an exponential smoothing procedure, which has been heavily used in the RBC literature to evaluate the simulated business cycles to the cycles in the real world. The second and third method are based on Nelson and Plosser (1982) notion of stochastic trends. The trend is now either modelled as a pure random walk which is uncorrelated with the cycle or found using the Beveridge-Nelson procedure where the cycle and trend are perfectly correlated.[15] The final method is based on frequency domain filtering, where it is not essential whether the trend is stochastic or deterministic in nature.

For the respective first five decompositions, we consider:

$$(4.1) \qquad y_t = g_t + c_t$$

where g_t is the trend and c_t is the cycle. We use seasonally adjusted data, based on the X-11 ARIMA methodology, which is the most commonly internationally used procedure for seasonal adjustment. Hence, the seasonal component of y_t is already filtered out, which simplifies several of the procedures, and makes the result comparable to other international studies that uses the same seasonal adjustment procedure. In the above representation, any noise component left in the seasonally adjusted data will be attributed to the cycle. The data and their abbreviations are described in table 2.1 above, and their sources and definitions are described in appendix A.

[15] A more general approach than using the ARIMA models is the unobserved components models, see e.g. Harvey (1989) and Watson (1986). Here each series is modelled as the sum of a set of uncorrelated components that each are parameterised as an ARIMA representation. For an application of unobserved components models to Norwegian quarterly data, see Skjerpen (1995).

For the sixth method, the frequency domain filtering method, we use the following decomposition:

$$(4.2) \qquad y_t = g_t + c_t + s_t + \varepsilon_t$$

where again g_t is the trend and c_t is the cyclical component, but now we use unadjusted data. Hence, we specify the seasonal component s_t in the model, and the noise component ε_t, is also specified explicitly in the model. Below we describe each method, and plot GDP with the trend and cycle (and noise) generated for each method. The cyclical component for a set of other series; consumption, investment, productivity, real wage, unemployment, CPI and M2, calculated using a linear trend with break, the Hodrick-Prescott filter, a random walk, the Beveridge-Nelson method and the frequency filtering technique, are displayed in appendix C. All test statistics are computed using GAUSS and RATS.

In addition to analysing the cyclical component in the time domain, we will also investigate the cyclical component in the frequency domain. Spectral or frequency analysis may provide a more useful way of analysing the business cycles properties since the frequency domain concentrates on the contributions made by the various periodic components in the series. Below we will therefore use spectral analysis to investigate the cyclical components that have been generated by the different detrendings techniques. However, most cycles in economic variables are rather irregular and will only be recognised as they have a tendency to occur at certain frequencies, (see Harvey 1993). The spectra of the cyclical component of GDP, real wage and CPI, calculated using a linear trend with break, the Hodrick-Prescott filter, the Beveridge-Nelson method and the frequency filtering technique, are displayed in appendix D. We start this chapter with some definitions which are central in spectral methods.

4.1 Spectral analysis, some definitions

A Fourier transform $f(\omega)$ expresses a stationary series y_t as the sum of cyclical components of different frequencies ω. The power spectrum (or power spectral density function, or spectrum) $S(\omega)$ is defined from the Fourier transform of the autocorrelations in the time series:

$$(4.3) \qquad S(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} e^{-i\tau\omega} C(\tau)$$

where $C(\tau)$ is the autocovariances of y_t, $C(\tau) = \mathrm{cov}(y_t, y_{t-\tau})$. From the assumption of wide sense stationarity of y_t, it follows that $S(\omega)$ will be both real and positive. As $e^{-it\omega}$ is periodic with period 2π, $S(\omega)$ will have the same periodicity. Further, as $S(-\omega) = S(\omega)$, it suffices to estimate $S(\omega)$ over the interval $[0,\pi]$. The period is inversely proportional to frequency. Inverting (4.3) yields:

$$(4.4) \qquad C(\tau) = \int_{-\pi}^{\pi} e^{i\tau\omega} S(\omega) d\omega$$

where in particular it follows that the variance of y_t can be defined in terms of frequency:

$$(4.5) \qquad C(0) = \int_{-\pi}^{\pi} S(\omega)d\omega$$

The spectrum can then be interpreted as a decomposition of the series variance by frequency. The small area under $S(\omega)$ between any two frequencies ω and $\omega + \Delta\omega$ will give the portion of the variance of y_t that is attributed to the cyclical components from the frequency band $[\omega, \omega+\Delta\omega]$. When y_t is an i.i.d. (white noise) process with variance σ^2, then $C(\tau) = 0$ for all $\tau \neq 0$, and the spectral density function will be a constant, $S(\omega) = (2\pi)^{-1}\sigma^2$. Hence, for a white noise process, all frequencies are equally important. Trend extraction focuses on removing power at zero frequency. The contribution of the trend is the difference between $S(0)$ and the contribution made by a white noise process, $(2\pi)^{-1}\sigma^2$. A spectral peak between 0 and π indicates important cycles or seasonal effects.

The estimation of the spectrum has proved to be complicated. For one reason, there are an infinite number of points on the continuous curve $S(\omega)$, $0 \leq \omega \leq \pi$, which shall be calculated from a finite amount of data. Early attempts at estimating the spectrum $S(\omega)$ were based on the discrete Fourier transform (DFT), defined as:

$$(4.6) \qquad f(\omega_j) = 1/T \sum_{t=1}^{T-1} y_t e^{-it\omega_j}$$

where $\omega_j = 2\pi j/T$, $j=0,1, 2, 3,..., T-1$ so $f(\omega)$ is calculated from 0 to $2\pi(T-1)/T$ fundamental frequencies. The calculation of these statistics are computational time consuming, and instead we use the fast Fourier transform (FFT) algorithm, that reduces the computational operation considerably when T is a power of 2.

An estimate of the spectrum is found from the periodogram or sample spectral density defined as:

$$(4.7) \qquad I(\omega) = \frac{T}{2\pi}|f(\omega)|^2 = \frac{1}{2\pi} \sum_{\tau=-T+1}^{T-1} s(\tau)e^{-i\omega\tau}$$

where $f(\omega)$ is defined from (4.6). $s(\tau)$ is the sample autocovariance such that $s(\tau) \rightarrow C(\tau)$ as $T \rightarrow \infty$. $I(\omega)$ is an asymptotically unbiased estimator of $S(\omega)$ if $f(\omega)$ is continuous. However, it is not an consistent estimator, as the variance of $I(\omega)$ does not go to zero as the sample size tends to infinity. Further, the covariance between estimates at two different frequencies tends to zero as the sample size goes to infinity, so for large sample sizes, the behaviour of $I(\omega)$ is highly erratic and it is possible to find spurious cyclical behaviour in the data. The usual approach has been to smooth the periodogram in order to get reasonable spectral estimates, (see e.g. Robinson 1983, and the references he states). An *estimate of the spectrum* can then be find by writing the periodogram in (4.7) as:

(4.8) $$W(\omega) = \frac{1}{2\pi}[s_0 + 2\sum_{\tau=1}^{T-1} k_M(\tau)s(\tau)\cos(\omega\tau)]$$

which follows from the fact that $e^{-i\tau\omega} = \cos(\omega\tau) - i\sin(\omega\tau)$ and some simple rules of trigonometry, (see e.g. Hamilton 1994). $k_M(\tau)$ is a lag window, with a sequence of weights such that $k_M(\tau) \to 0$ as $|\tau| \to \infty$. $k_M(\tau)$ is often defined so $k_M(\tau) = k(\tau/M)$, where M is called the truncation point. A popular estimate of the spectrum is the modified *Bartlett* estimate, where the effect on the variance of the $s(\tau)$ for large τ is damped using $k_M(\tau) = 1\text{-}\tau/(M)$, for $\tau = 1, 2,\ldots M\text{-}1$, or $k_M(\tau) = 0$ if $\tau > M\text{-}1$, where here $M = T$.

As we shall se later, a convenient way to extrapolate the cyclical component is to apply a linear filter $A(L) = \sum_{j=-\infty}^{\infty} a_j L^j$, with weights a_j to the observed series y_t, so $c_t = A(L)y_t$.

By analysing the filter in the frequency domain we can show that the spectrum for y_t and c_t is related by the expression:

(4.9) $$S_c(\omega) = |B(\omega)|^2 S_y(\omega)$$

where $B(\omega)$ is known as the frequency response (or transfer) function defined as:

(4.10) $$B(\omega) = \sum_{j=-\infty}^{\infty} a_j e^{i\omega j}$$

Time series must be made stationary before spectral analysis can be applied. In the case of a unit root, the large contribution to the spectrum at zero frequency can affect the other (non zero) frequencies, and create spurious cycles in the data. In figure 4.1a and 4.1b below, we plot the spectra of the log of GDP, real wage and CPI, and the spectra of their first differences. All series have an important component at the zero frequency, hence they are all trending. Taking first differences of GDP, real wage and CPI, most of the power at zero frequency is removed for GDP and real wage, whereas for the first differences of CPI (inflation), there is still a peak at the zero frequency. Hence, as documented in chapter two and three, although prices may be I(1) so inflation is stationary I(0), prices have a high degree of persistence, so shocks to inflation will take a long time to die out, (inflation is a long memory process).

Figure 4.1a. Spectrum of GDP, real wage and CPI

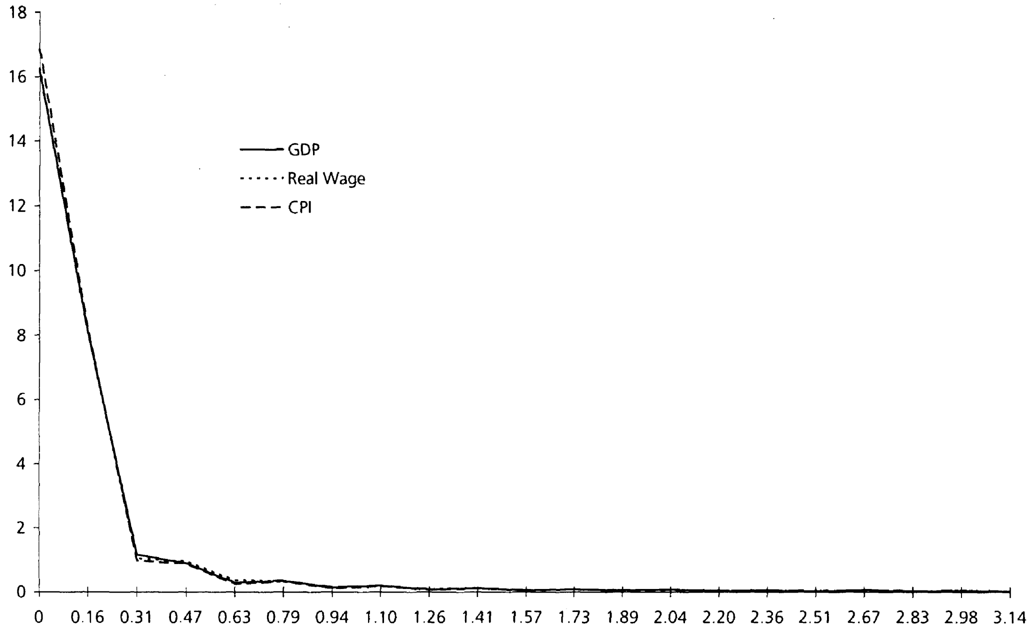
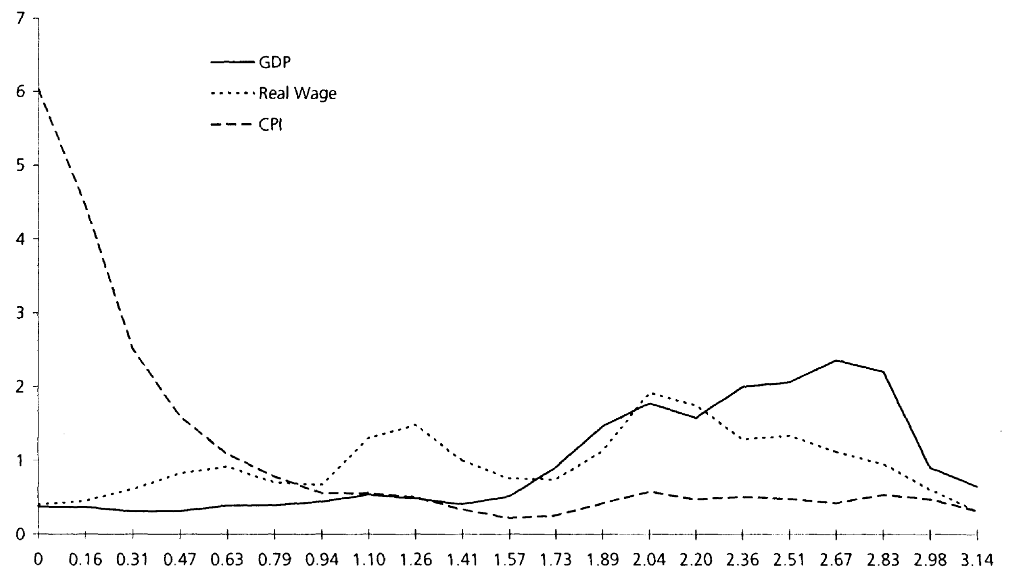


Figure 4.1b. Spectrum of the first differences of GDP, real wage and CPI



4.2 Deterministic trends - a polynomial function of time

The traditional method used to estimate business cycles, is to define a smooth (natural) growth path for the economy which will only be perturbed by transitory cyclical fluctuations. The secular component will reflect permanent changes in e.g. technology, where technology grows constantly over time.

The simplest procedure to numerical measurement of business cycles, is to characterize the trend as a deterministic (polynomial) function of time. By using a relative low order polynomial function of time compared to the number of observations, the trend will be smooth. This smooth trend can be thought of as the natural rate of growth that captures all the non stationarity in the time series. The residuals from regressing the economic series on time, can be interpreted as stationary cyclical movements around the trend. Hence, the trend is found by simple estimation techniques, where the cycle corresponds to the residual in the series and the secular and cyclical components will be independent of each other by definition (for an application, see e.g. Lesteberg and Wettergreen 1975).

A polynomial of first degree corresponds to a linear trend, whereas a polynomial of second degree corresponds to a trend that can display downturn after upturn and vice versa:

$$
\begin{aligned}
y_t &= g_t + \varepsilon_t \\
\hat{g}_t &= \hat{\alpha}_0 + \hat{\alpha}_1 t + \hat{\alpha}_2 t^2 + \ldots \qquad (4.11) \\
\hat{c}_t &= y_t - \hat{g}_t
\end{aligned}
$$

where ε_t can be estimated as a stationary ARMA process. The use of deterministic functions of time has been particularly popular when there is no prior theory that specifies the properties of the trend. Although we could not reject the unit root hypothesis in chapter 2 for any of the variables in favour of a linear trend (except maybe for import), we conduct the decomposition using both a linear and a quadratic trend for all the variables in the sample bearing in mind the low power of the unit root tests.

The advantage of this method is that it is easy and quick to apply, and gives an intuitive first approximation to study business cycles when we have no a priori economic theory. High order polynomials can approximate the trend in a non stationary series closely. The disadvantage is that it can infer spurious cycles if the data is generated by a random walk, see chapter 2.1.

Figure 4.2a shows GDP together with a linear trend (LT) and a quadratic trend (QT), wheras figure 4.2b displays the detrended GDP. The data are measured in logs, so a value of 0.1 in figure 4.2b implies a 10 pct. deviation from the trend in figure 4.2a.

Figure 4.2a. Trend components of GDP: Polynomial trends

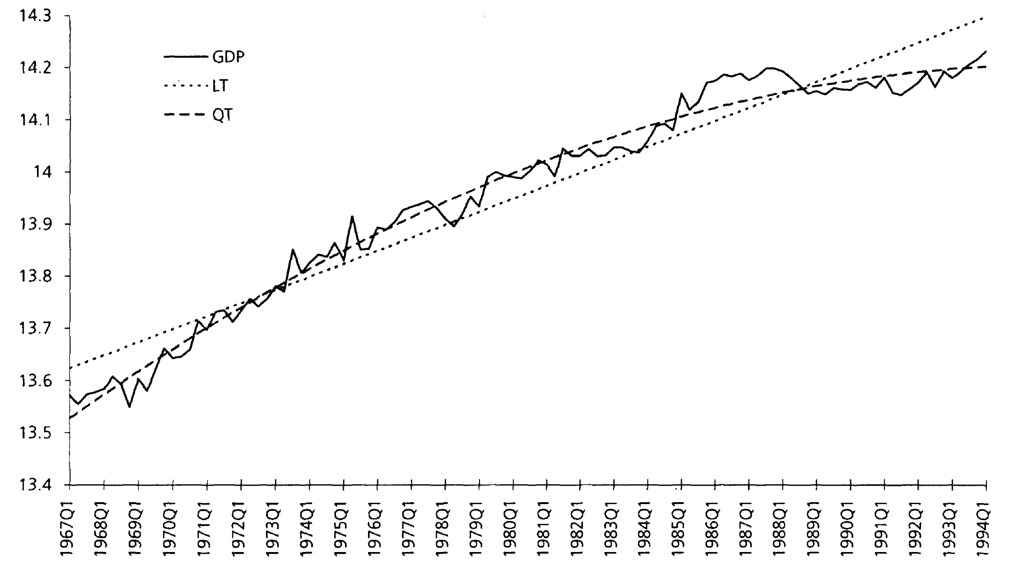
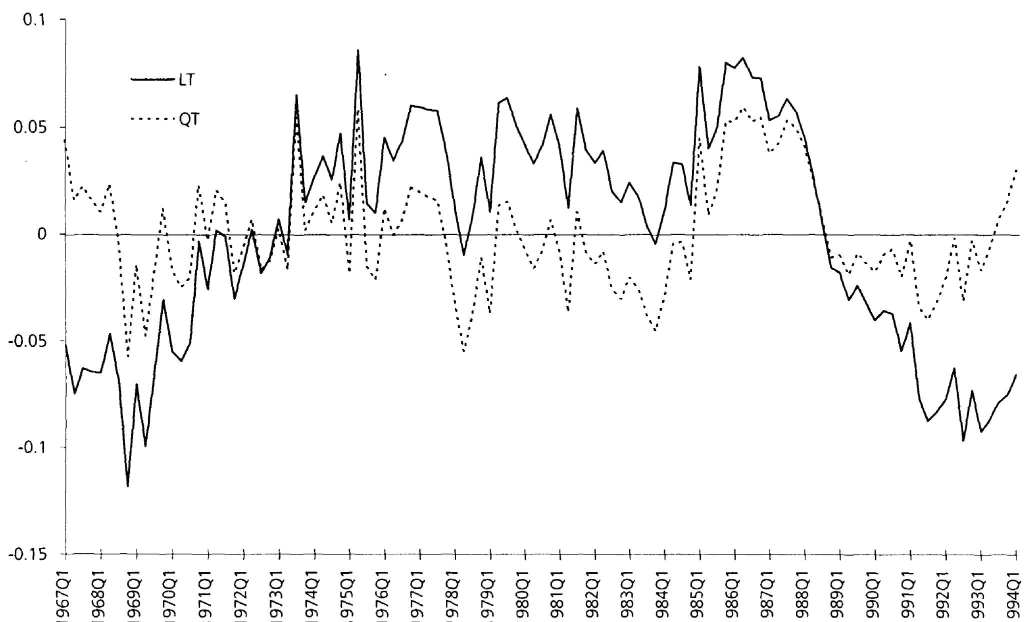


Figure 4.2b. Cyclical components of GDP: Polynomial trends



4.3 Deterministic trends with break

Historically, productivity growth has been far from smooth. As emphasized in chapter 2, some economic variables may in fact be well represented by a deterministic trend, if we allow for a large but infrequent break/shift in the trend.

$$
\begin{aligned}
y_t &= \alpha_0 + \alpha_1 t + \alpha_2 DS_t(k) + \alpha_3 DB_t(k) + \varepsilon_t \\
\hat{g}_t &= \hat{\alpha}_0 + \hat{\alpha}_1 t + \hat{\alpha}_2 DS_t(k) + \hat{\alpha}_3 DB_t(k) \\
\hat{c}_t &= y_t - \hat{g}_t
\end{aligned}
\tag{4.12}
$$

where again ε_t can be measured as a stationary ARMA process. $DS_t(k)$ and $DB_t(k)$ are dummy variables that capture the change in the trend at period k, where now a single *shift* in the trend (change in the growth rate) (case A in chapter 2.2) is captured by $DS_t(k)$ and a single *break* in the trend (shift in the mean) (case B in chapter 2.2) is captured by $DB_t(k)$:

$$
\begin{aligned}
&\text{A)} \quad DS_t(k) = (t-k)1(t>k) \\
&\text{B)} \quad DB_t(k) = 1(t>k)
\end{aligned}
\tag{4.13}
$$

where $1(t>k)$ is the indicator function. To ensure there are no misspecification between the trend break and trend shift model, we include both the trend break and the trend shift point in the estimation. The trend break and trend shift points are taken from table 2.3a in chapter 2.3, except for real wages where the dates are taken from table B.1 in appendix B. GDP together with the estimated linear trend with break (LTB) are graphed in figure 4.3a, whereas figure 4.3b shows detrended GDP.

Figure D.1 in appendix D shows the spectrum for GDP, real wage and CPI using LTB. it emphasize that the linear trend with break does not remove all the power at zero frequency for neither of the variables. Using a higher polynomial together with the estimated break date, will remove more of the power at the zero frequency.

The advantage of this method is that it is a more satisfactory approach than using a linear trend, when the data are in the borderline between being trend-stationary and difference-stationary. The method is easy to apply when the break dates are found. The disadvantage is that it is time demanding in terms of finding the exact break points. As with the deterministic polynomials in time, if there are more than one significant break point (or if the data are difference-stationary), the method may infer spurious cycles.

