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Moving beyond expectations

From cohort-component to microsimulation projections

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Abstract:
Population projections are predominantly made using the cohort-component method (CCM). The opportunities for further development within that framework are limited. Lately, with advances in technical and computational capacity, the microsimulation framework has become a serious contender. In contrast to CCM, it allows for rich com-plexity of behavior and provides insights on projection uncertainty. Still, demographers have been reluctant to apply this framework, which may be due to lack of guidance. We contribute by clarifying underlying CCM assumptions, translating a multi-regional version of the model into a dynamic spatial microsimulation model, and discuss the usefulness of prediction intervals for planning. Using data for Norway, we demonstrate that the re-sults for the two models are equivalent, even for very small subgroups, and converge with relatively few simulations. The model can easily be amended with additional indi-vidual heterogeneity, facilitating more accurate representations of population dynamics.

Keywords: Population projections; microsimulation; cohort-component method; uncertainty; multi-regional; small area

JEL classification: J11, C15, C63, C81

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Sammendrag

Befolkningframskrivinger er høyt etterspurt og brukes i forskning, markedsanalyse, offentlig politikk og planlegging. Framskrivinger beregnes hovedsakelig ved hjelp av kohort-komponentmetoden, som har flere gunstige egenskaper slik som gjennomsiktighet, at den er enkel i bruk og har tett kobling til demografisk teori. Imidlertid er en alvorlig begrensning at modellen fort blir uhåndterlig når antallet individuelle egenskaper øker. Kompleksiteten i modellen øker eksponentielt med hver ny egenskap som legges til. Ettersom befolkningsdynamikk er sammensatt, risikerer man å utelate viktig heterogenitet i demografisk atferd. Dette truer realismen til modellen og fører til at færre interessante og politikkrelevante spørsmål kan undersøkes.

Mikrosimuleringsmodellen har blitt en seriøs utfordrer ved at den håndtere kompleksitet godt. Nye individuelle karakteristikker og interaksjoner kan enkelt inkluderes i modellrammeverket. Tidligere var de kravene mikrosimulering stilte til datamengde og -kraft svært krevende. Tilgang til administrative registerdata og rask utvikling i tilgjengelig datakraft har redusert viktigheten av disse begrensningene. Likevel har de som holder på med befolkningsframskrivinger vært tilbakeholdne med å bytte modell, og kohort-komponent og mikrosimuleringslitteraturen har hovedsakelig utviklet seg parallelt. Manglende praktisk veiledning for overgang har vært pekt på som en viktig årsak til at få benytter mikrosimuleringsrammeverket.

I denne artikkelen har vi som mål å bygge bro mellom disse to litteraturene ved å vise hvordan man kan gå fra kohort-komponentmetode til mikrosimulering og likevel beholde styrkene til begge metoder. Vi tydeliggjør de underliggende antagelsene til modellene, gir en detaljert beskrivelse av modelleringsvalgene og validerer mikrosimuleringsmodellen ved å sammenligne med resultatene fra kohort-komponent modellen. Videre vil vi vise konvergeringsegenskapene til mikrosimuleringene og forklare hvordan de detaljerte resultatene fra mikrosimulering kan brukes til å forbedre politikkanalyse. Som case bruker vi Statistisk sentralbyrås offisielle kommunale befolkningsframskrivinger. Dette er et spesielt instruktivt eksempel siden de mange kommunene gjør at vi allerede er nær grensen for hva som er mulig av heterogenitet i kohort-komponent modellen. Det er også et nyttig eksempel ved at små befolkningsstørrelser i mange kommuner gjør det spesielt verdifullt å kvantifisere modellusikkerheten til resultatene, som produseres direkte fra kjøringene av mikrosimuleringsmodellen.

1 Introduction

Population projections are a key demographic tool used in academic research, market