Figure 4.3a. Trend component of GDP: Trend with break

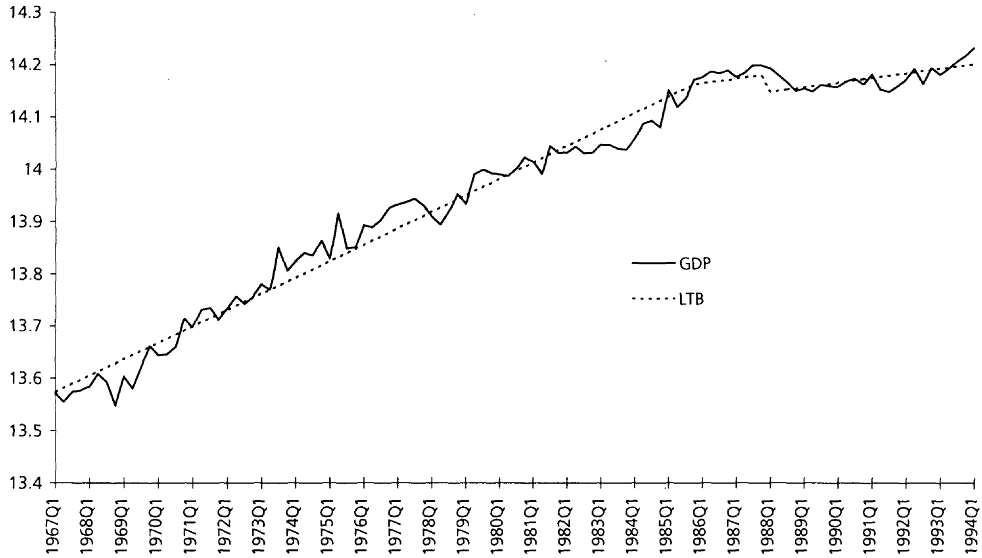
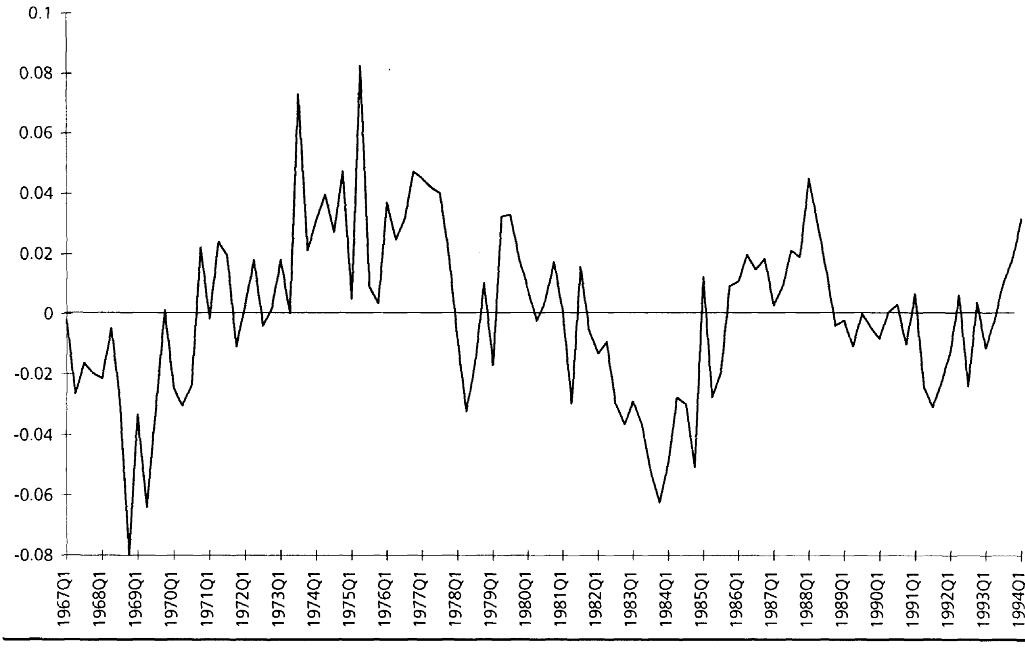


Figure 4.3b. Cyclical component of GDP: Trend with break



4.4 The Hodrick-Prescott filter

One commonly used approach to extract cycles in the real business cycle literature, is to use the so-called Hodrick-Prescott (HP) filter. This filter extracts a stochastic trend, that moves smoothly over time and is uncorrelated with the cycle. The HP filter is an optimal extractor of a stochastic trend, g_t, and is determined by a convex minimisation problem, for a given value of λ. The optimal value of g_t (g_t^{HP}) is found by minimising the following expression:

$$\min_{[g_t]_{t=1}^{T}} [\sum_{t=1}^{T}(y_t - g_t)^2 + \lambda\sum_{t=3}^{T}((g_t - g_{t-1}) - (g_{t-1} - g_{t-2}))^2]$$

(4.14)

$$\hat{c}_t = y_t - g_t^{HP}$$

The first term in the expression is the sum of squared deviations between the observed data and the trend, and is a measure of the goodness of fit of the trend to the original series. The second term is the sum of the squares of the secular component's second differences and measures the degree of smoothness of the trend. λ is the smoothing parameter, which penalises the variation in the growth rate of the trend (see Kydland and Prescott 1990, pp. 8-9). With λ=0 for all t, there is no penalty for the variation in the trend, and minimisation implies a goodness of fit problem, where the best fit is found when g_t exactly coincides with y_t, so c_t= 0. By increasing λ, the variation in the trend becomes penalised, and the secular component becomes smoother. When λ approaches infinity, the lowest minimum is achieved when the variability in the trend is zero, and the trend is perfectly log linear.

A convenient way to extrapolate the cyclical component is to apply a linear filter $A(L) = \sum_{j=-\infty}^{\infty} a_j L^j$, to the observed series y_t, so $c_t = A(L)y_t$. King and Rebelo (1993) show that the HP filter takes the form of a two sides symmetric filter, that is only dependent on λ:

$$A^{HP}(L) = \left[\frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2}\right] \tag{4.15}$$

where c_t is found by applying this filter to the observed series y_t, so $c_t = A^{HP}(L)y_t$.

As there are four differences in the numerator, it will render stationary any integrated series up to fourth order. It is convenient to analyse this filter in the frequency domain.

The Fourier transform of the cyclical component filter has the following form (again, see King and Rebelo 1993):

$$f^{HP}(\omega) = \frac{4\lambda(1-\cos(\omega))^2}{1+4\lambda(1-\cos(\omega))^2} \tag{4.16}$$

The filter removes all power at the zero frequency by placing zero weight at this frequency, $f^{HP}(0) = 0$. The weight increases as the frequencies increase, and at the high frequencies ($\omega=\pi$), the filter places almost unit weight $f^{HP}(\pi) = 16\lambda/(1+16\lambda)$. By increasing λ, the weights will increase for all frequencies but zero.

Several other researchers have recently examined the properties of the HP-filter. Cogley and Nason (1992) argue that applying the HP filter to a difference-stationary process, is similar to detrending a random walk. This makes it subject to the Nelson and Kang (1981) critique, which showed that detrending a random walk, would generate spurious periodicity in the residuals. The HP filter will then generate business cycle periodicity and comovement even if none is present in the original data. Harvey and Jaeger (1993), also makes the point that applying the HP filter to a random walk produces spurious cycles which corresponds to a period of 30 quarters. They show that spurious cycles may also emanate from I(2) processes. Jaeger (1994) emphasizes this point further.

Since the smoothness of the secular component will be sensitive to the value of λ that is chosen, a justification for the choice should be made. Kydland and Prescott (1990, p. 9) find a value of $\lambda=1600$ for quarterly data to be reasonable, and subsequent studies for US and several European countries have used this value. They determine the value of λ by assuming c_t and $\Delta\Delta g_t$ to be i.i.d. normally distributed variables, with mean zero, and variances given by σ^2_c and $\sigma^2_{\Delta\Delta g}$ respectively. The solution to (4.14) can then be shown to be equivalent to the conditional expectation of g_t given y_t, where $\lambda = \sigma^2_c/\sigma^2_{\Delta\Delta g}$. By interpreting the smoothness parameter as the relative variability between the cyclical component and the degree of smoothness, they argue that $\lambda=(5)^2/(1/8)^2=1600$, as 5 pct. seems a plausible measure of the mean deviation from the trend in a quarter, while 1/8 pct. seems a plausible measure of the quarterly growth rate of the series. However, c_t and $\Delta\Delta g_t$ are generally not normally distributed. The choice of λ then essentially becomes arbitrary. For instance, Danthine and Girandin (1989) argue for the value of $\lambda=1600$, as it gives a rather more stable measure of the regularity of the business cycle patterns than other values of λ. Blackburn and Ravn (1992), argue that the gain (the relative importance of the cyclical component), of $\lambda=1600$ for UK data, corresponds to an output cycle of 4-5 years.

We use $\lambda=1600$, as a benchmark value, so the results can be compared to other international studies. For comparison, we choose two other values for λ that confirm to two other stories. First we choose a value for λ that is rather small, which can account for the fact

Figure 4.4a. Trend components of GDP: Hodrick-Prescott filter

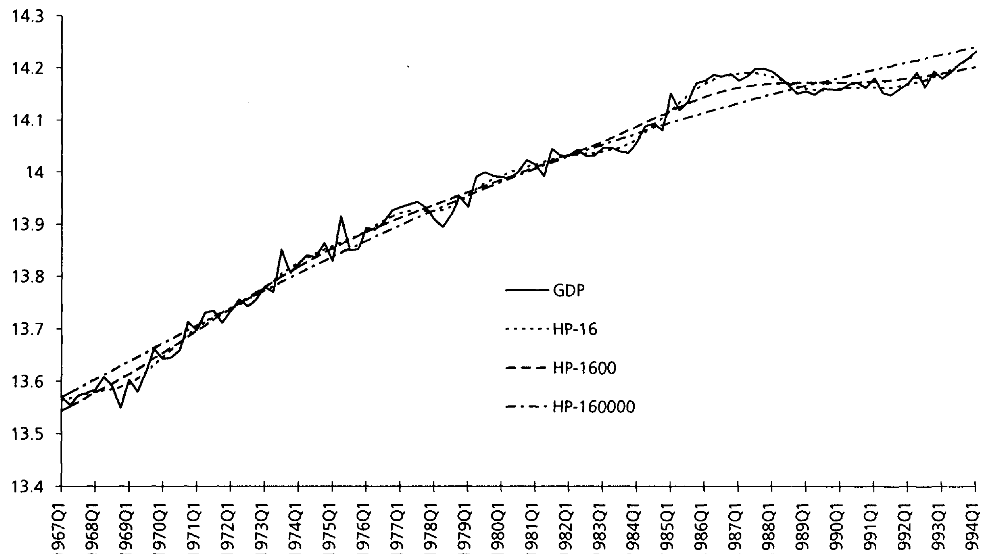
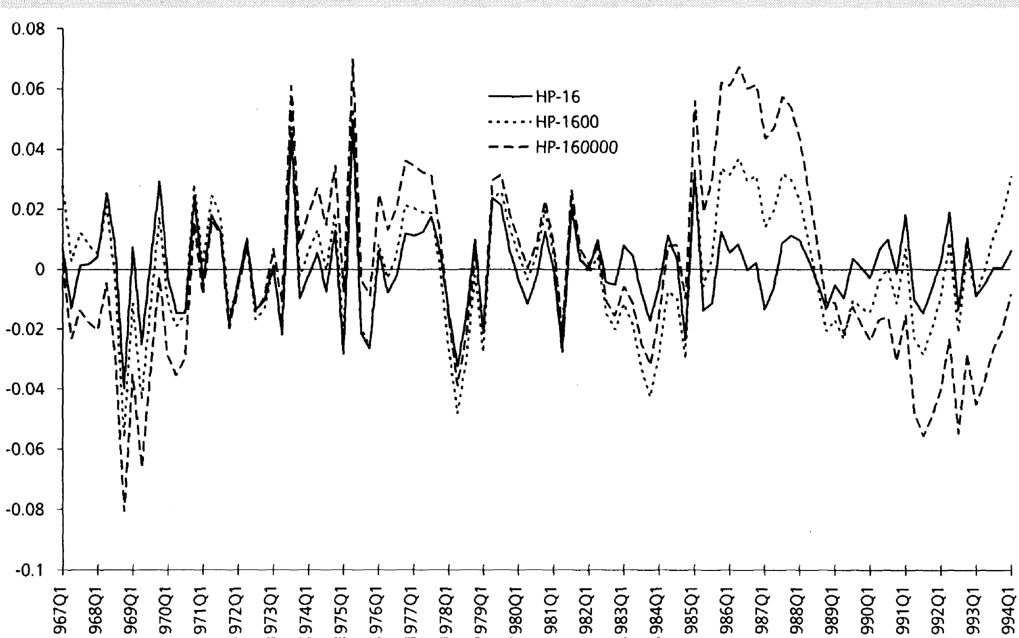


Figure 4.4b. Cyclical components of GDP: Hodrick-Prescott filter



of relative more volatility in the trend, like in the stochastic trend case. Nelson and Plosser (1982) argue that the ratio between the standard deviations of innovations in the growth component and the standard deviation in the cyclical component, should be no higher than 1, but with probably values of 1/4 or 1/5 rather than 1/40 as Hodrick and Prescott (1990) choose. We choose $\lambda=16$, corresponding to a ratio of 1/4. On the other contrast, to account for the fact that an open economy like Norway (being dominated by the oil sector), may experience rather more cyclical volatility than experienced in the aggregate US economy, we consider a λ that is considerable higher. The argument is that an economy that experiences 10 times more cyclical volatility than the US economy, may experience a ratio between the standard deviations of innovations in the growth component and the standard deviation in the cyclical component, that is 1/400 rather than 1/40, corresponding to a value for $\lambda=160000$.

In figure 4.4a we plot GDP together with the HP trend using $\lambda=16$ (HP-16), $\lambda=1600$ (HP-1600) and $\lambda=160000$ (HP-160000). Figure 4.4b plot the detrended GDP.

In figure D.2 and D.3 in appendix D, we compare the spectra for the three variables GDP, real wage and CPI, using $\lambda=16$ and $\lambda=1600$ respectively. Most of the power at the zero frequency is removed, especially using $\lambda=16$. For $\lambda=16$, the spectrum has a peak around frequency 0.6, corresponding approximately to 2.5 years. For $\lambda=1600$, the peak is centred around frequency 0.35, corresponding to 3.5-4 years. However, $\lambda=16$ has also affected the higher frequencies, by inducing more cycles there, see especially for real wage and GDP. However, as both GDP and CPI are probably difference-stationary, (and CPI may even be I(2)), the cycles may be spurious and reflect the properties of the HP filter instead of the underlying dynamics.

The advantage of the HP method is that it is easy to apply. It is used increasingly by economists, e.g. by OECD in "Economic Forecast", documenting 'desyncronisation of business cycles'. It removes most of the power at zero frequency. The disadvantage is that the choice of λ must be made a priori, but cycles generated by different values of λ will differ. Although the trend is stochastic in nature, applying the filter to a difference-stationary series may infer spurious cycles.

4.5 Random walk

A simple stochastic model can be defined by letting the series consists of a permanent component that follows a random walk with no drift and a white noise disturbance term, that is independent of the secular component:

$$y_t = g_t + \varepsilon_t$$

(4.18)
$$g_t = g_{t-1} + \varepsilon_{t-1} = g_0 + \sum_{j=1}^{t-1} \varepsilon_j$$

Figure 4.5a. Trend Component of GDP: Random walk

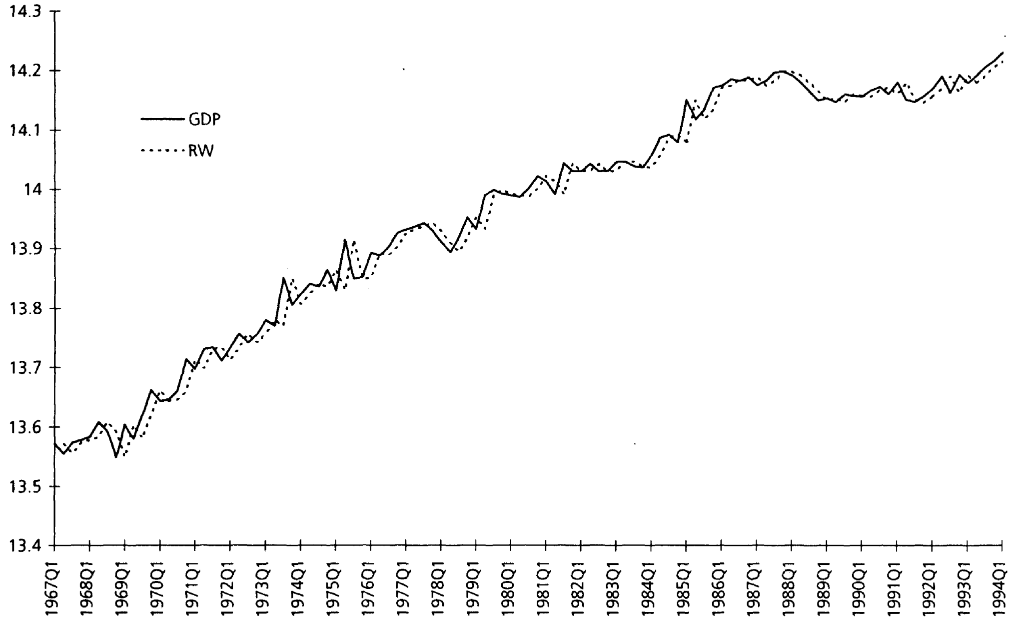
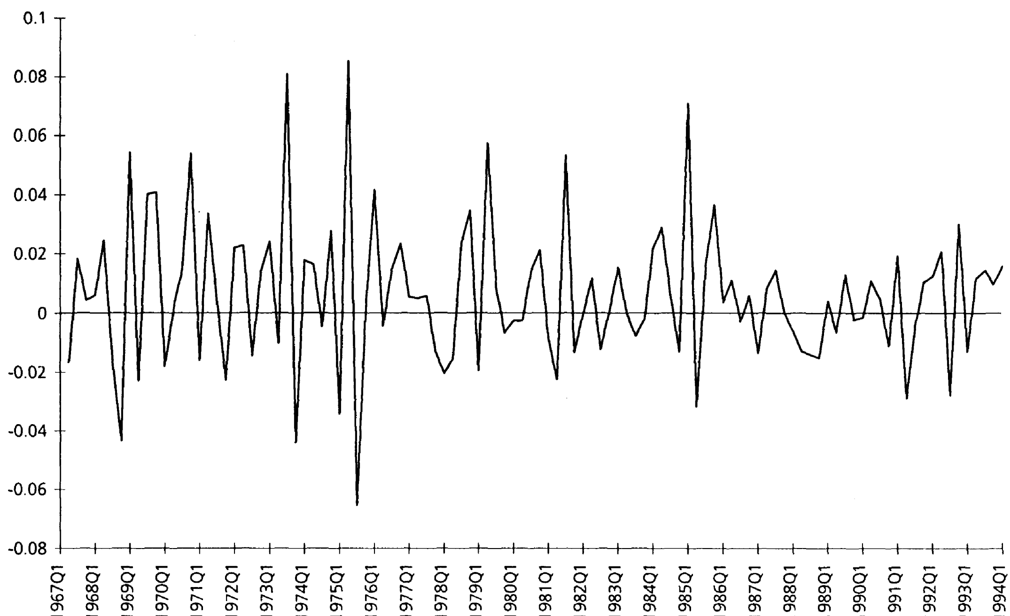


Figure 4.5b. First differences of GDP



The process is reflected in the observed series, which is described as having a unit root. This is a very simple and common representation, where in terms of (4.1), the cyclical component is simply represented as the growth rate of y_t:

$$c_t \equiv \Delta y_t = \varepsilon_t \tag{4.19}$$

Obviously, we restrict the variance of ε_t to be strictly positive, as otherwise, there would be a trivial case where the cyclical component was zero. Hence, what we essentially are studying here are the growth rates of y_t, which has nothing to do with business cycles as we have defined them. The spectra of the first differences of GDP, real wage and CPI that were seen in chapter 4.1 confirm this. Although the first difference filter (RW) removed most of the power at zero frequency for GDP and real wage, it also removed most of the power at the intermediate (business cycle) frequencies, especially compared to HP-16 and HP-1600, which had clear peaks at the frequencies centred around cycles lasting 2.5-4 years. GDP with its trend is plotted in figure 4.5a, wheras the cycle or random walk is seen in figure 4.5b.

The advantage of this method is that it is easy to apply and often used as a way of removing the zero frequency component. The disadvantage is that the filter attenuate some of the low frequencies component we may be interested to study. It will also induce a phase shift. The dynamics for both the secular and cyclical components are restrictive compared to ARIMA modelling.

4.6 The Beveridge and Nelson Procedure

Many econometric time series can be characterized by the class of nonstationary ARIMA processes, where the first differences of a process can be represented as a stationary process of autoregressive moving average form. The question is how one should decompose a nonstationary series into a permanent and a transitory component. In one decomposition, that is due to Beveridge and Nelson (1981), the permanent component is shown to be a random walk with drift and the transitory component is a stationary process with zero mean, which is perfectly correlated with the permanent component.

Assume $(1\text{-}L)y_t$ to be a stationary process, which by Wold's theorem can be written as the infinite moving average process $(1\text{-}L)y_t = \alpha_1 + A(L)\varepsilon_t$, where $A(L) = \Sigma A_j L^j$, and ε_t are uncorrelated, mean zero, random disturbances with variance equal to σ^2 as in (3.1). α_1 represents the long run mean of the series. The Beveridge-Nelson (BN) decomposition was obtained in (2.5) by defining $A^*(L) = (1\text{-}L)^{-1}(A(L)\text{-}A(1))$ and substituting for $A(L)$, so $(1\text{-}L)y_t = \alpha_1 + [A(1) + (1\text{-}L)A^*(L)]\varepsilon_t$. This reduces to a trend component and a cyclical component:

$$\begin{aligned} \Delta y_t &= \Delta g_t + \Delta c_t \\ \Delta g_t &= \alpha_1 + A(1)\varepsilon_t \\ \Delta c_t &= (1 - L)A^*(L)\varepsilon_t \end{aligned} \tag{4.20}$$

where A(1) is the sum of the moving average coefficients. From (4.20) it can be seen that the trend follows a random walk with drift, which can be solved to yield:

$$g_t = g_0 + \alpha_1 t + A(1)\sum_{s=1}^{t} \varepsilon_s \tag{4.21}$$

The trend consists of both a deterministic term ($g_0 + \alpha_1 t$) and a stochastic term ($A(1)\ \Sigma_1{}^t \varepsilon_s$). As discussed in chapter 3, for A(1)=0, the trend reduces to a deterministic case, whereas for A(1) ≠ 0, the stochastic part indicates the long run impact of a shock ε_t, on the level of y_t.

The cyclical component is stationary and is given by :

$$c_t = A^*(L)\varepsilon_t \tag{4.22}$$

Beveridge and Nelson (1981) showed that the stochastic trend defined in (4.21) could also be interpreted as the long- term forecast of the series adjusted for the mean rate of change, (see appendix E) and the cycle defined in (4.22) as the stationary process that reflects the deviations of the trend from the observed series:

$$g_t = \lim_{k\to\infty}\left[\hat{y}_{t+k} - \alpha_1 k\right]$$

$$c_t = -\lim_{k\to\infty}\left[\sum_{j=1}^{k} \Delta\hat{y}_{t+j} - \alpha_1 k\right] \tag{4.23}$$

$$\hat{y}_{t+j} = E(y_{t+j} \mid \ldots, y_{t-1}, y_t)$$

The BN decomposition implies that innovations in g_t and c_t will be perfectly correlated. The permanent component will have the same rate of drift (α_1) as the observed values. Further, the variance of the innovations $(\Sigma_{i=0}A_i)\varepsilon_t$ in the permanent component is given by $(\Sigma_{i=0}A_i)^2\sigma^2$, which will be larger (smaller) than the variance of the innovations ε_t in the observed data y_t, (σ^2), if $(\Sigma_{i=0}A_i)^2$ is larger (smaller) than one. Note also that when the permanent component is restricted to be a random walk with drift, (A_0= 1 and all the A_i's = 0 for i>0), the variance of the permanent component equals the variance in the observed series, and the cyclical component will be zero for all t.

To be able to identify the cyclical and permanent component, we must specify models that can be written as the stationary Wold moving average process in (3.1). There are two stages involved in the trend-cycle decomposition in the BN method. First an ARIMA model (p,d,q) have to be estimated to the series y_t where p is the number of AR lags, d is the number of differencing and q is the number of MA lags. Then c_t has to be numerically estimated.

Beveridge and Nelson (1981) proposed to truncate the infinite number of forecast of y_t in (4.23) at a suitable large number of k. A quick computational approach was suggested by Cuddington and Winter (1989). g_t is calculated directly from the expression in (4.21) by estimating A(1) from a truncated Wold representation of Δy_t:

(4.24)
$$\Delta y_t = \alpha_1(1-\phi_1-\ldots-\phi_p)+\phi_1\Delta y_{t-1}+\ldots+\phi_p\Delta y_{t-p}+\varepsilon_t+\theta_1\varepsilon_{t-1}+\ldots+\theta_q\varepsilon_{t-q}$$
$$A(1)=\frac{(1+\theta_1+\ldots+\theta_q)}{(1-\phi_1-\ldots-\phi_p)}$$

The obvious difficulty, is that the initial value of g_t in (4.21) is unknown, so the procedure is only correct up to an additive factor. Newbold (1990) solves this by suggesting another computational method for establishing the cycle and the trend.

Define:

(4.25) $$z_t = \Delta y_t - \alpha_1$$

so that:

(4.26) $$\hat{z}_t(j) = \Delta \hat{y}_t(j) - \alpha_1$$

and c_t in (4.23) can be written as[16]:

(4.27)
$$c_t = -\lim_{k\to\infty}\Big[\sum_{j=1}^{k}(\hat{z}_t(j))\Big]$$
$$c_t = -\sum_{j=1}^{q}(\hat{z}_t(j)) - (1-\phi_1-\ldots-\phi_p)^{-1}\sum_{j=1}^{p}\sum_{i=j}^{p}\phi_i\hat{z}_t(q-j+1)$$

where $\hat{z}_t(s)=z_{t+s}, s\leq 0$.

For a pure AR(p) process, the first term in (4.27) equals zero, and:

(4.28) $$\hat{z}_t(q-j+1) = \hat{z}_t(-j+1) = z_{t-j+1},\quad j\geq 1$$

[16] See Newbold (1990 pp. 454-455) for a proof.

Hence (4.27) can be written in the compact form as :

$$(4.29) \qquad c_t = -(1 - \phi_1 - \ldots - \phi_p)^{-1} \sum_{j=1}^{p} \sum_{i=j}^{p} \phi_i (\Delta y_{t-j+1} - \alpha_1)$$

As pointed out by Newbold (1990), provided the error in estimating ε_t is small, it is possible to estimate the trend component by using (4.21), if g_0 is established by applying (4.27) for one period only (by determining c_0). The approach we take in this paper is therefore the following: For a pure AR process, we will use (4.29) to estimate the cyclical component directly, whereas for a mixed ARMA process, we will apply (4.27) for one period only to establish g_0, and thereafter use (4.21) to determine the remaining observations of g_t. c_t is found as the deviations from the observed series.[17]

One obvious difficulty in using the BN method, is that we will have to choose between several ARIMA models. Although different ARIMA models may fit the short run properties of an observed series, the forecast functions from these models may differ substantially. Since the trend-cycle decomposition in the BN method relies on the forecast properties of the ARIMA models, these models may give very different trend-cycle decompositions. As a result of this, we have specified several ARIMA models, of which we here present two results. We choose one low order 'best fit' ARIMA model (BN-low), based among other on the Schwarz and Akaike criteria. For comparison, we choose a model with very long AR lags (BN-high), chosen from the criteria by the Ljung-Box Q statistics.[18] GDP and the estimated trends using BN-low and BN-high are graphed in figure 4.6a, whereas detrended GDP is shown in figure 4.6b. As we loose some observations at the beginning of the sample due to the AR lags estimated in the ARIMA models, BN-low starts in 1967Q3 and BN-high starts in 1972Q2 for GDP.

For GDP and real wage, the BN method based on the low order ARIMA model produce a quite different spectra than the BN method based on the high order ARIMA model, (figure D.4 and D.5 respectively). Whereas BN-low removes almost all of the power at the low and intermediate frequencies (as for the first difference filter in figure 4.1b), BN-high has a peak at the zero and intermediate frequencies. For CPI, the BN-high model and the BN-low model both give spectra that have most power at the lowest frequencies.

The advantage of the Beveridge-Nelson method is that it is an appropriate method to extract cycles when a series is difference-stationary and hence using a linear trend may infer spurious cycles. It allows the series to contain a unit root that can be highly volatile. The disadvantage of the method is that it is time demanding, as we have to choose between different ARIMA models that may give quite different results. Further, misrepresenting an I(2) process as an I(1) process may generate excess volatility in the trend.

[17] We compared the two methods for some simple processes, and the differences were negligible.
[18] See appendix E for details on estimation and model selection.

Figure 4.6a. Trend components of GDP: Beveridge-Nelson procedure

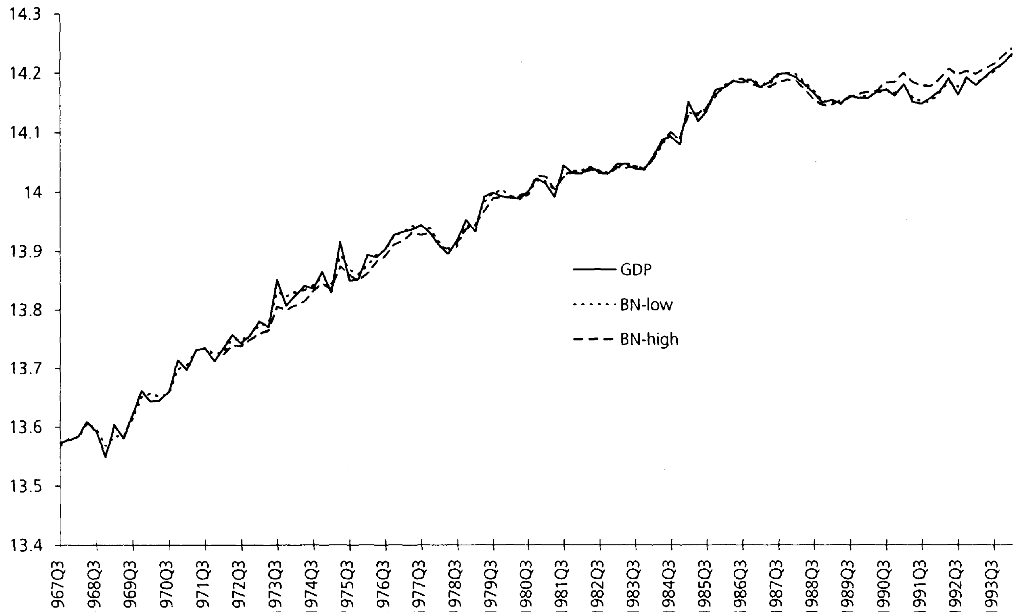
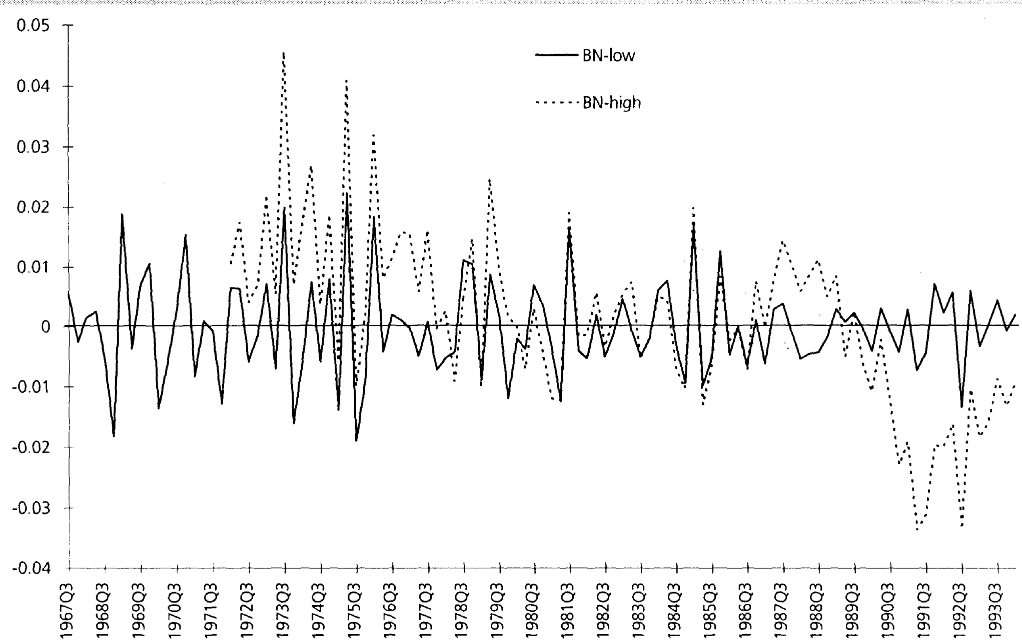


Figure 4.6b. Cyclical component of GDP: Beveridge-Nelson procedure



4.7 Frequency filtering techniques

In this chapter we filter the variables to isolate the 'business cycle' frequencies explicitly. Here we identify the different components in the time series from which frequency band in the spectrum that have concentrated most of their power. The trend would typically have most of its power located in a low frequency band, the 'business cycle' would have most of its power in a 'intermediate' frequency band, and the irregular cycle would be attributed to a high-frequency band. One way to identify the different components would be to apply a band-pass filter to the original data in the frequency domain. The band pass filter will be constructed to filter out all the cyclical components in a series, except those components that correspond to a chosen frequency band. Given that our intention is to discover any link between the business cycle component in the main economic variables, this method has an intuitive appeal as we will be able to extract the frequency components we are interested in directly without restricting ourselves to an economic or statistical theory.

However, as the term 'business cycle frequencies' does not imply a precise definition of the relevant frequencies, we have to take an explicit stand on the upper and lower limits of the frequency band that will include the so called 'business cycles'. Economist have previously argued to have identified different types of cycles in the data, e.g. the *Kondratieff cycles* (40-60 years), the *Juglar cycles* (7 -11 years), and the *Kitchin cycles* (2-4 years), (which are all named after the authors that invented them). The Kondratieff cycles are the main long wave cycles, whereas the Juglar and Kitchin cycles are associated with major and minor business cycles.[19]

Today, business cycles are more commonly thought of as the short wave cycles which, according to the NBER classification, show up with an average periodicity of about 4-6 years, (see e.g. Zarnowitz and Moore 1986). We define business cycles as having a periodicity from 1.5 to 8 years, so we do not rule out any cycles with a periodicity that is longer than the average cycle. Cycles with a periodicity of more than 8 years we attribute to the trend and cycles with period less than 1.5 year, we attribute to the irregular component. Also, as we use nonadjusted data, the seasonal component (corresponding to one year) will be wiped out and attributed to the noise component.

As we defined in section 4.1, by applying a linear time invariant filter $A(L) = \sum_{j=-\infty}^{\infty} a_j L^j$,

to the observed series, we can find the cyclical component as $c_t = A(L)y_t$. We also showed that in the frequency domain, the spectrum for y_t and c_t ($S_y(\omega)$ and $S_c(\omega)$ respectively) were related through the expression $S_c(\omega) = |B(\omega)|^2 S_y(\omega)$, where $B(\omega)$ was defined as the transfer function, expressed by the weights a_j's. Estimation of the spectrum was based on the fast Fourier transform (FFT) $f(\omega)$.

[19] Schumpeter (1939, pp. 161-174) developed a scheme in which he could identify six Juglar cycles to one Kontratieff cycle, and three Kitchin cycles to one Juglar cycle. However, there has been little formal evidence to verify his claim.

Whereas an ideal filter in an infinite sample would eliminate all frequencies other than those at the chosen business cycle frequencies, applying the filter to a finite sample will lead to some 'leakages' outside the band. In a finite sample, the FFT used for calculations will treat the series as periodic and assume that the last observation corresponds to the observation preceding the first observation. This effect of having a linear time trend in the data can distort the time series and create spurious cycles in the data. To eliminate this distortion, we allow each series to be linearly detrended before we apply the filter.[20] To use the FFT we further pad the data with zeros up to four times its length, until the number of elements is equal to a power of two.

To investigate whether the results are sensitive to the way we removed the trend in the data before applying the band-pass filter, we experienced with other ways to remove the low frequency component. We constructed moving average filters that pass through the data using 16, 12 or 8 leads and lags. Hence 4, 3 and 2 years of observations are lost respectively at the beginning and at the end of the sample. The quality of the approximation depends of the length of the moving average. Whereas a first order difference filter will remove much of the low frequency components we are interested to keep, the moving average filtes constructed here are more optimal in the sense that they remove little except the values at the zero frequency. However, the results prefiltering with a linear trend do not differ much from the results obtained using the moving average for most series. To keep exposition simple, we therefore only report the results having prefiltered the data with a liner trend.

The computational procedure is as follows. We first apply the FFT to the original data. We then construct the band pass filter for the frequency domain. The filter is designed so that all cycles with a period between 6 and 32 quarters pass through the filter unchanged, whereas all other cycles are wiped out. The Fourier transform of the cyclical component $f_c(\omega)$ is then found by multiplying the filter with the Fourier transform of the original series $f_y(\omega)$:

$$f_c(\omega) = |B_c(\omega)|^2 f_y(\omega) \tag{4.30}$$

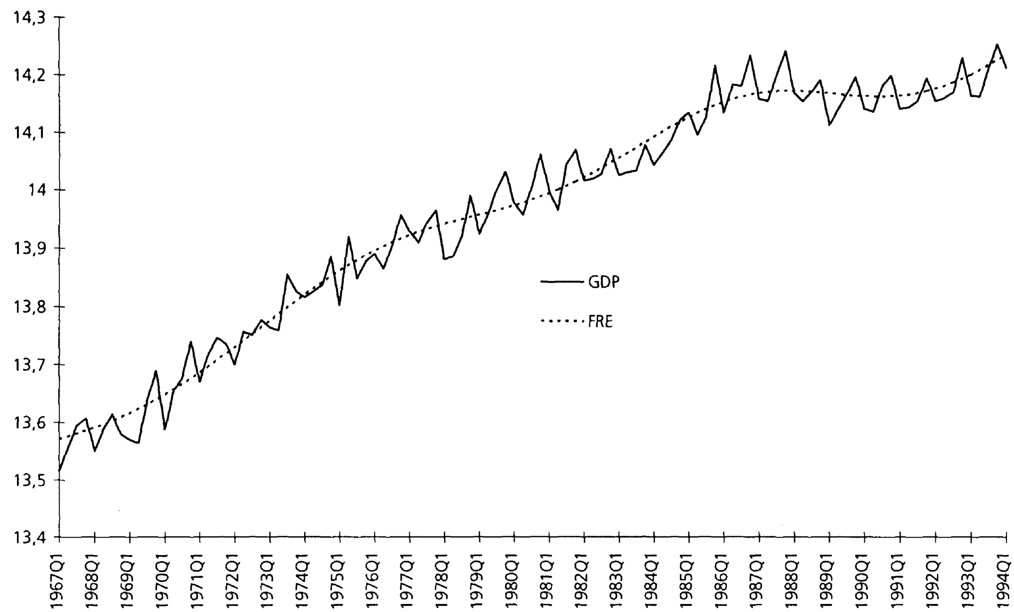
where the transfer function to the band pass filter $B_c(\omega)$, is defined as :

$$B_c(\omega) \begin{array}{l} = 1 \text{ if } \dfrac{2\pi}{32} \leq \omega \leq \dfrac{2\pi}{6} \;\wedge\; 2\pi(1-\dfrac{1}{6}) \leq \omega \leq 2\pi(1-\dfrac{1}{32}) \\ = 0 \text{ otherwise} \end{array} \tag{4.31}$$

Given the correspondence between the period of the cycle (quarter per cycle) and the frequencies, (periodicity=$2\pi/\omega$), the lower limit of the frequency band that corresponds to 32 quarters, equals $\omega=2\pi/32$, whereas the upper limit that corresponds to a cycle with period of 6 quarter equals $\omega=2\pi/6$. However, with ω_j defined over

[20] For a discussion of this issue, see Stock and Watson (1990) and Hassler et al. (1992).

Figure 4.7a. Trend component of GDP: Band Pass filter



0, $2\pi/T,\ldots,2\pi(T-1)/T$, the spectrum is periodic with a period of 2π, and the values for $\pi\leq\omega_j\leq2\pi$ equals $-\pi\leq\omega_j\leq0$. We therefore construct a two-sided symmetric filter over the whole period 2π. Finally, the filtered cyclical component is found in the time domain by calculating the inverse FFT. The trend generated by the frequence filter (FRE), the business cycle and the irregular component are graphed in figure 4.7a, 4.7b and 4.7c respectively.

The spectra for the cycle of GDP, real wage and CPI are graphed in figure D.6, and show that we have removed most of the power at the zero frequency, and that the spectra have peaks at the intermediate ('business cycle') frequencies corresponding from 2.5 to 8 years.

The advantage of the method is that it is easy to apply if one is familiar with frequency domain techniques. We can a priori take a stand on the periodicity of the business cycle component. We can also use data that are not seasonally adjusted. The disadvantage is that one may have some consideration for removing the low frequency component before filtering in the frequency domain.

Figure 4.7b. Cyclical component of GDP: Band Pass filter

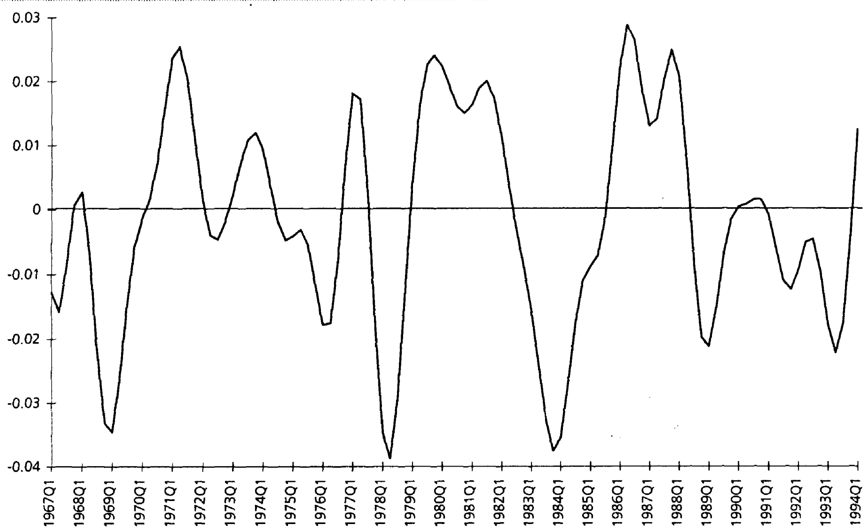
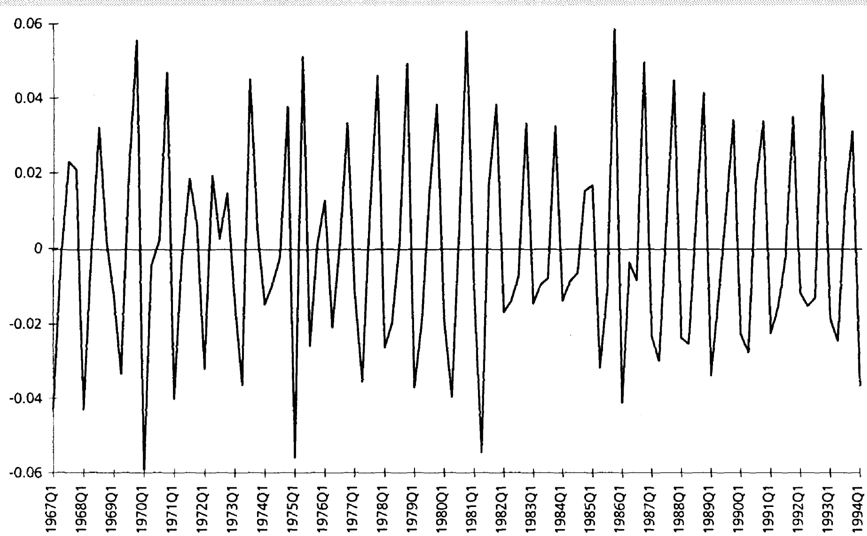


Figure 4.7c. White noise component of GDP: Band Pass filter



We sum up the main findings to now. From the figures in chapters 4.1-4.7 and appendix C, we saw that the cycles generated by the linear trend with break (LTB), the Hodrick-Prescott filter with a smoothing parameter equal to 1600 (HP-1600) and the frequency domain filter (FRE) seemed similar, although in most cases the cycles generated by LTB and HP-1600 were much more jagged than the cycles generated by FRE, (see especially for productivity). The Beveridge-Nelson procedure using a high order ARIMA model (BN-high) generated cycles that resembled those generated by LTB, HP-1600 and FRE, although the cycles were more volatile and with a shorter periodicity, (except maybe for CPI and M2, where BN-high and BN-low (constructed from a low order ARIMA model) constructed similar cycles, generated by an excessive moving trend, (cf. chapter 4.6)). For all series except CPI and M2, BN-low generated a noisy cyclical pattern.

Analysing GDP, real wage and CPI in the frequency domain, we saw that the spectra using LTB, HP-1600 and FRE, also resembled each other in that they displayed a peak at the intermediate frequencies (the so called 'business cycle frequencies'), that correspond to a periodicity of 4 to 8 years. However, HP-1600 and FRE removed much more of the power at the zero frequency than LTB. For BN-high, there was an even larger peak at the zero frequency in the spectra, and the weights given to the 'business cycle frequencies', were much smaller than those generated by LTB, HP-1600 and FRE. This indicates that BN-high generated less cyclical behaviour in the variables compared to what the other methods (LTB, HP-1600 and FRE) did. BN-low placed even lower weight at the business cycle frequencies than BN-high. In a more general ARIMA representation like the unobserved component model, Skjerpen (1995) found also little evidence of a cyclical component in GDP and some other main Norwegian quarterly macroeconomic variables.

4.8 Cycles in the Norwegian economy 1967-1994

Below we will identify and characterize some economic cycles in Norway from 1967-1994. We will use an informal procedure, where we investigate and compare the cycles in the figures in chapters 4.1-4.7 and appendix C, to draw inference on economic movements over the sample period. A more formal analysis of the stylized facts of economic variables in the Norwegian economy will be given in chapter 5, where we concentrate on measures of volatility and correlations.

For expository convenience, we have divided the whole sample period into three subperiods; 1967-1979, 1979-1985 and 1985-1993. Below we will summarize the main patterns of economic fluctuations in the Norwegian economy in each of these periods.

1967-1979

Early in this period, the Norwegian economy went through a cycle with low growth rates in output, where money supplies were falling and prices were increasing. Norway had discovered huge oil resources by the end of the 1960s, so when the first adverse oil price shock occurred in 1973/1974, national net wealth increased. Most detrendings methods indicate that from the mid 1970s to the end of the 1970s the economy was booming. GDP, consumption and investment experienced a cyclical upturn in this period, although the amplitude and the dates of the upturns and subsequent downturns suggested by the different detrendings methods are varying somewhat. Generally we can also say that the fluctuations in GDP seemed more noisy and with a shorter periodicity than the fluctuations in e.g. consumption and investment. This may be the result of a very active countercyclical policy during this period. The fluctuations in prices remained relatively low and stable during most of the 1970s, probably due to the fact that the government conducted income policies and direct price control on several occasions from 1970 to the end of the 1970s. Unemployment also remained relatively stable until the end of the 1970s. Real wage showed evidence of large fluctuations early in this period. This is a result of the direct price and income controls implemented, in addition to the effects of the raw material price shock in 1973/1974 as Norway mainly exports raw materials and semi-manufactures.

1979-1985

By the time the second oil price shock occurred in 1979/1980, Norway was self sufficient with oil resources. In the first few years in this period, the economy experienced a set of smaller cyclical downturns and upturns, where prices increased early in the period (using the Beveridge and Nelson procedure). Most methods indicate that from 1982-1985, there was a large temporary recession in GDP, consumption and investment, but again GDP displayed shorter and more noisy cycles than the other variables. During the same period, cyclical unemployment and prices increased temporarily, whereas the money supply fell. Real wage and productivity show also evidence of a cyclical downturn in this period.

1985-1993

The financial deregulation in the Norwegian economy in the middle 1980s, triggered off a period of economic prosperity, that lasted until 1987/1988. GDP, consumption and investment were all booming, while unemployment rates fell. During this period, prices were also falling, whereas money supply increased slightly. From 1988/1989, a financial crisis occurred, which triggered off a subsequent recession, (the largest in the whole sample according to the Beveridge-Nelson methodology). GDP, consumption and investment fell back again, and unemployment rates rose to new high levels. Cyclical prices and money increased only slightly in this period, but real wages were falling (note that using the Beveridge-Nelson methodology, cyclical prices and money are increasing drasticly from 1988 and onwards). From 1993/1994, consumption, investment and GDP seem to be on an upward trend again, with unemployment rates falling slightly.

5. Stylized facts of business cycles in Norway

In this section we try to establish the stylized facts for Norwegian business cycles. We follow Lucas (1977) definition of business cycles, as "movements about trend in Gross National Product." A business cycle would be associated with the *comovements* between the deviations from trend (the business cycle) of gross national product - and the business cycles in various aggregate time series. These comovements would be stable over time and countries. Recent business cycle studies like Danthine and Girardin (1989), Kydland and Prescott (1990), Blackburn and Ravn (1992), Englund, Persson and Svensson (1992) and Fiorito and Kollintzas (1994) have typically followed Lucas (1977) and argued that a variable contains a business cycle if it displayes a significant cross correlation with GNP (GDP). We follow this terminology and specify a business cycle as *procyclical (countercyclical)* if the cross correlation with GDP is positive (negative). If on the other hand the correlation coefficient is close to zero, the series is said to be uncorrelated with the cycle, or *acyclical*. Further, if the highest correlation between a variable and GDP occurs when the variable is shifted backwards (forwards) relative to GDP, then the variable is said to be *leading (lagging)* GDP. Below, the facts are presented both in terms of volatility and correlations.

Rather than presenting stylized facts of business cycles using one decomposition method as in Kydland and Prescott (1992) that use a Hodrick- Prescott filter, we summarize the properties of the cyclical components in the time domain based on all the six different methods of trend-cycle decompositions presented in chapter 4. The idea is that we will be able to study whether the stylized facts of business cycles are independent of the method of decomposition used. Blackburn and Ravn (1991) and Canova (1993) have applied a similar 'multi method' approach when analysing stylized facts of business cycles in the UK and the US respectively. This paper differs in that we have applied a comprehensive analysis to the underlying dynamic processes in the series which can be used to discriminate between the results. In particular, when we have information with regard to the underlying dynamic process in the series from chapter 2 and 3, we use this information to discriminate between the business cycle properties obtained using the different methods.

The methods used in this chapter are again: A linear trend (LT), a quadratic trend (QT), a trend with a break/shift (LTB), the Hodrick- Prescott filter with $\lambda=16$ (HP-16), $\lambda=1600$ (HP-1600), $\lambda=160000$ (HP-160000), a random walk (RW), The Beveridge-Nelson decomposition with a high order ARIMA specification (BN-High) and a low order ARIMA specification (BN-Low) (see appendix E for details on the estimation), and the frequency domain method (FRE). In chapter 5.1 we first emphasize some of the differences between the decomposition methods used, in terms of the cyclical components that the different methods generate in the time domain. Some stylized facts of business cycles in Norway are then suggested. The results are presented as a set of summary statistics - sample moments and cross correlations. Chapter 5.2 investigates whether the different methods generate stylized facts that are invariable over time. The covariations of the cyclical components are finally analysed through spectral analysis in chapter 5.3. Chapter 5.4 studies the business cycle in some other countries.

5.1 Broad facts in the time domain

The Norwegian economy is small and rich in oil resources. 80 pct. of total GDP is generated from mainland activities, whereas the remaining 20 pct. is generated from oil and gas activities. In the analysis below, we use GDP from mainland Norway as the reference series, (denoted GDP). Results using total GDP can be obtained from the author on request, although the main findings are largely unchanged using either mainland- or total GDP as the reference series.

The growth rates in many economic variables changed drastically in the 1970s. A typical business cycle in the Norwegian economy would be led by changes in demand for Norwegian traditional exports, which would be followed by changes in investment, unemployment and consumption (cf. Wettergreen 1978). After the discoveries of hugh oil resources in the 1970s, the economy adjusted to a new situation. Today 35-40 pct. of total exports are exports of oil and gas. The economy also experienced important changes during the 1980's due to finance and capital deregulations that generated a consumption boom from the mid 1980s. As agents' behaviour may have changed drastically over a period with institutional and structural changes, the stylized facts obtained in this chapter may be sensitive to such a change. However, some stylized facts seem to stand out as robust over the whole period, and by studying the sample over subperiods, additional information can be obtained. For some of the variables, the results using different methods will vary. However, by using the information we have found in the previous chapters, some conclusions can be drawn.

In table 5.1 we report the first five autocorrelations of GDP that each method generates. This will give us an idea of duration or persistence of the business cycles. As can be seen from the table, the autocorrelation pattern for GDP varies with the methods used. Generally, we can divide the methods into two groups with regard to what type of serial correlation they generate in the business cycle. One type generates slowly decaying positive autocorrelations, indicating a 'persistent' pattern for the cycle.[21] The other group shows a shifting pattern of negative and positive autocorrelations giving a small persistent, noisy pattern for the cycle. In the first group we find all the deterministic trends, together with some of the smoothest stochastic trends and the frequency domain filter approach; LT, QT, LTB, HP-160000 and FRE. The highest degree of persistence in the cycle is found in the three deterministic trends and HP-160000, whereas the cycle generated by FRE decays most quickly. In the second group we find some of the most stochastic volatile trends; HP16, RW and BN-low. All three methods generate negative first order autocorrelations, with a first order autocorrelation as low as -0.44 for the RW. Note that by choosing a λ as low as 16, HP-16 generates a pattern in the autocorrelations in line with the stochastic trends generated by the low order ARIMA models. The autocorrelation patterns generated by HP-1600 and the BN-high methods are somewhere in between the two groups, although their first order autocorrelations are positive.[22] The cycle generated by HP-1600 decays most quickly whereas the BN-high method generates a more oscillating pattern for the cycle.

Comparing these findings to the graphs of spectra discussed in chapter 4, we see that those methods that generated most power at the high frequencies, are also those methods that give the most noisy pattern for the cycle (RW, HP-16 and BN-low). This is obvious, as the autocovariances and spectrum are uniquely determined by each other. In this section we will however focus on the autocovariances, as they give information on serial dependence in the variables which is a more useful way to distinguish the different detrendings methods here, (whereas the spectrum give information about important periodic components in the data).

Table 5.1. Autocorrelations (r_i) of the cyclical component of GDP

	LT	QT	LTB	HP 16	HP 1600	HP 160000	RW	BN High	BN Low	FRE
1	0.88	0.56	0.61	-0.21	0.33	0.69	-0.44	0.40	-0.41	0.89
2	0.84	0.52	0.58	-0.09	0.30	0.67	0.04	0.54	-0.13	0.62
3	0.80	0.46	0.53	0.02	0.24	0.61	0.17	0.58	0.14	0.33
4	0.72	0.26	0.37	-0.27	-0.01	0.46	-0.19	0.42	-0.17	0.11
5	0.68	0.23	0.37	-0.02	0.02	0.43	0.10	0.45	0.05	-0.02

[21] The term 'the persistent pattern for the cycle' here refers to the high degree of serial correlation in the cycle, and must not be confused with *how* persistent the effects of the shocks to the level of the economic series are, as was discussed in chapter 3.

[22] For the high order BN method, an ARIMA model with 20 AR lags had to be estimated before the first order autocorrelation was positive, and the other autocorrelations showed a pattern of persistence. See appendix E for a further discussion of the use of ARIMA models for the BN method.

In table 5.2, we investigate the absolute standard deviation of the *cyclical component*, expressed in percentages. This will give us an idea of the amplitude of the cycle. The results are unsurprising. The less volatile the trend, (as e.g. the LT, where the standard deviation in the innovation in the trend is zero), the more volatility is attributed to the cycle (plus noise) in the series. This was further emphasized using the spectral analysis in chapter 4. There we saw that some of the most persistent cycles generated by methods such as LTB, is composed of more low frequency components than for instance the RW and low order BN method, where the cycles consist mostly of high frequency components, or noise.

As expected, for almost all variables standard deviations are largest for most of the methods in the first group mentioned above, (the deterministic-plus smooth stochastic trends), especially using LT, QT, LTB and HP-160000. Volatility is lowest in the second group, (that is the most volatile stochastic trends), especially for BN-low and HP-16. The results for FRE, HP-1600, BN-high and RW vary somewhat with the variables under study. However, generally, we can say that BN-high and RW are the least volatile of them and belong in the second group, whereas HP-1600 and FRE are the most volatile of them and belong to group one. Note nevertheless that for some variables (government consumption, CPI and M2) HP-1600 and FRE are less volatile than BN-high and BN-low, whereas for government consumption, GDP and productivity, HP-1600 and FRE are less volatile than RW.

In chapter 2, for none of the variables could we reject the hypothesis of a unit root in favour of a deterministic linear trend. Hence, volatility measures reported by LT in table 5.2 are probably too high. For investment, government consumption, real wages and unemployment, we found evidence of a linear trend with break or shift in chapter 2, and using the LTB specification reduces volatility compared to the LT specification. The different stochastic trends generate quite different volatility in the series. For instance, for M2 and CPI, BN-low and BN-high generate four to six times more volatility in these variables than what RW does. However, as discussed in chapter 4.6, when a variable generates high persistence, (A(1) and V is in excess of one in table 3.1), the cyclical behaviour of the variable will be generated by an excessive moving trend using the BN method, where the variance of the innovations in the trend is larger than the variance of the innovation in the series itself. This is seen especially for CPI and M2 in table 3.1, where the variance of an innovation in the trend is about 16 times the variance of an innovation in the series using BN-low and BN-high.

The spread of the volatility measures generated by the different methods for each series is rather large. For instance percentage standard deviation in GDP is 0.81 pct. using BN-low and 5.16 pct. using LT. Percentage standard deviation for consumption is 0.81 pct. for BN-low and 5.13 pct. for LT, whereas for investment, the range is from 1.19 pct. using BN-low to 15.36 pct. using LT. Volatility for productivity lies in the range 1.24 pct. using BN-low to 3.97 pct. using LT, whereas for real wage the range is from 0.31 pct. with BN-low to 3.93 pct. for LT. Percentage standard deviation for CPI varies from 0.51 pct. with HP-16 to 6.55 pct. with LT, whereas the range for M2 is from 0.78 pct. with HP-16 to 6.62 pct. using BN-low.

Table 5.2. Standard deviations of the cyclical component, in percentage[1,2]

	LT	QT	LTB	HP 16	HP 1600	HP 160000	RW	BN High	BN Low	FRE
GDP	5.16	2.67	2.78	1.51	2.13	3.13	2.46	1.45	0.81	1.66
C	5.13	3.17	2.75	1.26	2.32	3.45	2.1	1.82	0.81	1.95
G	5.43	1.85	1.27	0.84	1.11	2.41	1.3	1.23	NA	1.10
I	15.36	8.45	6.89	2.30	5.12	9.37	3.72	4.94	1.19	4.49
X	6.66	6.28	5.61	2.60	4.30	5.93	3.60	2.85	NA	3.91
M	8.25	7.18	6.36	2.81	5.49	7.31	3.95	4.43	NA	5.07
PR	3.97	2.19	2.61	1.63	2.01	2.52	2.51	1.33	1.24	1.40
U	88.00	58.00	52.00	18.00	44.00	62.00	28.00	33.00	11.00	42.00
RWG	3.93	2.94	2.81	1.34	2.05	2.96	0.93	1.41	0.31	1.63
CPI	6.55	5.09	2.98	0.51	1.54	4.55	1.41	3.28	5.05	2.59
M2	6.99	4.41	2.29	0.78	1.68	4.35	1.15	4.95	6.62	2.03
R	0.67	0.40	0.39	0.25	0.36	0.49	0.36	0.25	NA	0.35
OP	57.91	32.69	24.3	10.54	20.29	34.52	14.41	6.62	NA	19.31

[1] For all variables except the unemployment rate (U) and the interest rate (R), we report the percentage standard deviation. For U and R, we report 100 times the percentage point change.
[2] For a definition of the variables, see table 2.1.

The ranking of the variables according to their percentage standard deviations varies also between the methods used. Nevertheless, regardless of the methods, volatility is highest for the oil price series, (disregarding the unemployment series and the interest rates that are not measured in logarithms, but in percentage point times 100). All methods (ignoring BN-low for now) also include investment, imports, and exports among the six most volatile series, although the ranking of these three vary somewhat between the methods used. Further, LT, QT, HP-160000, FRE and BN-high all consider CPI and M2 among the six most volatile series, LTB consider CPI and real wages among the six most volatile series, whereas HP-16, HP-1600 and RW consider GDP and either productivity or consumption among the six most volatile series. Finally, most of the methods suggest government consumption to be the least volatile variable, except LT and RW that suggest real wages to be the least volatile and HP-16 that suggest CPI to be the least volatile series. Of the eight variables reported by BN-low, CPI and M2 are the most volatile, whereas real wage has the lowest volatility.

In table 5.3, we report the standard deviation of each series relative to GDP. The distinction between the different methods in terms of what they report on the variation in each variable relative to the variation in GDP is more difficult to interpret below, as the different methods have ranked the volatility of GDP and the other variables differently. However, the smallest percentage standard deviations of most series relative to GDP are found for RW and HP-16, as they rank GDP to be among the more volatile of the series than the other methods do, (see table 5.2).

Consumption is about as volatile as GDP, with the range varying from 0.83 using HP-16 to 1.26 using BN-high. Investment is more volatile than GDP, with the range varying from 1.47 with BN-low to 3.41 for BN-high. Productivity and real wage are mostly less

Table 5.3. Standard deviations of the cyclical component relative to GDP[1]

	LT	QT	LTB	HP 16	HP 1600	HP 160000	RW	BN High	BN Low	FRE
C	0.99	1.19	0.99	0.83	1.09	1.10	0.85	1.26	1.00	1.17
G	1.05	0.69	0.46	0.56	0.52	0.77	0.53	0.85	NA	0.66
I	2.98	3.16	2.48	1.52	2.40	2.99	1.51	3.41	1.47	2.70
X	1.29	2.35	2.02	1.72	2.02	1.89	1.46	1.97	NA	2.36
M	1.60	2.69	2.29	1.86	2.58	2.34	1.61	3.05	NA	3.05
PR	0.77	0.82	0.94	1.08	0.94	0.81	1.02	0.92	1.53	0.84
U	17.05	21.72	18.7	11.92	20.66	19.81	11.38	22.76	13.58	25.30
RWG	0.76	1.10	1.01	0.89	0.96	0.95	0.38	0.97	0.38	0.98
CPI	1.27	1.91	1.07	0.34	0.72	1.45	0.57	2.26	6.23	1.56
M2	1.35	1.65	0.82	0.52	0.79	1.39	0.47	3.41	8.17	1.22
R	0.13	0.15	0.14	0.17	0.17	0.16	0.14	0.18	NA	0.21
OP	11.22	12.24	8.74	6.98	9.53	11.03	5.86	4.57	NA	11.63

[1] For a definition of the variables, see table 2.1.

volatile than GDP. The range for CPI relative to GDP is from 0.34 using HP-16 to 6.23 using BN-low, whereas for M2, the range is from 0.47 for RW to 8.17 for BN-low.

The cross correlations of the cyclical components in the Norwegian variables are reported in table 5.4. For some variables, the stylized facts of cross correlations are suggestive although the absolute value of the correlation coefficient may differ between the methods used. This is so for consumption, imports, investment, productivity and unemployment. *Consumption* is clearly procyclical, but the range varies from 0.17 to 0.88.[23] *Import* is also clearly procyclical, with the range of correlations varying from 0.13 to 0.72. For both consumption and import, the procyclical behaviour is smallest for the 'stochastic methods': HP-16, RW, BN-low and also BN-high. *Productivity* is also procyclical, but now the range of correlations is narrower, from 0.59 to 0.88.

Unemployment shows a countercyclical behaviour in the range from -0.13 to -0.87, where all the methods except both the BN methods, RW and LT indicate that unemployment leads the cycle by one quarter. Again, RW, HP-16 and both the BN-methods generate the lowest autocorrelations in absolute value. That unemployment is countercyclical and leading the cycle by one quarter, implies that in a typical recession, employees are laid off before output (GDP) is cut.

Investment is procyclical for all methods except BN-low.[24] Three methods also suggest it is leading the cycle. The maximum range of correlations vary from -0.17 to 0.9. However, given that we rejected the unit root hypothesis in favour of the linear trend with shift in chapter 2, we support the evidence given by LTB and FRE and suggest that investment is highly procyclical.

[23] The range reported here and below refers to the maximum correlation among the five leads and lags that are found in table 5.4.

[24] However, BN-low indicates that investment is slightly procyolical when it is lagging GDP with two quarters, the correlation coefficient being 0.13.

The results for *export* are more difficult to interpret, the range of correlations varying from -0.46 to 0.44. When export is modelled using a linear or quadratic deterministic trend or a smooth stochastic trend, (LT, QT, HP-160000, HP-1600 and FRE), it displays a countercyclical pattern leading the cycle by four quarters. Using a deterministic trend with break or a more noisy stochastic trend (LTB, HP-16, RW or BN-high) it displays a procyclical pattern, lagging the cycle by up to two quarters. However, the contemporaneous correlations are small and using the large sample standard deviation ($\sqrt{108}$), in most cases the correlations are insignificant indicating that export is acyclical. Export is analysed further in chapter 5.2 where we analyse the cross correlations in different subperiods.

Real wage displays a similar conflicting pattern, the range being from -0.3 to 0.6. Using LTB, BN-high and FRE, real wage is countercyclical, lagging the cycle. For the other methods real wage is procyclical, and disregarding LT, the correlation is highest for BN-low and RW. Given that we believe we have captured the business cycle component with FRE, and that LTB is appropriate (as we could reject the hypothesis of a unit root in favour of a deterministic trend with shift in chapter 2), it seems reasonable to suggest that the business cycle behaviour of real wage relative to GDP is countercyclical, and the procyclical behaviour captured by HP-16, BN-low and RW is high frequency 'white noise' correlation. This has also been confirmed by the peak in the spectra for GDP and real wages at the high frequencies using HP-16, BN-low and RW, in chapter 4. However as the countercyclical correlations are rather low, we are suspicious that the cross correlations can have changed over the sample period and analyse the cross correlations further in different subsamples in chapter 5.2.

For the *Consumer Price Index*, LT, QT, HP-160000 and BN-low all display a procyclical pattern, whereas LTB, HP-16, HP-1600, RW, BN-high and FRE give a countercyclical pattern, the range being from -0.62 to 0.69. Given that we have some evidence that we cannot reject the unit root hypothesis for CPI, the smooth deterministic trends that show a procyclical pattern here seem inappropriate for capturing the business cycle component. From the spectra in chapter 4 and appendix D, we can also suggest that FRE, BN-high and LTB all show a countercyclical pattern at the business cycle frequencies, with CPI lagging the cycle by four to five periods. RW and HP-16 which let through more of the high frequencies components are also both countercyclical, although now CPI is neither leading nor lagging the cycle. Hence, it seems reasonable to suggest that the correlation at the business cycle components and some of the noise components are countercyclical, lagging the cycle by about a year.

Finally, note that BN-low in contrast to BN-high suggests a procyclical pattern for CPI, although from the plot of CPI in appendix C, we see that the cycles in CPI generated by BN-low and BN-high (during their common sample) are similar, and quite different than what the other methods suggest. From table 5.2 (and 5.3), we can also see that both BN-low and BN-high generate the most volatile behaviour of the cycle in CPI (relative to GDP) of all the methods. Recall also that in chapter 2 we found that CPI may be better represented as I(2) (or I(1) with drift) than as I(1), so both BN-high and BN-low may have misrepresented the behaviour of CPI. The high value of A(1) reported in table 3.1 on CPI by both BN-low and BN-high also indicates that the cycle is generated by making the trend component excessive volatile. The difference in correlations is therefore most likely due to the fact

Table 5.4. Contemporaneous and maximum cross-correlation of GDP with:[1,2]

	LT	QT	LTB	HP 16	HP 1600	HP 160000	RW	BN High	BN Low	FRE
C	0.88	0.64	0.57	0.45	0.56	0.72	0.41	0.50	0.17	0.64
	-	-	-	-	-	-	-	0.57	0.19	-
								(+4)	(+4)	
G	0.81	0.00	-0.03	-0.01	0.02	0.37	0.10	-0.30	NA	0.15
	-	-	-0.09	-0.20	0.11	0.39	0.14	-0.40		0.30
			(+1)	(+1)	(-4)	(+3)	(+3)	(+2)		(-5)
I	0.90	0.64	0.65	0.28	0.52	0.74	0.30	0.64	-0.05	0.65
	-	-	-	-	-	0.76	-	0.66	-0.17	0.72
						(-2)		(+1)	(-1)	(-1)
X	-0.33	-0.11	0.31	0.00	-0.08	-0.11	0.02	0.12	NA	-0.01
	-0.46	-0.17	0.44	0.28	-0.28	-0.25	0.23	-		-0.37
	(-4)	(-4)	(+2)	(+2)	(-4)	(-4)	(+2)			(-4)
M	0.72	0.66	0.51	0.31	0.57	0.67	0.29	0.13	NA	0.68
	-	-	-	-	-	-	-	0.33		-
								(+5)		
PR	0.88	0.59	0.78	0.83	0.68	0.71	0.84	0.63	0.65	0.65
	-	-	-	-	-	-	-	-	-	0.68
										(+1)
U	-0.87	-0.66	-0.52	-0.23	-0.53	-0.72	-0.13	-0.43	-0.13	-0.72
	-	-0.68	-0.59	-0.33	-0.57	-0.74	-0.25	-0.61	-	-0.74
		(-1)	(-1)	(-1)	(-1)	(-1)	(+2)	(+5)		(-1)
RWG	0.60	0.09	-0.09	0.35	0.14	0.21	0.42	-0.14	0.41	-0.14
	-	0.16	-0.30	-	0.20	0.30	-	-0.27	-	-0.21
		(-5)	(+4)		(-5)	(+2)		(+4)		(+2)
CPI	0.58	0.11	-0.53	-0.29	-0.19	0.23	-0.15	-0.35	0.12	-0.14
	0.69	0.26	-0.62	-	-0.32	0.45	-	-0.44	-	-0.51
	(-4)	(-4)	(+5)		(+5)	(-4)		(+4)		(+5)
M2	0.76	0.28	0.02	0.13	0.26	0.45	0.11	-0.49	0.02	0.37
	0.78	0.35	0.10	-	0.28	0.55	0.14	-	0.05	0.46
	(-2)	(-3)	(-5)		(-2)	(-3)	(-2)		(+3)	(-1)
R	0.57	0.12	0.04	0.02	0.11	0.41	-0.07	0.15	NA	0.27
	0.62	0.29	0.24	0.25	0.33	0.54	-0.29	0.24		0.58
	(-2)	(-2)	(-2)	(-2)	(-2)	(-2)	(+2)	(-2)		(-2)
OP	0.55	-0.11	-0.08	0.10	-0.02	0.13	0.04	-0.16	NA	0.04
	0.58	-0.34	-0.17	0.34	-0.21	0.18	0.16	-		0.06
	(+2)	(-5)	(+5)	(-2)	(-5)	(+5)	(+2)			(-2)

[1] Each cell contains in the first row the contemporaneous cross correlation between GDP and the individual series. The second row contains the maximum correlation, (if different from the contemporaneous correlations), between GDP(t) and the individual series(t-k), (k=-5,-4,...0,...,4,5), with the chosen number of lead (-) / lag (+) in parenthesis below. Hence, a value -5/(+5) in parenthesis, implies that the series leads /(lags) GDP by 5 quarters.

[2] For a definition of the variables, see table 2.1.

that the cycle generated in GDP by BN-high (which is more in line with what e.g. FRE suggests) is different from the cycle generated by BN-low, especially from the latter part of the 1980s, (see figure 4.6b). To understand the correlation between GDP and CPI better, we investigate the cross correlations in different subperiods in chapter 5.2.

M2 is procyclical leading the cycle using all methods, except for BN-high which shows a countercyclical behaviour, the range varying from -0.49 to 0.78. The procyclical correlation is smallest for LTB and BN-low that indicate that M2 is acyclical. The countercyclical behaviour of M2 relative to GDP found when using BN-high, may again be due to the fact that M2 is better represented as I(2) (or I(1) with drift), than I(1), so the BN method generates excess volatility in the trend. From table 5.3, we see that BN-high (and BN-low) generate the highest volatility in M2 relative to GDP, and table 3.1 suggests that the volatility in the trend is about 16 times the volatility in the series itself. As BN-high and BN-low generate a similar pattern for M2 in figure C.7b in appendix C, the difference in the correlation coefficient with GDP is due to their different treatments of GDP, (again see figure 4.6b). However, the remaining stochastic methods and FRE suggest a procyclical pattern, where M2 is leading the cycle.

Government consumption is procyclical using FRE, where the business cycle component is emphasized. However, using either LTB or HP-1600, government consumption is acyclical using the large sample standard deviation ($\sqrt{108}$). The range of correlations vary from -0.4 to 0.8.

Interest rates are procyclical leading the cycle for all methods except RW, although for many of the methods the correlation coefficient is close to zero. Interest rates are analysed further in chapter 5.2, where the cross correlations in different subperiods are analysed. *Oil price* shows no conclusive evidence of being either procyclical or countercyclical, and FRE indicates it is acyclical.

We conclude this chapter by summing up the main findings. Despite the fact that some methods generate quite different trends and cycles, much can be learned by analysing the dynamics of the secular and cyclical component separately and in conjunction. We claim that based on the results documented here, we have come across some stylized facts:

First, we have shown how the different methods generate different serial correlation in the cycle, with the most smooth deterministic trends generating most serial correlation in the cycle and the most 'noisy' stochastic trends the least serial correlation in the cycle.

Second, we have seen that the standard deviation of the different variables will vary with the different methods suggested. The more volatile the trend is, the less volatile will the cycle be. Nevertheless, most methods rank the cyclical component of oil prices, investment, export and import as the most volatile series, whereas the cyclical component of government consumption is mostly ranked as the least volatile series. The ranking of volatility in the variables measured relative to volatility in GDP varies between the methods, although some of the stochastic 'noisy' methods still consistently give the lowest standard deviations for the series relative to GDP. Generally, consumption and productivity are equally volatile as GDP, whereas import, export and investment are up to three times as volatile as GDP. For some variables the range of relative volatility is large, and e.g. M2 varies from being about 0.47 as volatile as GDP to being more than 8.17 times as volatile as GDP.

Third, the cross-correlations also varies between the methods used. Although some variables are persistently procyclical (consumption, import and productivity) or persistently countercyclical (unemployment), the absolute magnitude of the correlations varies, being usually highest with the deterministic smooth trends and lowest with the stochastic noisy trends. Some variables (investment and M2), indicate a procyclical pattern using most of the methods. For other variables (export, real wage and CPI), half of the methods suggest a procyclical pattern, and the other half a countercyclical pattern. By emphasizing the business cycle frequencies, export is acyclical, whereas both real wages and CPI will be countercyclical. Interest rates are procyclical although the correlations are not very significant. For oil prices and government consumption, the correlations are inconclusive and insignificant, indicating that both are acyclical.

In the above analysis we have captured the sample moments based on the whole sample period. As we suspect the moments may have changed over time due to structural changes, i.e. policy changes or changes in the behaviour of the agents, we now turn to study the sample moments (of the cross correlations) over time in Norway, when they are computed as a fixed fraction of the full sample.

5.2 Principal regularities and stability analysis

Lucas (1977) definition of stylized facts of business cycles requires that although they may not have an uniform periodicity or amplitude, the comovements reported must be regular over time. In this chapter we try to cast some more light on some of the stylized facts of the sample moments reported above, by analysing them over subperiods of time. Generally, it is believed that if the correlations between two variables are changing over time, we have no evidence of a structural relationship between the two variables. However, the following example taken from Bårdsen et al. (1995), illustrates that bivariate correlations may hide structure despite the fact that the correlation coefficient changes over time. Define the correlation coefficient between x and y as $r_{yx}^2 = \hat{\alpha}\hat{\beta}$, where $\hat{\alpha}$ is the OLS coefficient from the bivariate regression of y on x, and $\hat{\beta}$ is the OLS coefficient from the inverse bivariate regression of x on y. Although r^2_{yx} is unstable, $\hat{\alpha}$ may be stable over the whole sample and have a structural interpretation, if all instability comes from $\hat{\beta}$ in the inverse regression. Although this is a simple example, the same ideas can be used in multiple regressions, where r_{yx} is interpreted as a partial correlation coefficient. However, the purpose here is not to reveal underlying structural relationship, other than those implied by the correlation coefficient.

Blackburn and Ravn (1991) analyse their cyclical properties found by recomputing the sample moments recursively over time. A recursively calculated moment will keep the initial (or terminal) date fixed, and will finally reach full sample size. We study instead 'rolling' correlations, where the moments are computed as a fixed fraction of the full sample, that is shifted over time. The fraction used to calculate the rolling correlations corresponds to 8 years, (32 quarters), the maximum expected length of a business cycle. The first observation in the rolling correlation, will be placed in the mid point in the first eight year sample. The correlations of GDP with export, real wage, CPI and interest rates are analysed below. The results are presented as a set of figures.

Figure 5.1. Rolling correlations of export

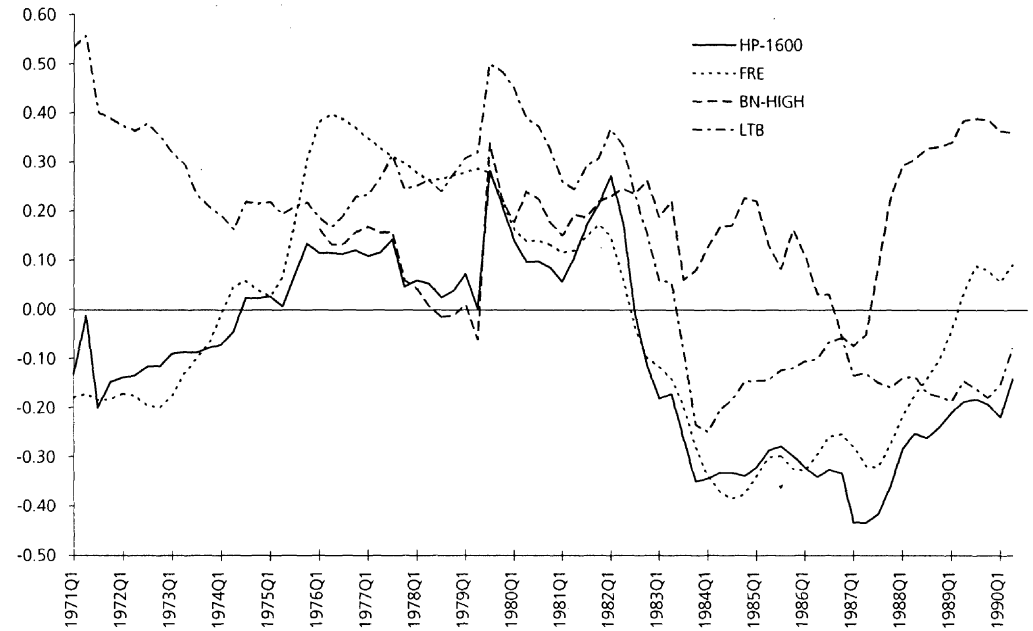
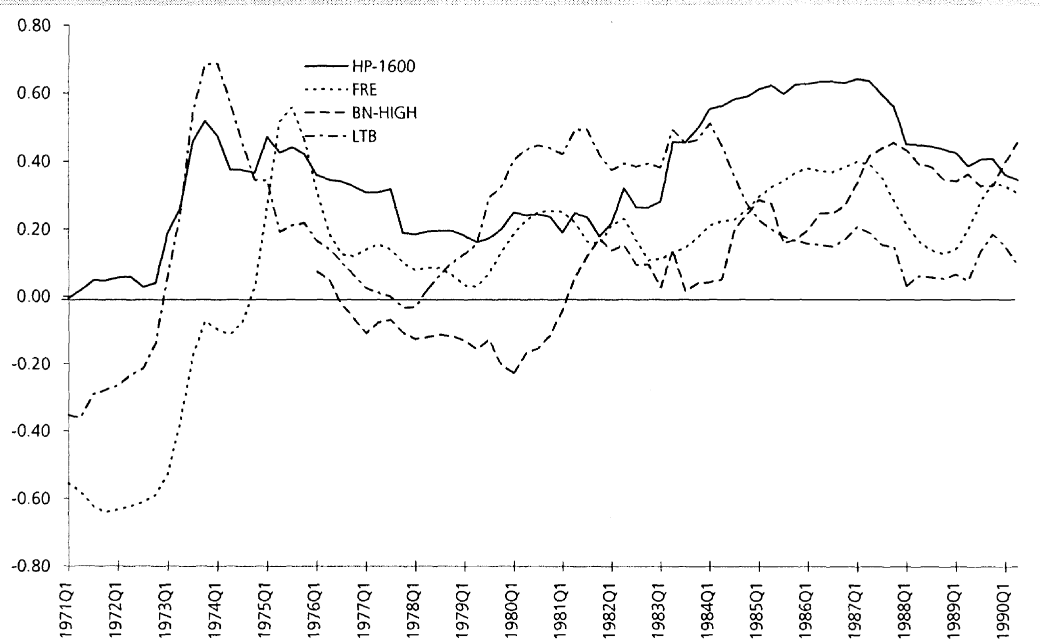


Figure 5.2. Rolling correlations of real wage



In table 5.4, half of the methods (e.g. HP-1600 and FRE) suggested export behaved countercyclically, whereas the other half (e.g. BN-high and LTB) indicated that export behaved procyclically. By analysing the correlation coefficient in sub periods in figure 5.1, we find that export behaves procyclically in some periods and countercyclically in other periods. In the early 1970s, HP-1600 and FRE indicate that export is slightly countercyclical, whereas LTB shows export to be procyclical.[25] All four methods show export to be procyclical in most periods from the middle 1970s to the early 1980s, although the coefficient is fluctuating. Thereafter export behaves countercyclically for all methods but BN-high, which is only negative a short period in the middle 1980s. From the late 1980s the correlation coefficient of all methods are on an upward trend. Hence, little can be concluded about the correlation between exports and GDP over the whole sample from figure 5.1, and FRE which emphasize the business cycle component, shows export to be both countercyclical and procyclical.

Real wage seems to behave procyclically from the 1980s for all methods reported in figure 5.2 (HP-1600, FRE, BN-high and LTB). However, in the early 1970s, FRE and LTB indicate that real wages are countercyclical, whereas in the middle/late 1970s, BN-high suggests real wages to behave countercyclically. Overall then, the countercyclical behaviour indicated by BN-high, FRE and LTB in table 5.4, seems basically to stem from the 1970s.

The countercyclical behaviour of CPI suggested in table 5.4 by HP-1600 and FRE, is supported by their rolling correlations from the middle of the 1970s and onwards in figure 5.3. The correlations generated by BN-low and BN-high behave quite differently from those generated by HP-1600 and FRE. Until the middle of the 1980s, the rolling correlations generated by BN-high and BN-low indicate that CPI is procyclical. Only from the middle of the 1980s, is the rolling correlation generated by BN-high negative, explaining why in table 5.4, BN-low predicts a procyclical pattern for CPI, whereas BN-high predicts a countercyclical pattern for CPI.

Finally, we analyse interest rates. The procyclical behaviour suggested in table 5.4 is confirmed by the rolling correlations in figure 5.4. Both FRE and HP-1600 show interest rates to be procyclical, and FRE suggest this behaviour has become more positive over time. BN-high on the other hand, shows the correlations to be fluctuating around zero.

We conclude by summing up the main findings here. Depending on the methods used, traditional export shows both a procyclical and a countercyclical pattern, and studies over sub periods of the sample indicate that it is most likely procyclical in the 1970s and countercyclical in the 1980s, (especially when adjusted for a break in the trend in the 1970s). The real wage is either countercyclical or procyclical. Studies over sub periods of the sample, show that the real wage behaves countercyclically in the early 1970s

[25] Note that BN-high uses a shorter sample than the other methods, so that the first rolling correlation starts in 1976, as we loose some observations at the beginning due to the ARMA specifications.

Figure 5.3. Rolling correlations of CPI

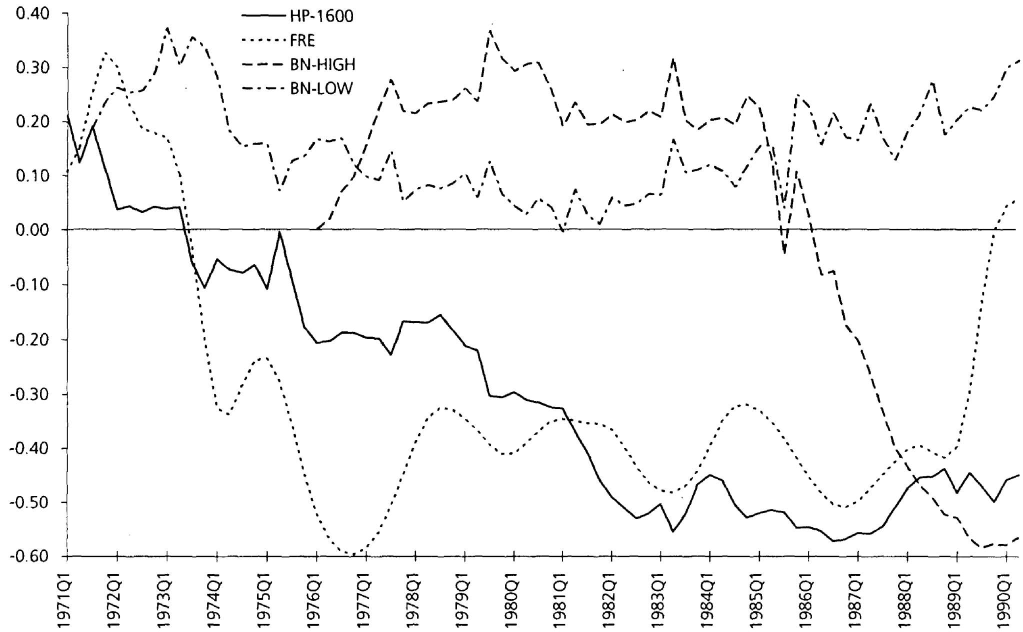
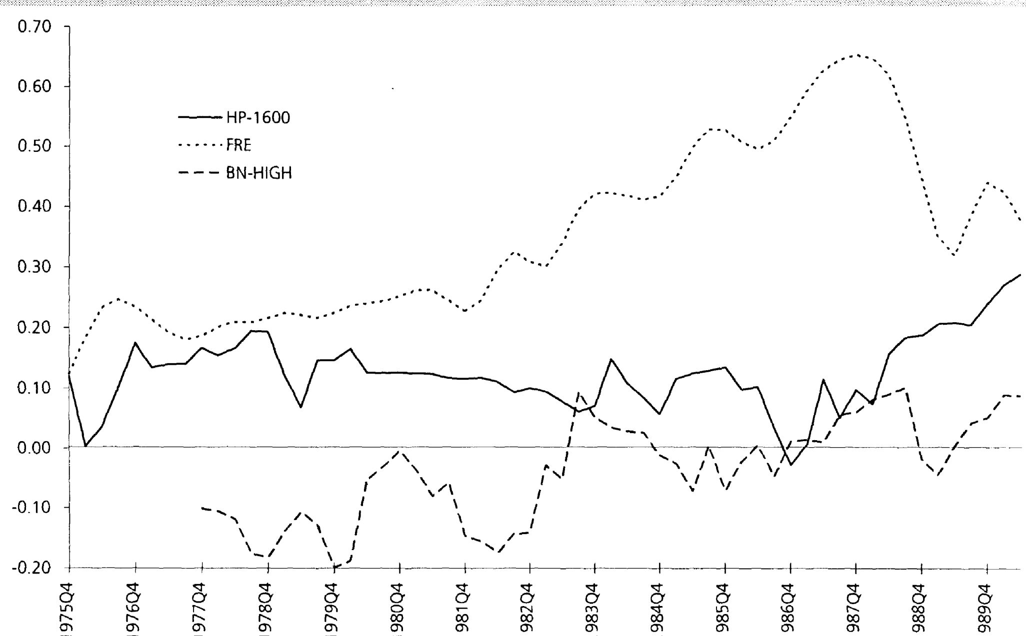


Figure 5.4. Rolling correlations of interest rate



whereas from the 1980s, real wage has been mostly procyclical. Most methods (over the whole sample or in sub periods) indicate that price is countercyclical and lagging the cycle by more than a year. That price has behaved in a countercyclical way in the post war period has been one of the main stylized facts established in other international studies of business cycles, see e.g. Kydland and Prescott (1990). The interest rate is mostly procyclical, at least during most of the 1980s.

5.3 The (squared) coherence function

Whereas we previously have presented the 'stylized facts' as a set of summary statistics in the time domain, we now look at the relations between two stationary variables in the frequency domain, where we emphasize the coherence function. The (squared) coherence function between two series c_{1t} and c_{2t} at frequency ω, is defined as:

$$(5.1) \qquad Ch_{c_1c_2}(\omega) = \frac{|S_{c_1c_2}(\omega)|^2}{S_{c_1}(\omega)S_{c_2}(\omega)}$$

where $S_{c_1c_2}(\omega)$ is the cross spectrum of two series c_{1t},c_{2t}, defined as:

$$(5.2) \qquad S_{c_1c_2}(\omega) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty} e^{-i\tau\omega} C_{c_1c_2}(\tau)$$

where $C_{c1c2}(\tau)$ is the cross-covariances of c_{1t} and c_{2t}. The cross spectrum may be estimated using a periodogram like in (4.7), but replacing $s(\tau)$ with an estimate of the corresponding cross-variance, $r_{c1c2}(\tau)$. Below we concentrate on CPI and real wage, and plot the coherence between GDP and CPI, and GDP and real wage in figure 5.5 and 5.6 respectively, using LTB, HP-1600, BN-high, BN-low, RW and FRE. Generally it is difficult to interpret these figures, as the coherence may not have a distinct peak that dominates.

BN-high, FRE, LTB, HP-1600 and RW showed a countercyclical pattern between GDP and CPI in table 5.4. From figure 5.5, we can see that for BN-high and LTB, this countercyclical correlation is mainly generated by a high coherence at the low and intermediate frequencies, with declining weights to the higher frequencies. The coherence using FRE has also a peak at the lowest frequencies, although now the highest frequencies are much more emphasized. For RW and HP-1600 the whole spectra of frequencies are emphasized, whereas for BN-low (which is the only one in figure 5.5 that generated a procyclical correlation in table 5.4), all but the lowest frequencies are emphasized. As LTB and BN-high display the most significant contemporaneous correlation coefficients in table 5.4, it is reasonable to suggest that the countercyclical correlations between GDP and CPI stem mainly from the low frequencies.

Figure 5.5. Coherence between GDP and CPI using different detrendings methods

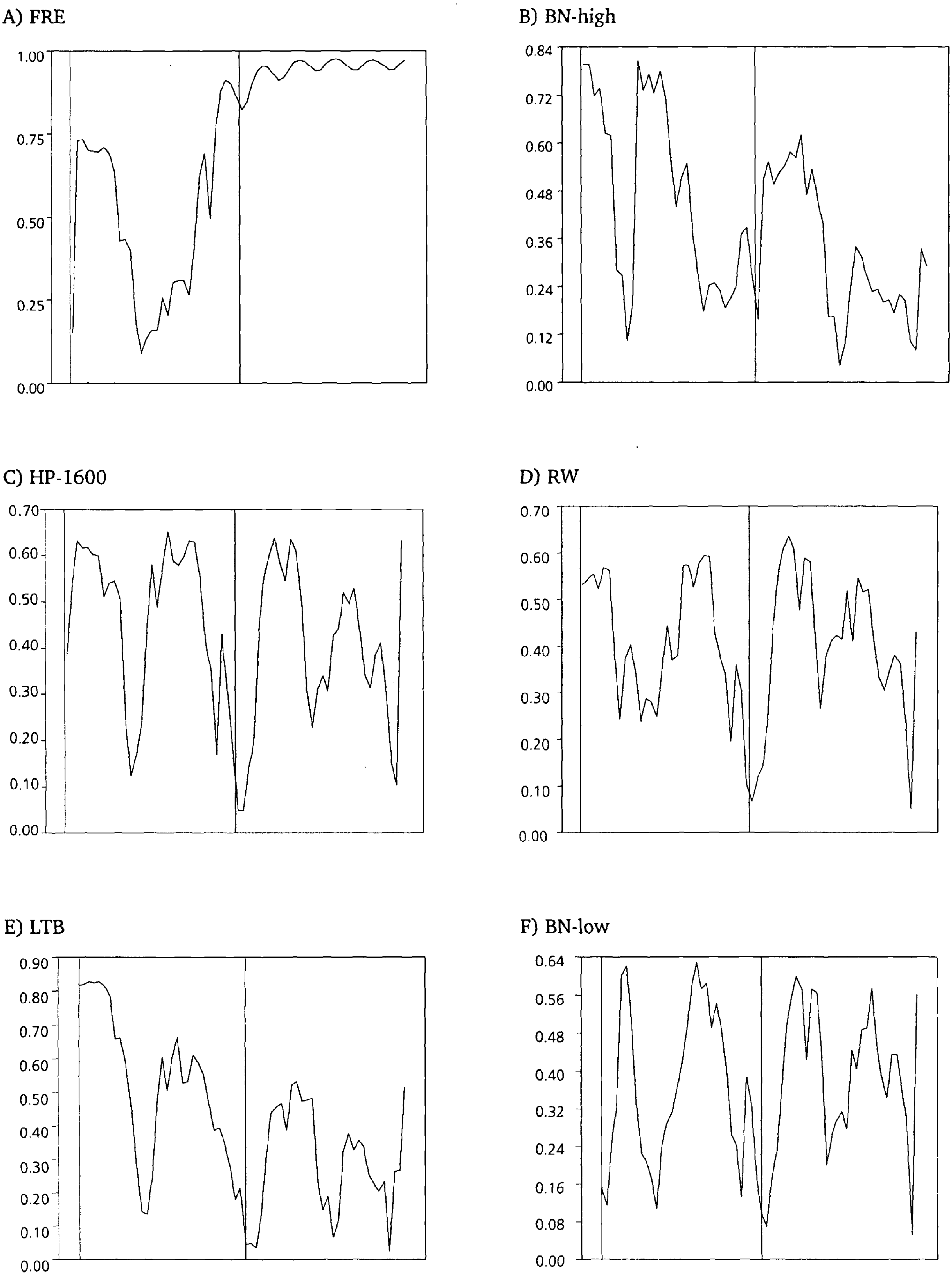
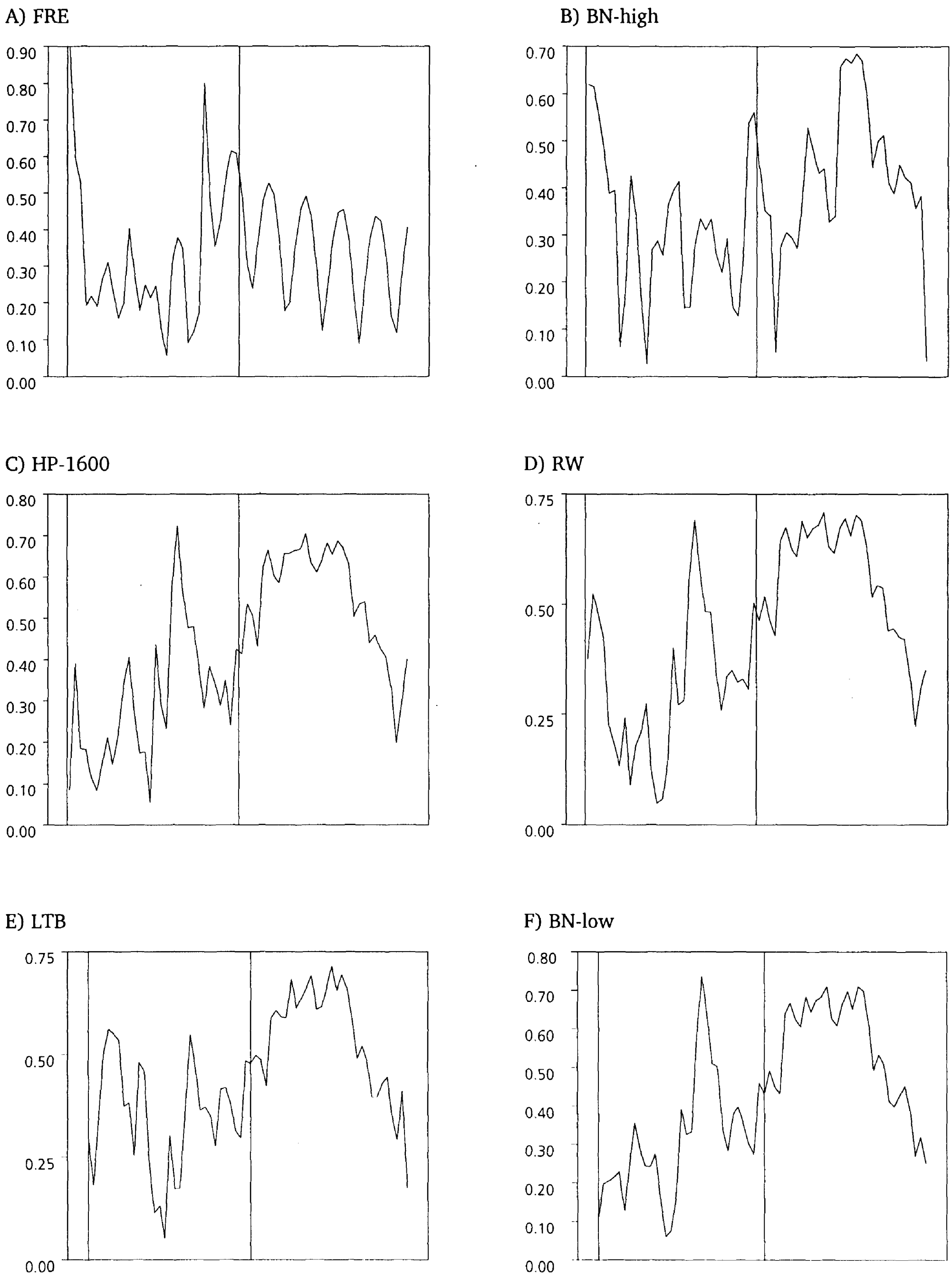


Figure 5.6. Coherence between GDP and real wage using different detrendings methods



In table 5.4, BN-high, FRE and LTB indicated a countercyclical pattern between GDP and real wage, whereas BN-low, HP-1600 and RW showed a procyclical correlation between GDP and real wage. In figure 5.6, BN-high, FRE and LTB emphasize the low or intermediate frequencies in the coherence between GDP and real wage, although for LTB the high frequencies are also important. For BN-low, HP-1600, and RW the high frequencies are emphasized. We therefore confirm the conclusions from chapter 5.1 and 5.2, namely that the procyclical pattern shown by BN-low, HP-1600 and RW is due to high frequency (white noise) correlations, whereas the countercyclical pattern indicated by BN-high, FRE and LTB stem mainly from the lower frequencies.

5.4 International business cycles

Finally, we study the cycle in some selected countries; Denmark, Finland, Germany, Norway, Sweden, the UK and the US. All the cycles are computed using the frequency filtering method, as defined in chapter 4.7. This seems appropriate, as we want to analyse the correlations at the frequency components without saying anything about the underlying process. The sample varies somewhat, (see appendix A for definitions). Below we have plotted the cycle in countries which (by visual inspection) have the most synchronised cycle. That is, we plot Germany and Denmark together with Norway, the UK together with the US, and Sweden together with Finland.

In table 5.5 we report the bivariate correlations of output between these seven countries, with the shortest sample used in each correlation. Norway behaves procyclically with all the other countries in the sample, but the contemporaneous correlations are relative

Figure 5.7. International business cycles: US and UK

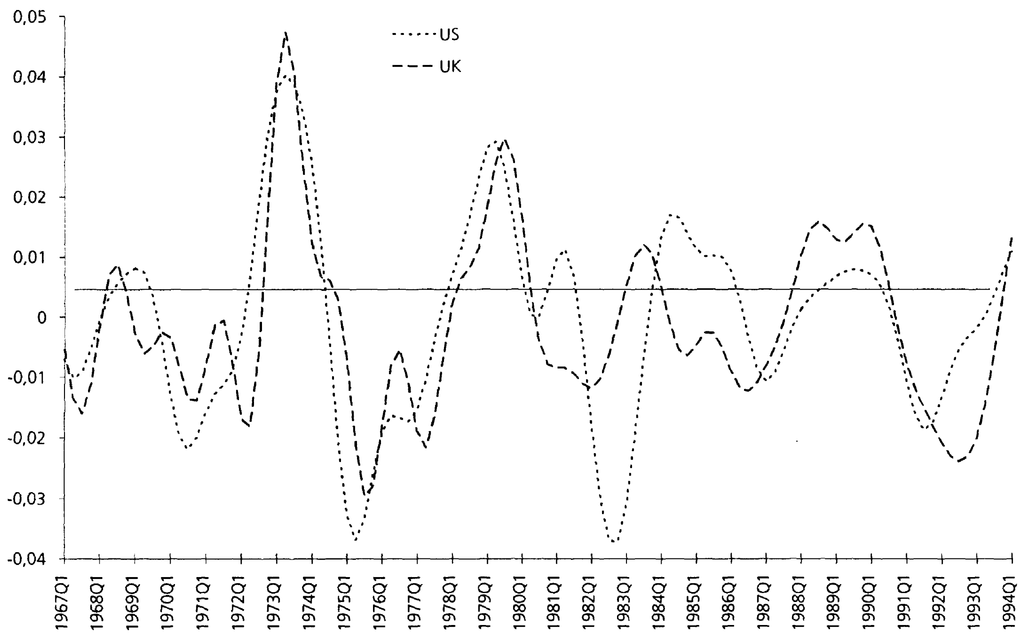


Figure 5.8. International business cycles: Germany, Denmark and Norway

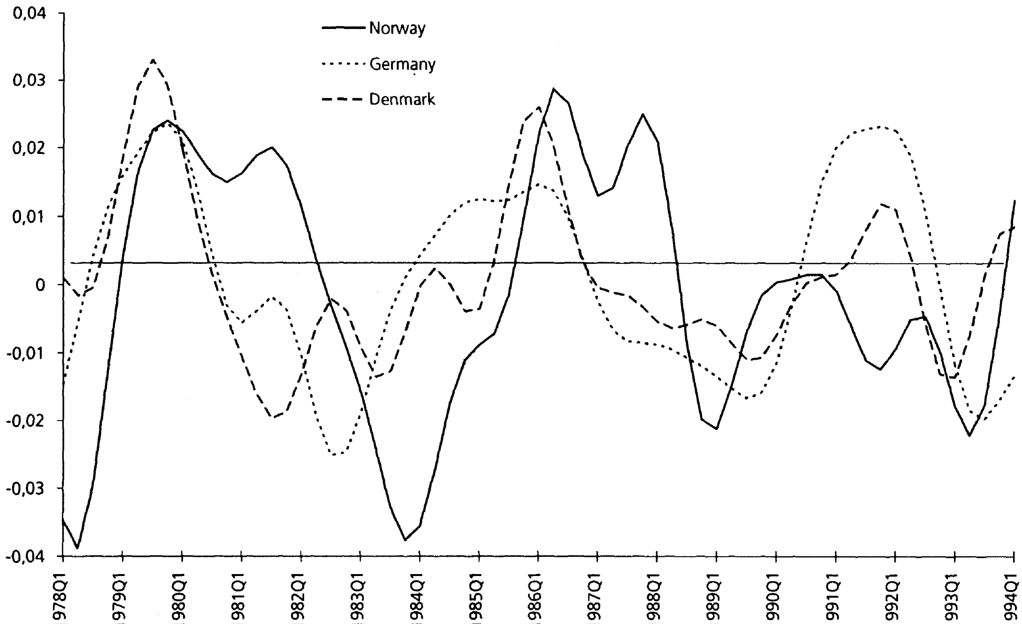


Figure 5.9. International business cycles: Finland and Sweden

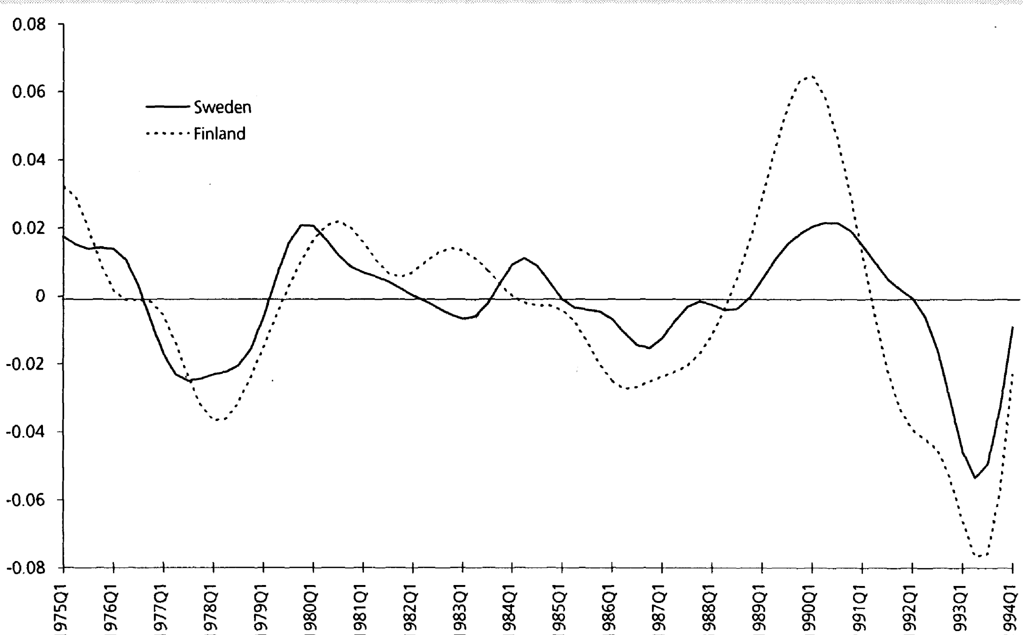


Table 5.5. Contemporaneous and maximum cross-correlation of output between:[1,2]

	Finland	Germany	Norway	Sweden	UK	US
Denmark	-0.19	0.64	0.28	0.13	0.27	0.35
	-0.23	-	0.49	0.18	-	0.45
	(-2)		(+3)	(+1)		(-3)
Finland	1	-0.11	0.12	0.79	0.37	-0.09
		-0.50	-	-	0.66	-0.31
		(-5)			(-3)	(+5)
Germany		1	0.22	0.29	-0.11	0.30
			0.40	0.34	-0.55	0.39
			(+5)	(-1)	(+4)	(-2)
Norway			1	0.17	0.04	0.02
				0.18	0.15	0.43
				(-1)	(-4)	(-5)
Sweden				1	0.29	0.02
					0.70	0.42
					(-5)	(-5)
UK					1	0.67
						-

[1] All the cyclical components are calculated by the frequency domain method as described in chapter 4.7

[2] Each cell contains in the first row the contemporaneous cross correlation between GDP in both countries. The second row contains the maximum correlation, (if different from the contemporaneous correlations), between GDP(t) in the country reported in the column on the left hand side of the table, and GDP(t-k), (k=-5,-4,...0,...,4,5) in the countries reported in the top row in the table, with the chosen number of lead (-) / lag (+) in parenthesis below. For instance, the value 0.49 (+3) in the cell between Denmark and Norway, indicate that the maximum correlation between Denmark and Norway is 0.49, and that Denmark is lagging Norway with three quarters. The value 0.18 (-1) in the cell between Norway and Sweden indicate that the maximum correlation between Sweden and Norway is 0.18 and that Norway is leading Sweden with one quarter.

low. The business cycle in Norway is highest correlated with the business cycle in Denmark, Germany and the US when Norway is leading the cycle, and the correlations lie in the range 0.4-0.5 (the range here and below refers to the maximum correlations).

Whereas the correlation between Sweden and Norway is relative low (0.18), the cycle in Sweden and Finland behaves highly procyclical, with a contemporaneous correlation coefficient as high as 0.79. Both Finland and Sweden are also highly correlated with UK (the correlation coefficient is 0.66 between Finland and UK, and 0.7 between Sweden and UK) when Finland and Sweden lead the cycle with three and five quarters respectively. Among the other countries, Denmark and Germany behave procyclically, with a contemporaneous correlation coefficient of 0.64. UK and US behave also procyclically, with a equally high contemporaneous correlation coefficient of 0.67. None of these countries are leading the cycle. Whereas the UK and the US behave procyclically, UK and Germany behave countercyclically, when Germany is lagging the cycle with a year (the correlation coefficient is -0.55). However, Germany and UK behave procyclically, when Germany is leading the cycle with five quarters (the correlation coefficient is 0.37, but it is not reported here). Germany and US are also procyclical, if Germany is leading the cycle with two quarters.

6. Concluding remarks

Until the 1970s, empirical analysis of business cycles saw the decomposing of a time series into a secular (trend) component and a cyclical component as a straightforward exercise. The economic mechanisms underlying short- and long-run movements would be quite different, and the cyclical and trend component could be studied separately. The secular component would typically be a deterministic time trend, and the business cycles would be stationary fluctuations around this trend. Recent advances in time-series econometrics have taught us that this traditional trend-cycle decomposition is not so straightforward, as many macroeconomic time series may in fact be better represented with a stochastic trend (unit root) than a deterministic time trend. When there is a unit root in the series, the cycles and the trend can no longer be seen as separate and independent components, since the fluctuations in the series itself represent accumulations of permanent shocks. Detrending data that contain a stochastic trend may instead infer spurious cycles in the data.

These findings have spurred interest in questions about the long run effects (persistence) of macroeconomic shocks. Whereas shocks to a series that is stationary around a deterministic trend are only transitory, shocks to a random walk will have a permanent effect and persist forever. However, more recently, it has been argued that the persistent effect of the shocks/innovations in a time series may be severely exaggerated if one misinterpret a deterministic time trend with a structural break, as a stochastic trend.

In the first three chapters of this study, we analysed the underlying dynamics in macroeconomic variables in Norway, by testing for unit roots when we allowed the alternative to be a deterministic trend with an endogenous structural break in the slope or the level of the trend. Measures of persistence of shocks in economic variables were performed, also when we adjusted for a possible break in the trend. Essentially we found that for the unemployment rate, government consumption, investment and real wage, we could reject the unit root hypothesis in favour of the linear trend with a break alternative. Further, whereas these variables initially showed a high degree of persistence, correcting for a break in the trend reduced persistence measures considerably. In the end, only CPI and M2 showed clear evidence of persistence in Norway.

The final chapters of this study calculated and presented the stylized facts of business cycles in Norway, using several stochastic and deterministic trend alternatives. For some variables (e.g. consumption, import, investment and productivity), the stylized facts were suggestive, indicating that the business cycles in these variables were positively correlated with the business cycle in GDP regardless of the method used. For other variables, the results varied considerably with the decomposition method used. For instance, traditional export, real wage and CPI showed both a procyclical and a countercyclical pattern depending on the decomposition methods used. Only when we got information on the underlying dynamics of the secular component in these variables, could we infer anything about the business cycles.

The sensitivity of business cycles to the measurement of the trend implies that one should be careful not to routineously detrend a time series without examining the secular component in the data. In the worst case one may infer spurious cycles, but it is equally possible that one emphasize cycles of an other frequency than what one initially set out to extract. An alternative to the univariate approach would be to use a multivariate model that take into account information contained in other variables. One type of model that has been widely used is the vector autoregression model that find information of the secular component in a series by imposing long run or cointegration restrictions among variables. This is the approach we would turn to from here.

References

Andvig, Jens C. (1981): Ragnar Frisch and Business Cycle Research during the Interwar Years, *History of Political Economy* **13**, 695-725.

Banerjee, Anindya, Robin L. Lumsdaine and James H. Stock (1992): Recursive and Sequential Tests of the Unit-Root and Trend-Break Hypotheses: Theory and International Evidence, *Journal of Business and Economic Statistics* **10**, 271-287.

Beveridge, Stephen and Charles R. Nelson (1981): A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle', *Journal of Monetary Economics* **7**, 151-174.

Blackburn, K. and M.O. Ravn (1991): Univariate Detrending of Macroeconomic Time Series, Aarhus University, Centre for international Economics Working Paper 22.

Blackburn, K. and M.O. Ravn (1992): Business Cycles in the UK: Facts and Fictions, *Economica* **59**, 383-401.

Blanchard, Oliver J. (1981): What is Left of the Multiplier-Accelerator? *American Economic Review Proceedings* **71**, 150-154.

Blanchard, Oliver J. and Stanley Fischer (1989): *Lectures on Macroeconomics*, Cambridge Massachusetts: The MIT Press.

Blanchard, Oliver J. and Danny Quah (1989): The Dynamic Effects of Aggregate Demand and Supply Disturbances, *American Economic Review* **79**, 655-673.

Bronfenbrenner, Martin (1989): *Is the Business Cycle Obsolete?* New York: John Wiley & Sons.

Burns, Arthur F. and Wesley C. Mitchell (1946): *Measuring Business Cycles*, New York: NBER.

Bårdsen, Gunnar, Paul G. Fischer and Ragnar Nymoen (1995): Business cycles: Real facts or fallacies? Presented at Symposium at the Centennial of Ragnar Frisch, March 3-5, 1995, Oslo.

Campbell, John Y. and Gregory Mankiw (1987a): Are output Fluctuations Transitory?, *Quarterly Journal of Economics* **102**, 857-880.

Campbell, John Y. and Gregory Mankiw (1987b): Permanent and Transitory Components in Macroeconomic Fluctuations, *American Economic Review Papers and Proceedings* **77**, 111-117.

Campbell, John Y. and Gregory Mankiw (1989): International Evidence on the Persistence of Economic Fluctuations, *Journal of Monetary Economics* **23**, 319-333.

Canova, Fabio (1993): Detrending and Business Cycle Facts, CEPR Discussion Paper 782.

Christiano, Lawrence (1992): Searching for a Break in GNP, *Journal of Business and Economic Statistics* **10**, 237-250.

Christiano, Lawrence J. and M. Eichenbaum (1990): "Unit Roots in Real GNP: Do We Know, and Do We Care?" in Allan H. Meltzer, (ed.): *Unit Roots, Investment Measures and Other Essays,* Carnegie-Rochester Conference Series on Public Policy **32**, Amsterdam: North Holland, 7-61.

Cochrane, John H. (1988): How Big is the Random Walk Component in GNP?, *Journal of Political Economy* **96**, 893-920.

Cochrane, John H. (1991): A critique of the application of unit root tests, *Journal of Economic Dynamics and Control* **15**, 275-284.

Cogley, Timothy (1990): International Evidence on the Size of the Random Walk in Output, *Journal of Political Economy* **98**, 501-518.

Cogley, Timothy and James M. Nason (1992): Effects of The Hodrick-Prescott Filter on Trend and Difference-stationary Time Series: Implications for Business Cycle Research, The University of British Columbia, Department of Economics Discussion Paper 23.

Cuddington, John T. and Alan L. Winters (1987): The Beveridge-Nelson Decomposition of a Time Series: A Quick Computational Approach, *Journal of Monetary Economics* **19**, 125-127.

Danthine, J.P. and M. Girandin (1989): Business Cycles in Switzerland: A comparative Study, *European Economic Review* **33**, 31-50.

Dickey, David A. and Wayne A. Fuller (1979): Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association* **74**, 427-431.

Englund, Peter, Torsten Persson and Lars E. O. Svensson (1992): Swedish business cycles: 1861-1988, *Journal of Monetary Economics* **30**, 343-371.

Fiorito, Riccardo and Tryphon Kollintzas (1994): Stylized facts of business cycles in the G7 from a real business cycles perspective, *European Economic Review* **38**, 235-269.

Friedman, Milton and Anna J. Schwartz (1963): *A Monetary History of the United States 1867-1960*, Princeton: Princeton University Press for NBER.

Frisch, Ragnar (1933): Propagation Problems and Impulse Problems in Dynamic Economies, in *Economic Essays In Honour of Gustav Cassel*, London: George Allen & Unwin Ltd, 171-205.

Fuller, Wayne A. (1976): *Introduction to Statistical Time Series*, New York: Wiley.

Ghysels, E. (1990): Unit root tests and the statistical pitfalls of seasonal adjustment: The case of U.S. post war gross national product, *Journal of Business and Economic Statistics* **8**, 2, 145-152.

Ghysels, E. and Pierre Perron (1993): The effect of seasonal adjustment filters on tests for a unit root, *Journal of Econometrics* **55**, 57-98.

Hamilton, James D. (1994): *Time Series Analysis*, Princeton, New Jersey: Princeton University Press.

Harvey, Andrew C. (1989): *Forecasting, structural time series and the Kalman filter*, Cambridge: Cambridge University Press.

Harvey, Andrew C. (1993): *Time Series Models,* 2nd edition, London: Harvester Wheatsheaf.

Harvey Andrew C. and Albert Jaeger (1991): Detrending, Stylized Facts and the Business Cycle, *Journal of Applied Econometrics* **8**, 231-247

Hassler, John, Petter Lundvik, Torsten Persson and Paul Søderlind (1992): The Swedish Business Cycle: Stylized facts over 130 years, Monograph no. 21, Institute for International Economic Studies, Stockholm

Jaeger, Albert (1994): Mechanical Detrending by Hodrick-Prescott Filtering: A Note, *Empirical Economics* **19**, 493-500.

Jaeger, A and R.M. Kunst (1990): Seasonal adjustment and measuring persistence in output, *Journal of Applied Econometrics* **5**, 47-58.

Judge, George G., W.E. Griffiths, R. Carter Hill, Helmut Lutkepohl, and Tsoung-Chao Lee (1985): *The Theory and Practice of Econometrics*, New York: John Wiley and Sons.

King, Robert G. and Sergio T. Rebelo (1993): Low frequency filtering and real business cycles, *Journal of Economic Dynamics and Control* **17**, 207-231.

Koopmans, Tjalling C. (1947): Measurement without theory, *Review of Economic Statistics* **29**, 161-172.

Kydland, Finn E. and Edward C. Prescott (1980): "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy", in S. Fischer (ed.): *Rational Expectations and Economic Policy,* Chicago: University of Chicago Press.

Kydland, Finn E. and Edward C. Prescott (1982): Time To Build and Aggregate Fluctuations, *Econometrica* **50**, 1345-70.

Kydland, Finn. E. and Edward C. Prescott (1990): Business Cycles: Real Facts and a Monetary Myth, *Federal Reserve Bank of Minneapolis Quarterly Review,* (Spring), 3-18.

Lesteberg, Halvard and Kjell Wettergreen (1975): "Konjunkturbølger i Vest-Europas industriproduksjon 1955-1975, (The Business Cycle and Industrial Production in Western Europe 1955-1975)", in Statistics Norway: *Nasjonalregnskap, modeller og analyse. En artikkelsamling til Odd Aukrusts 60-årsdag (National Accounts, Models ands Analysis. To Odd Aukrust in Honour of his Sixtieth Birthday),* Social and Economic Studies 26.

Long, John B. and Charles I. Plosser (1983): Real Business Cycles, *Journal of Political Economy* **91**, 39-69.

Lucas (1977): "Understanding Business cycles", in Karl Brunner and Allan Meltzer: *Stabilisation of the domestic and international economy*, Carnegie-Rochester Conference Series **5**, Amsterdam: North Holland.

Mitchell, Wesley (1913): *Business cycles,* Berkeley: University of California Press.

Mitchell, Wesley (1927): *Business cycles: The problem and its setting,* NBER.

Moore, Geoffrey H. (1961): *Business Cycle Indicators*, Volume I, Princeton: Princeton University Press.

Morgan, Mary S. (1990): *The history of econometric ideas,* Cambridge: Cambridge University Press.

Nelson, Charles R. and Heejoon Kang (1981): Spurious Periodicity in Inappropriately Detrended Time Series, *Econometrica* **49**, 741-751.

Nelson, Charles R. and Charles I. Plosser (1982): Trends and Random Walks in Macroeconomic Time Series, *Journal of Monetary Economics* **10**, 129-162.

Newbold, Paul (1990): Precise and efficient computation of the Beveridge-Nelson decomposition of economic time series, *Journal of Monetary Economics* **26**, 453-457.

Perron, Pierre (1989): The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis, *Econometrica* **57**, 1361-1401.

Perron, Pierre (1993): The HUMP-Shaped Behaviour of Macroeconomic Fluctuations, *Empirical Economics* **18**, 707-727.

Priestley, M.B (1982): *Spectral Analysis and Time Series*, New York: Academic Press.

Prescott, Edward (1986): Theory Ahead of Business Cycle Measurement, *Federal Reserve Bank of Minneapolis Quarterly Review* **10**, 9-22

Quah, Danny (1990): Permanent and Transitory Movements in Labor Income: An explanation for 'Excess Smoothness' in Consumption, *Journal of Political Economy* **98**, 449-475.

Quah, Danny (1992): The Relative Importance of permanent and Transitory Components: Identification and Some Theoretical Bounds, *Econometrica* **60**, 107-118.

Raj, Baldev (1992): International Evidence on Persistence in Output in the Presence of an Episodic Change, *Journal of Applied Econometrics* **7**, 281-293.

Rappoport, P. and L. Reichlin (1989): Segmented Trends and Non-Stationary Time Series, *Economic Journal* **99**, Conference Supplement, 168-177.

Robinson, Peter M. (1983): "Review of Various Approaches to Power Spectrum Estimation" in D.R. Brillinger and P.R. Krishnaiah, (eds.): *Handbook of Statistics* **3**, Elsevier Science Publishers B.V, 343-368.

Rudebusch, G.D. (1992): Trends and Random Walks in Macroeconomic Time Series: A Re-examination, *International Economic Review* **33**, 661-680.

Schumpeter, J.A. (1939): *Business Cycles-A Theoretical, Historical and Statistical Analysis of the Capitalist Process,* New York and London: McGraw-Hill Book Company, Inc.

Schwert, G.W (1989): Tests for Unit Roots: A Monte Carlo Investigation, *Journal of Business and Economic Statistics* **7**, 147-159.

Serletis, Apostolos (1994): International Evidence on Breaking Trend Functions in Macroeconomic Variables, *Applied Economics* **26**, 175-179.

Shapiro, Matthew and Mark W. Watson (1988): "Sources of Business Cycle Fluctuations" in Stanley Fischer (ed.): *Macroeconomics Annual*, Cambridge: MIT Press, 111-148.

Skjerpen, Terje (1995): Is there a Business Cycle Component in Norwegian Macroeconomic Quarterly Time Series? Discussion Papers 140, Statistics Norway.

Slutsky, Eugen (1937): The Summation of Random Causes as the Source of Cyclic Processes, *Econometrica* **5**, 105-135.

Stock, James H. and Mark E. Watson (1990): Business Cycle Properties of Selected U.S. Economic Time Series, 1959-1988, Working Paper No. 3376, NBER.

Swensen, Anders Rygh (1995): Simple examples on smoothing macroeconomic time series, Documents No. 95/1, Statistics Norway.

Watson, Mark W. (1986): Univariate Detrending Methods with Stochastic Trends, *Journal of Monetary Economics* **18**, 49-75.

West, Kenneth D. (1988): On the Interpretation of Near Random-Walk Behaviour in GNP, *The American Economic Review* **78**, 202-209.

Wettergreen, Kjell (1978): Konjunkturbølger fra utlandet i Norsk økonomi, (International Cycles in the Norwegian Economy), *Social and Economic Studies* 36, Statistics Norway.

Zarnowitz, Victor and Geoffrey H. Moore (1986): "Major Changes in Cyclical Behavior" in Robert J. Gordon (ed.): *The American Business Cycle*, Chicago: The University of Chicago Press, 519-582.

Zivot, Eric and Donald W. K. Andrews (1992): Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis, *Journal of Business and Economic Statistics* **10**, 251-270.

Appendix A: Data sources and definitions

(A) NORWAY

All series used in the analysis except those used in chapter 4.7 for the frequency filtering techniques, are quarterly seasonal adjusted national accounts from *KVARTS Database, Statistics Norway*. The seasonally adjustment procedure used is X-11 ARIMA. The data used in chapter 4.7 are unadjusted quarterly data from the KVARTS Database. Throughout the analysis, the list of variables below refers to the seasonally adjusted and unadjusted data interchangeably. The sample period is from 1967Q1 -1994Q1, except for nominal interest rates where the sample runs from 1971Q4- 1994Q1. All variables are measured in natural logarithms except for the unemployment rate and the interest rate that are measured in levels.

1. **(GDP)** Gross Domestic Product in *mainland* Norway at constant 1991 prices

2. **(C)** Private consumption expenditure at constant prices

3. **(G)** Government consumption expenditure at constant prices

4. **(I)** Gross Fixed Capital formation in *mainland* Norway at constant prices

5. **(X)** Export of goods and services at constant prices

6. **(M)** Import of goods and services at constant prices

7. **(PR)** Productivity in *mainland* Norway; GDP/H; Gross Domestic Product in *mainland* Norway (GDP) deflated by total hours worked in *mainland* Norway (H)

   (H) Total hours worked in *mainland* Norway

8. **(U)** Unemployment rate

9. (**RWG**) Real wages in *mainland* Norway; W/PY; Nominal wages (W) in *mainland* Norway deflated by the implicit price deflator of GDP *mainland* Norway, (PY)

   (**W**) Nominal wages pr. employee (pr. hours.)- *mainland* Norway
   (**PY**) Implicit deflator of Gross Domestic Product- *mainland* Norway

10. (**CPI**) Consumer Price Index

11. (**M2**) Money supply M2 at current prices

12. (**R**) 3-months nominal interest rate (NIBOR)

13. (**OP**) Oil prices in Norwegian currency

(B) INTERNATIONAL ECONOMY

Denmark: GDP, constant prices, (n.s.a.), (1977Q1-1994Q1), *Datastream*
Finland: GDP, constant prices, (n.s.a.), (1975Q1-1994Q1), *Datastream*
West Germany: GDP, constant prices, (n.s.a.), (1978Q1-1994Q1), *Datastream*
Sweden: GDP, constant prices, (n.s.a.), (1970Q1-1994Q1), *Konjunkturinstitutet*
UK: GDP, constant prices, (n.s.a.), (1967Q1-1994Q1), *Datastream*
US: GDP, constant prices (s.a.), (1967Q1-1994Q1), *Datastream*

Appendix B: Sequential unit root tests

Table B.1. Sequential unit roots tests; p=8[1,2]

Series:	ADF t_{ADF}	(A) Shift in trend k^*	$F^A_{DU\text{-}k^*}$	$t^A_{ADF\text{-}k^*}$	(B) Break in trend (shift in mean) k^*	$F^B_{DU\text{-}k^*}$	$t^B_{ADF\text{-}k^*}$
GDP	-1.75	1986Q1	7.35	-3.26	1988Q1	13.14	-3.72
C	-1.71	1985Q4	9.80	-3.60	1988Q1	14.20	-3.71
G	-0.48	1982Q3	17.63[b]	-4.22[c]	1975Q2	9.07	-2.48
I	-1.77	1986Q3	18.34[b]	-4.68[b]	1988Q2	22.78[a]	-4.40
X	-1.71	1982Q3	6.95	-3.09	1974Q3	15.97	-3.31
M	-2.89	1986Q2	2.35	-3.24	1988Q2	6.87	-3.90
PR	-1.15	1985Q2	3.69	-2.24	1987Q2	5.73	-2.50
U	-2.18	1986Q2	8.14	-3.63	1988Q2	18.00[c]	-4.52[c]
RWG	-1.06	1976Q4	34.30[a]	-5.30[a]	1973Q2	19.76[b]	-3.79
CPI	1.04	1987Q1	12.96	-2.67	1988Q2	3.89	-0.02
M2	1.04	1988Q1	15.99[c]	-3.22	1988Q2	16.93[c]	-0.53
R	-1.00	1986Q4	8.15	-2.60	1979Q1	6.26	-1.67
OP	0.03	1982Q2	12.68	-3.70	1985Q3	8.57	-2.74

[1] For a definition of the variables, see table 2.1.
[2] k* indicates the break date suggested by $F^{A,B}_{DU\text{-}k^*}$.
[a] Rejection of the unit root hypothesis at the 2.5 pct. level
[b] Rejection of the unit root hypothesis at the 5 pct. level
[c] Rejection of the unit root hypothesis at the 10 pct. level

Table B.2. Sequential unit roots tests; p=8, not seasonally adjusted data[1,2]

	ADF	(A) Shift in trend			(B) Break in trend (shift in mean)		
Series:	t_{ADF}	k^*	$F^A_{DU\text{-}k^*}$	$t^A_{ADF\text{-}k^*}$	k^*	$F^B_{DU\text{-}k^*}$	$t^B_{ADF\text{-}k^*}$
GDP	-1.55	1986Q1	6.68	-3.04	1988Q2	14.36	-3.76
C	-1.43	1986Q1	7.53	-3.10	1988Q2	17.09[c]	-3.87
G	-0.53	1982Q4	17.70[b]	-4.24[c]	1975Q2	7.99	-2.37
I	-2.05	1986Q3	17.85[b]	-4.78[a]	1988Q2	25.08[b]	-4.69[c]
X	-1.66	1982Q3	7.12	-3.07	1974Q2	15.81	-2.90
M	-2.81	1986Q3	4.67	-3.59	1988Q2	9.50	-4.07
PR	-1.91	1973Q4	5.14	-2.98	1987Q4	6.89	-3.27
U	-2.43	1986Q4	10.45	-4.10	1988Q2	19.07[b]	-4.79[c]
RWG	-1.46	1977Q4	29.06[a]	-5.40[a]	1974Q4	16.06	-4.19
CPI	0.62	1987Q2	13.42	-2.96	1988Q3	5.08	-0.58
M2	0.47	1988Q1	17.95[b]	-3.67	1988Q2	13.93	-0.99
R	0.01	1986Q4	8.76	-2.70	1979Q1	5.80	-1.60
OP	-1.23	1982Q1	12.74	-3.80	1986Q1	9.90	-3.24

[1] For a definition of the variables, see table 2.1.

[2] k* indicates the break date suggested by $F^{A,B}_{DU\text{-}k^*}$.

[a] Rejection of the unit root hypothesis at the 2.5 pct. level

[b] Rejection of the unit root hypothesis at the 5 pct. level

[c] Rejection of the unit root hypothesis at the 10 pct. level

Appendix C: Plot of cyclical components using different detrending methods

Figure C.1a. Private consumption

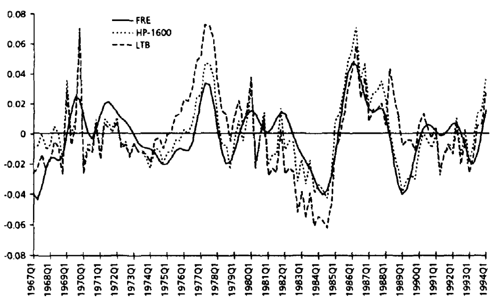


Figure C.1b. Private consumtion

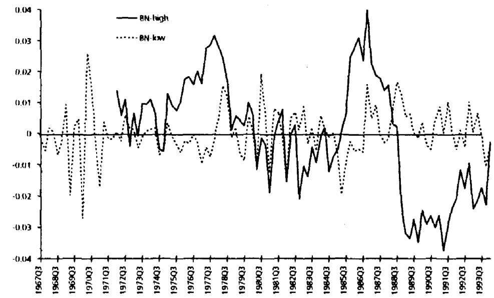


Figure C.2a. Investment

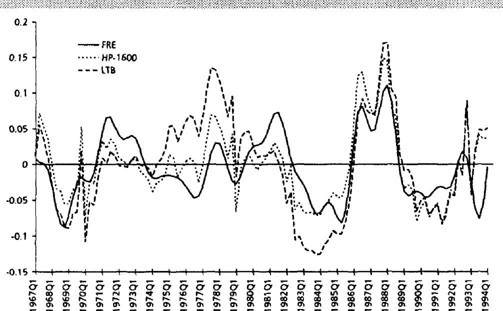


Figure C.2b. Investment

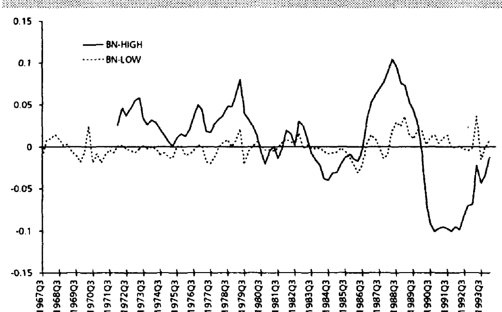


Figure C.3a. Productivity

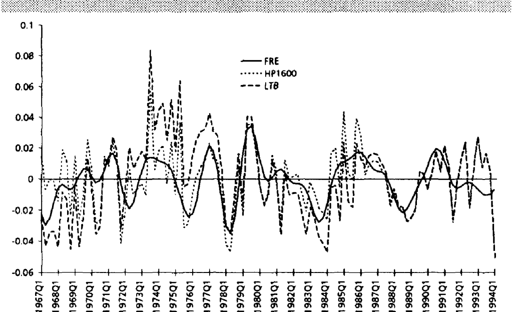


Figure C.3b. Productivity

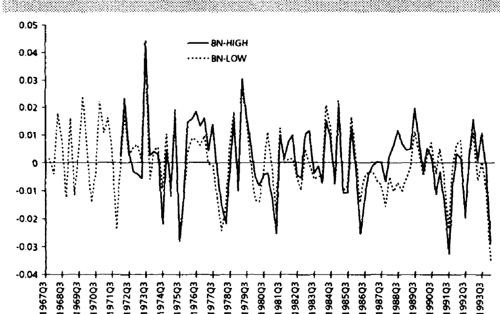


Figure C.4a. Real wage

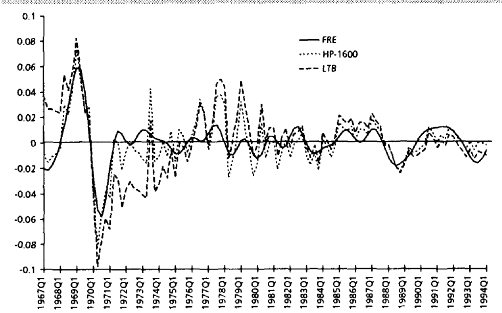


Figure C.4b. Real wage

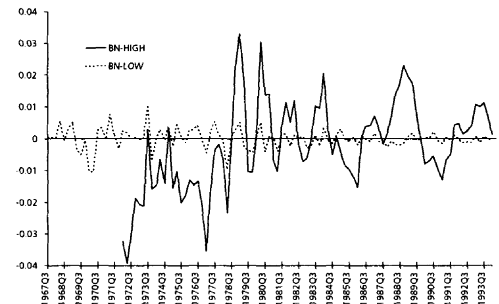


Figure C.5a. Unemployment

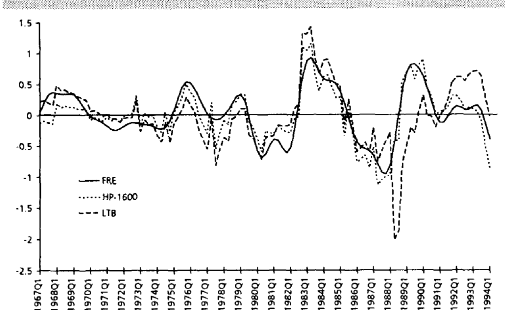


Figure C.5b. Unemployment

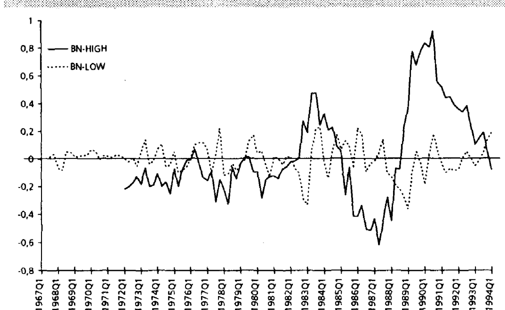


Figure C.6a. CPI

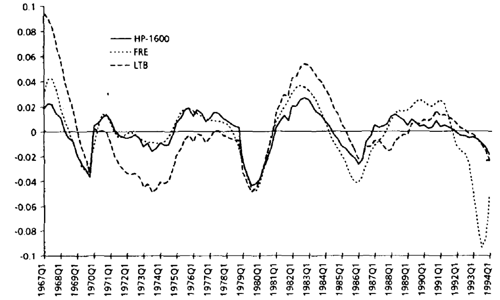


Figure C.6b. CPI

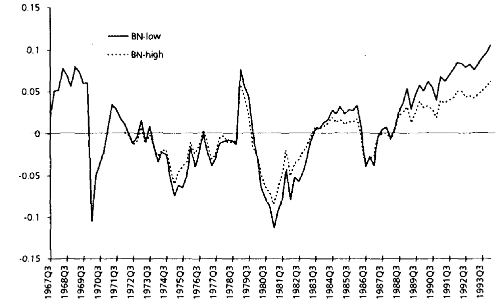


Figure C.7a. M2

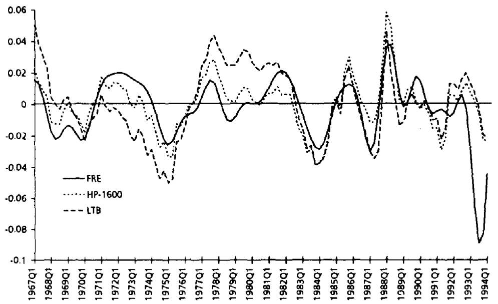
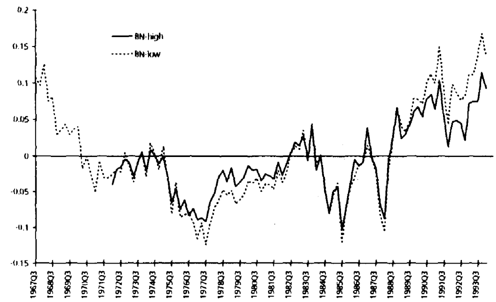


Figure C.7b. M2



Appendix D: Spectral analysis of the business cycle components

Figure D.1. Spectrum of detrended GDP, real wage and CPI using a linear trend with break

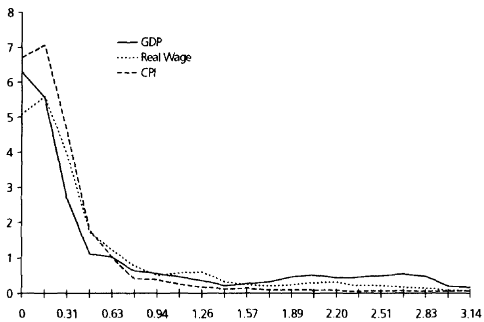


Figure D.2. Spectrum of detrended GDP, real wage and CPI using the HP-filter, λ=16

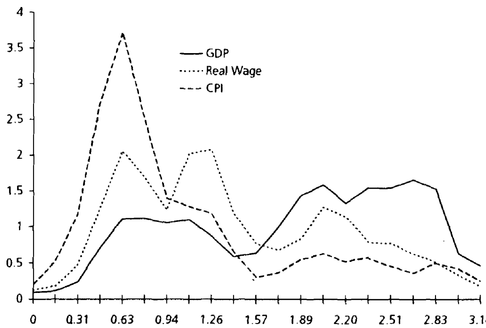


Figure D.3. Spectrum of detrended GDP, real wage and CPI using the HP-filter, λ=1600

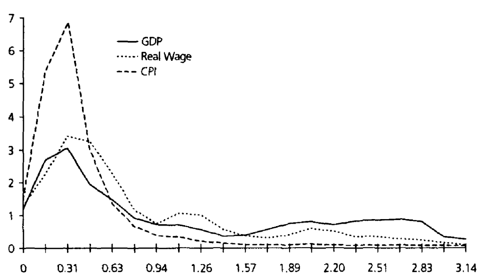


Figure D.4.. Spectrum of detrended GDP, real wage and CPI using BN-low

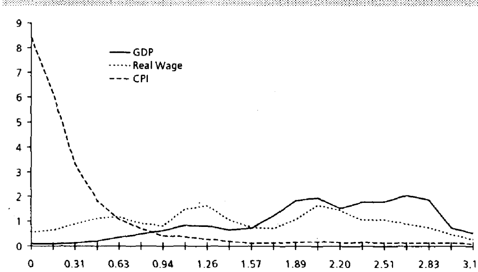


Figure D.5. Spectrum of detrended GDP, real wage and CPI using BN-high

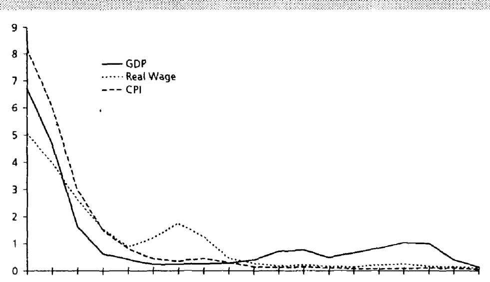
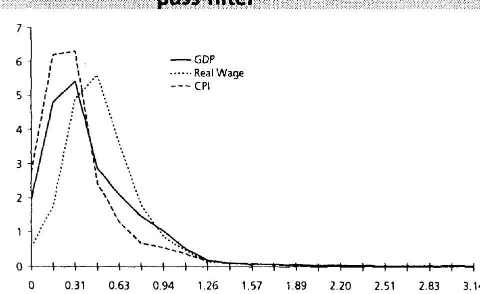


Figure D.6. Spectrum of detrended GDP, real wage and CPI using a band pass filter



Appendix E: The Beveridge and Nelson procedure

Interpretation:
Assume that the first differences of y_t are stationary, and denote $w_t = (1-L)y_t$. As in (3.1), by Wold's theorem, w_t can be written as the infinite moving average process, $w_t = \alpha_1 + A(L)\varepsilon_t$, where $A(L) = \Sigma_{j=0}A_jL^j$, and ε_t are uncorrelated, mean zero, random disturbances. From this expression we see that expectation of y_{t+k}, conditional on data up to period t, can be written as the accumulations of the w's:

E.1
$$\begin{aligned}\hat{y}_t(k) &\equiv E(y_{t+k}|\ldots,y_{t-1},y_t)\\ &= y_t + \hat{w}_t(1) + \hat{w}_t(2) + \ldots + \hat{w}_t(k)\end{aligned}$$

where:

$$\hat{w}(i) = E(w_{t+i}|\ldots,w_{t-1},w_t)$$

The conditional forecast $E(w_{t+i}|\ldots,w_{t-1},w_t)$ can then be expressed as:

E.2
$$\hat{w}_t(i) = \alpha_1 + A_i\varepsilon_t + A_{i+1}\varepsilon_{t-1} + \ldots + A_k\varepsilon_{t-(k-j)}$$

where j represents the infinite number of MA terms in (3.1), so that for i<j we start collecting the past disturbances, whereas for i>j, the forecasts of the disturbances are unknown with mean zero. Combining (E.1) and (E.2), the very long term forecast of y_t $(k\rightarrow\infty)$ can be expressed as:

(E.3) $$\hat{y}_t(k) \approx k\alpha_1 + y_t + (\sum_{i=1}^{\infty} A_1)\varepsilon_t + (\sum_{i=1}^{\infty} A_2)\varepsilon_{t-1} + \ldots$$

The forecast profile is asymptotic to a linear function of forecast horizon k, where α_1 is taken to be the rate of drift of the series (the slope), and the remaining stochastic process is interpreted as the level (intercept), of the forecast horizon. It is this level in the series that Beveridge and Nelson (1981) interpret as the permanent component or trend of y_t. In this sense, the trend will be stochastic, and can be expressed as the long run forecast of the series, adjusted for the mean rate of change.

$$g_t \equiv \hat{y}_t(k) - k\alpha_1$$

(E.4) $$g_t = y_t + \lim_{k\to\infty}[\hat{w}_t(1) + \ldots + \hat{w}_t(k) - k\alpha_1]$$

$$g_t = y_t + (\sum_{i=1}^{\infty} A_i)\varepsilon_t + (\sum_{i=2}^{\infty} A_i)\varepsilon_{t-1} + \ldots$$

From (E.4) one can interpret the permanent component as the current observed value of y_t plus all future forecastable changes adjusted for the mean rate of drift. By taking first differences of the expression of g_t in (E.4), one can see that the permanent component follows a random walk with drift α_1 as expressed in (4.21).

The second component in (E.4) (the difference between the permanent component g_t and the current component y_t), is interpreted as the cyclical component. It is a stationary process, the sum of all the forecastable changes in y_t, which in terms of the notation in (4.1) can be written as :

$$-c_t \equiv \lim_{k\to\infty} [\hat{w}_t(1) + \ldots + \hat{w}_t(k) - k\alpha_1]$$

(E.5)

$$-c_t = (\sum_{i=1}^{\infty} A_i)\varepsilon_{t-1} + (\sum_{i=2}^{\infty} A_i)\varepsilon_{t-1} + \ldots$$

which reduces to (4.22).

Estimation:
The first stage in the BN method, is to choose an appropriate ARIMA model for the variables when they are well represented as I(1). Several authors have found that the first differences of GDP in several countries may be well fitted by a low order ARMA model (ie. Campbell and Mankiw 1989). Blackburn and Ravn (1992) and Canova (1993) use low order ARMA processes when applying the BN method. However, as pointed out by Cochrane (1988), although different ARMA models may fit the short run properties of

the first differences of an observed series, the forecast functions from these models may differ substantially. Since the trend-cycle decomposition in the BN method relies on the forecast properties of the ARIMA models, these models may give very different trend-cycle decompositions. Cochrane (1988) shows how low-order ARIMA models will systematically overestimate the random walk component in the trend. An output series may have positive autocorrelations at the low lags, and small (negative) autocorrelations at the higher lags. A simple time series model will not be able to capture both this kind of behaviour. The maximum likelihood estimates match the short run behaviour and misrepresent the long run behaviour.[26] As the innovative variance of the random walk is a property of the very long-run behaviour of the series, one should estimate high-order models that adequately captures this long run behaviour.

In this paper we model the first differences of the data as high order ARMA models. For comparison, we will also consider some low order ARMA processes for some of the variables. One problem when estimating high order ARMA models, is to choose the appropriate number of AR lags. The more lags that are included, the more will we emphasise the cyclical component. Estimating the same lag length for all the variables (as in e.g. Canova (1993)), may be inappropriate, since the variables differ considerably with regard to their dynamic behaviour. As seen in table 3.1 in chapter 3.2, persistence measured by the V-ratio differs considerably for the variables. Further, the appropriate choice of k that can distinguish the permanent component (random walk) from the cyclical component varies between the variables, (ie. contrast the behaviour of consumption and CPI in table 3.1 as k increases). As mentioned in chapter 3.1, the exact mean square error of the estimated V^k is minimised using a large value of k when V is small and a small value of k when V is large.

In the BN method we estimate ARIMA models for the series to find A(1) to establish the trend. Given the way A(1) is related to V, (see (3.4)), we may argue that for some of the variables, when V and A(1) are essentially high, a low AR lag length will be more appropriate when we shall distinguish between the temporary and permanent components, whereas when V and A(1) are low, a higher order AR lag length will be more appropriate to distinguish the temporary component from the permanent component. Note that when A(1) is in excess of one, the variance of an innovation in the permanent component will be larger than the variance of an innovation in the series. The cyclical behaviour in this variable will be generated by excessive movements in the trend. Hence, we will be careful to choose a restrictive AR lag length for those variables with A(1) (and V) above one.

The estimation procedure is then as follows. We consider ARMA models varying from ARMA(8,0) up to ARMA(20,0) processes for the first differences of all the variables.[27] We

[26] See Cochrane (1988 pp. 918-919) for a proof of this.

[27] ARMA models using high order AR lags with one MA term for the first differences were also considered, but the results did not differ much from those obtained using only pure high-order AR models of the first differences. For simplicity, we only report the high order AR models for the first differences of the variables.

then let the Ljung-Box Q statistics suggest the AR model that give the highest probability that the error is white noise. A(1) is then calculated for each of the processes chosen, and evaluated how well it corresponds to the V-ratio in table 3.1.

For several of the variables, the Ljung-Box Q statistics suggested that the ARMA(16,0) process was appropriate, which also provided an estimate for A(1) that corresponded to the values of V with the appropriate choice of k as reported above. For GDP, consumption, productivity and real wages, the Q-statistics picked an ARMA(20,0) model, which also seemed a more appropriate model as the temporary component did not seem to be quite important before 20 lags were considered, (the autocorrelation structure of GDP for lags less than 20 suggested a random walk pattern, and the first autocorrelation of GDP was not positive before 20 lags were considered). For CPI, M2, interest rates and government consumption we chose an ARMA(8,0) process. The Q-statistics picked an ARMA(8,0) model for CPI and interest rates, whereas for government consumption and M2, an ARMA(8,0) was only preferred second to an ARMA(20,0). However, for all these models, using more than 8 autoregressive lags generates a large cyclical component, which is generated by excessive volatility in the trend. The models chosen for *BN-high* and the respective Q-statistics are reported in table E.1, where the significance level reported indicates the probability that the error is white noise. The values for A(1) corresponding to the chosen models are reported together with estimates of V^k in table 3.1 in chapter 3.2.

Table E.1. Test criteria for *BN-high* when the first differences are modelled as:

VARIABLES	ARMA(p,q)	Q	Q-sign
GDP	(20,0)	13.39	0.99
C	(20,0)	13.42	0.99
G	(8,0)	22.87	0.82
I	(16,0)	10.35	0.99
X	(16,0)	16.74	0.94
M	(16,0)	15.72	0.96
PR	(20,0)	17.11	0.93
U	(16,0)	10.83	0.99
RWG	(20,0)	21.47	0.76
CPI	(8,0)	17.53	0.97
M2	(8,0)	23.09	0.81
R	(8,0)	13.55	0.99
OP	(16,0)	3.58	0.99

For comparison, we also considered low order ARMA processes for the fist diffences of some variables. For the low-order processes, we specified models from ARMA(0,0) up to ARMA(3,3) processes. In choosing between the low-order ARMA models, a typical Box-Jenkins (1976) model selection procedure may be applied. At a first stage, the identification process, the correlogram, or the first sample autocorrelations of the first differences are investigated. This is carried out in table E.2, panel A. Table E.2, panel B reports the partial autocorrelations, (see ie. Harvey, (1993)). In the second stage, the model suggested by the autocorrelations is estimated. Diagnostic checks of the residuals are carried out in stage three.

As several models may satisfy the above three criteria, I apply in addition two further criteria, the Akaike and Schwarz criteria, where the goodness of fit is examined. The Akaike criteria, (AIC), selects a model that minimises -2ln(maximum likelihood) + 2 k, where k is the number of parameters. For a Gaussian process this criteria reduces to minimising, (see ie. Judge et. al, (1985))

$$\text{(E.6)} \qquad AIC(k) = \ln \tilde{\sigma}^2 + 2k / T$$

where T is the number of observations, and $\tilde{\sigma}^2$ is the ML estimate of the residual variance, evaluated keeping the initial number of observations fixed as k increases. Hence, the idea is that the criteria penalises the number of parameters in the model. The other criteria reported here, the Schwarz criteria, (SC), is based on a Bayesian argument and is constructed to minimise -2ln(maximum likelihood) + ln(T)k, where T and k are defined as above. For a Gaussian process, this reduces to:

$$\text{(E.7)} \qquad SC(k) = \ln \tilde{\sigma}^2 + \frac{k \ln T}{T}$$

where $\tilde{\sigma}^2$ is defined as for AIC. Generally, the Schwarz criteria will choose a shorter lag length than the Akaike criteria, and as AIC will tend to pick over parameterised models we let the Schwarz criteria select the models. In table E.3, we report the processes selected by the Schwarz criteria and also report the Akaike and the corresponding Ljung-Box Q-statistics for these processes. All models selected satisfy the stationarity and invertible criteria. In table E.4 we report the estimated models that we have selected and will use for *BN-low.* The estimated models are first picked from the Schwarz criteria. If they seem reasonable based on the AIC, the Ljung-Box criteria and the sample autocorrelation pattern, they are selected for *BN-low.* The result using other models can be obtained from the author on request.

Table E.2a. Sample autocorrelations of the change in the natural logs of selected variables[1]

Series: Lags:	GDP	C	I	PR	U[2]	RWG	CPI	M2
r_1	-0.44	-0.34	-0.15	-0.33	-0.06	-0.12	0.55	0.52
	(0.096)	(0.096)	(0.096)	(0.096)	(0.096)	(0.096)	(0.096)	(0.096)
r_2	0.04	0.10	0.27	-0.13	0.25	-0.15	0.53	0.21
	(0.113)	(0.107)	(0.098)	(0.106)	(0.097)	(0.098)	(0.122)	(0.119)
r_3	0.17	0.22	-0.01	0.15	0.07	0.08	0.42	0.23
	(0.114)	(0.108)	(0.105)	(0.108)	(0.102)	(0.099)	(0.141)	(0.123)
r_4	-0.19	-0.13	0.06	-0.15	-0.08	-0.17	0.29	0.07
	(0.116)	(0.112)	(0.105)	(0.109)	(0.103)	(0.100)	(0.153)	(0.127)
r_5	0.10	-0.02	0.10	0.06	0.11	0.07	0.28	0.13
	(0.119)	(0.113)	(0.105)	(0.111)	(0.103)	(0.103)	(0.162)	(0.126)
r_6	-0.06	-0.02	0.08	0.04	-0.05	0.06	0.25	0.20
	(0.12)	(0.114)	(0.106)	(0.112)	(0.104)	(0.103)	(0.166)	(0.128)
r_7	0.16	0.16	0.14	-0.01	-0.01	-0.11	0.18	0.05
	(0.12)	(0.114)	(0.107)	(0.112)	(0.105)	(0.104)	(0.168)	(0.131)
r_8	-0.17	-0.19	0.03	-0.19	-0.10	-0.18	0.12	0.11
	(0.122)	(0.116)	(0.108)	(0.112)	(0.105)	(0.105)	(0.168)	(0.131)

[1] Standard errors using Bartlett approximation are reported in brackets
[2] The unemployment rate is measured in levels rather than logs

Table E.2b. Sample partial autocorrelations of the change in the natural logs of selected variables

	GDP	C	I	PR	U[1]	RWG	CPI	M2
r_1	-0.44	-0.34	-0.15	-0.33	-0.06	-0.12	0.55	0.52
r_2	-0.19	-0.02	0.25	-0.27	0.24	-0.17	0.32	-0.08
r_3	0.14	0.29	0.07	0.01	0.10	0.05	0.08	0.21
r_4	-0.05	0.04	0.00	-0.14	-0.14	-0.18	-0.08	-0.18
r_5	0.00	-0.13	0.09	-0.00	0.06	0.05	0.04	0.25
r_6	-0.07	-0.16	0.10	0.01	0.01	0.02	0.08	-0.01
r_7	0.19	0.21	0.13	0.06	-0.04	-0.07	-0.05	-0.07
r_8	-0.06	-0.01	0.03	-0.22	-0.13	-0.24	-0.08	0.14

[1] The unemployment rate is measured in levels rather than logs

Table E.3 Test criteria for ARMA specifications of the change in the natural logs

Test criteria: Series:	ARMA(p,q)	Akaike[1]	Schwarz[1]	Q(30)[2]
GDP	(1,3)	7.59	7.47	24.82 (0.73)
C	(0,2)	7.89	7.82	26.85 (0.63)
I	(2,0)	6.61	6.54	14.68 (0.99)
PR	(0,1)	7.52	7.47	35.51 (0.22)
U [3]	(0,2)	2.54	2.47	28.12 (0.56)
RWG	(0,1)	7.89	7.84	25.87 (0.68)
CPI	(1,1)	9.76	9.69	19.01 (0.94)
M2	(1,2)	9.24	9.14	28,53 (0.54)

[1] The Akaike and Schwarz criteria are calculated using equation E.6 and E.7 respectively.

[2] Significance level in brackets.

[3] The unemployment rate is measured in levels.

Table E4 Low order estimated ARIMA models used for *BN-low* in the analysis above[1]

ΔGDP ARMA(1,3)	$\Delta y_t = 0.006 - 0.744\,\Delta y_{t-1} + \varepsilon_t + 0.318\varepsilon_{t-1} - 0.308\,\varepsilon_{t-2} + 0.244\,\varepsilon_{t-3}$ (0.001) (0.109) (0.136) (0.099) (0.103)
ΔC ARMA(0,2)	$\Delta y_t = 0.007 + \varepsilon_t - 0.439\varepsilon_{t-1} + 0.434\varepsilon_{t-2}$ (0.002) (0.088) (0.088)
ΔI ARMA(2,0)	$\Delta y_t = 0.003 - 0.108\,\Delta y_{t-1} + 0.255\Delta y_{t-2} + \varepsilon_t$ (0.004) (0.095) (0.095)
ΔPR ARMA(0,1)	$\Delta y_t = 0.005 + \varepsilon_t - 0.543\varepsilon_{t-1}$ (0.001) (0.086)
ΔU ARMA(0,2)	$\Delta y_t = 0.034 + \varepsilon_t - 0.055\varepsilon_{t-1} + 0.319\,\varepsilon_{t-2}$ (0.033) (0.093) (0.094)
ΔRWG ARMA(0,1)	$\Delta y_t = 0.004 + \varepsilon_t - 0.17\varepsilon_{t-1}$ (0.002) (0.096)
ΔCPI ARMA(1,1)	$\Delta y_t = 0.016 + 0.883\,\Delta y_{t-1} + \varepsilon_t - 0.481\,\varepsilon_{t-1}$ (0.003) (0.074) (0.129)
ΔM2 ARMA(1,2)	$\Delta y_t = 0.026 + 0.943\,\Delta y_{t-1} + \varepsilon_t - 0.262\,\varepsilon_{t-1} - 0.505\,\varepsilon_{t-2}$ (0.004) (0.092) (0.135) (0.117)

[1] All variables are measured in logs except the unemployment rate.

De sist utgitte publikasjonene i serien Sosiale og økonomiske studier *Recent publications in the series Social and Economic Studies*

66 L. Grøtvedt: Støy og helse: Analyse av støyopplevelser i Norge *Noise and Health: Study on Noise Annoyance in Norway*. 1988. 71s. 45 kr. ISBN 82-537-2574-4

67 Ø. Olsen, K. Roland: Modeling Demand for Natural Gas: A Review of Various Approaches *Etterspørsel etter naturgass: En oversikt over ulike modellopplegg*. 1988. 81s. 40 kr. ISBN 82-537-2665-1

68 Miljøstatistikk 1988: Naturressurser og miljø *Environmental Statistics: Natural Resources and the Environment*. 1988. 291s. 70 kr. ISBN 82-537-2664-3

69 B. Aardal, H. Valen: Velgere, partier og politisk avstand. 1989. 329s. 125 kr. ISBN 82-537-2762-3

70 Sosialt utsyn 1989 *Social Survey*. 1989. 230s. 125 kr. ISBN 82-537-2776-3

71 A.L. Ellingsæter: Normalisering av deltidsarbeidet: En analyse av endring i kvinners yrkesaktivitet og arbeidstid i 80-årene *Normalization of Part-Time Work: A Study of Women's Employment and Working Time Patterns in the 1980s*. 1989. 127s. 75 kr. ISBN 82-537-2779-8

72 D. Album: Individ, arbeid og inntekt: En fordelingsanalyse *Individuals, Jobs and Earnings: A study of Distribution*. 1989. 198s. 85 kr. ISBN 82-537-2850-6

73 K.A. Brekke, A. Torvanger (red.): Vitskapsfilosofi og økonomisk teori. *Philosophy of Science and Economic Theory*. 1990. 315s. 115 kr. ISBN 82-537-2857-3

74 H. Valen, B. Aardal, G. Vogt: Endring og kontinuitet Stortingsvalget 1989. 1990. 172s. 100 kr. ISBN 82-537-2963-4

75 O. Aukrust: Økonomisk forskning og debatt. *Economic research and debate*. Utvalgte artikler 1942-1989. 1990. 383s. 125 kr. ISBN 82-537-2984-7

76 G. Haraldsen, H. Kitterød: Døgnet rundt. Tidsbruk og tidsorganisering 1970-90. Tidsnyttingsundersøkelsene. 1992. 185s. 189 kr. ISBN 82-537-3639-8

77 J.-E. Lystad: Norsk hotellnæring 1950-1990. 1992. 174s. 115 kr. ISBN 82-537-3677-0

78 O. Ljones, B. Moen, L. Østby (red.): Mennesker og modeller: Livsløp og kryssløp. 1992. 336s. 165 kr. ISBN 82-537-3699-1

79 I. Gabrielsen: Det norske skattesystemet 1992 *The Norwegian Tax System*. 1992. 175s. 115 kr. ISBN 82-537- 3728-9

80 E. Bowitz: Offentlige stønader til husholdninger En økonometrisk undersøkelse og modellanalyse. 1992. 119s. 100 kr. ISBN 82-537-3785-8

81 S. Blom, T. Noack, L. Østby: Giftermål og barn – bedre sent enn aldri? 1993. 167s. 115 kr. ISBN 82-537-3808-0

82 R. Aaberge, T. Wennemo: Inntektsulikhet og inntektsmobilitet i Norge 1986-1990. 1993. 46s. 90 kr. ISBN 82-537-3911-7

83 I. Svendsen: Empirical Tests of the Formation of Expectations – A Survey of Methods and Results. 1993. 52s. 75 kr. ISBN 82-537-3948-6

84 B.E. Naug: En økonometrisk analyse av utviklingen i importandelene for industrivarer 1968-1990 *An Econometric Analysis of the Development of Manufacturing Import Shares 1968-1990.* 1994. 78s. 95 kr. ISBN 82-537-3955-9

85 E. Bowitz, Å. Cappelen: Prisdannelse og faktoretterspørsel i norske næringer *Price Formation and Factor Demand in Norwegian Industries.* 1994. 177s. 125 kr. ISBN 82-537-4024-7

86 K. Mohn: Modelling Regional Producer Behaviour – A Survey *Modellering av regional produsentatferd – En litteraturoversikt.* 1994. 71s. 95 kr. ISBN 82-537-4042-5

87 K.A. Magnussen: Old-Age Pensions, Retirement Behaviour and Personal Saving: A Discussion of the Literature *Alderspensjon, pensjoneringsatferd og privat sparing: En diskusjon av litteraturen.* 1994. 69s. 95 kr. ISBN 82-537-4050-6

88 K. Mohn, L.S. Stambøl, K.Ø. Sørensen: Regional analyse av arbeidsmarked og demografi: Drivkrefter og utviklingstrekk belyst ved modellsystemet REGARD *Regional Analysis of Labour Market and Demography with the Model REGARD.* 1994. 165s. 125 kr. ISBN 82-537-4082-4

89 N.M. Stølen: Wage Formation and the Macroeconomic Functioning of the Norwegian Labour Market *Lønnsdannelse og den makroøkonomiske funksjonsmåten til det norske arbeidsmarkedet.* 1995. 306s. 180 kr. ISBN 82-537-4141-3

90 Ø. Kravdal: Sociodemographic Studies of Fertility and Divorce in Norway with Emphasis on the Importance of Economic Factors *Sosiodemografiske studier av fruktbarhet og skilsmisse i Norge med vekt på betydningen av økonomiske faktorer.* 1994. 267s. 155 kr. ISBN 82-537-4088-3

91 T. Kornstad: Empirical Life Cycle Models of Labour Supply and Consumption *Empiriske livsløpsmodeller for arbeidstilbud og konsum.* 1995. 115s. 110 kr. ISBN 82-537-4166-9

92 H.C. Bjørnland: Trends, Cycles and Measures of Persistence in the Norwegian Economy *Trender, konjunktursvingninger og varighet av sjokk i norsk økonomi* 1995. 109s. 110 kr. ISBN 82-537-4220-7

C

Returadresse:
Statistisk sentralbyrå
Postboks 8131 Dep.
N-0033 Oslo

Publikasjonen kan bestilles fra:
Statistisk sentralbyrå
Salg- og abonnementservice
Postboks 8131 Dep.
N-0033 Oslo

Telefon: 22 00 44 80
Telefaks: 22 86 49 76

eller:
Akademika - avdeling for
offentlige publikasjoner
Møllergt. 17
Postboks 8134 Dep.
N-0033 Oslo

Telefon: 22 11 67 70
Telefaks: 22 42 05 51

ISBN 82-537-4220-7
ISSN 0801-3845

Pris kr 110,00

Statistisk sentralbyrå
Statistics Norway

9 788253 742205

© Falch Hurtigtrykk, Oslo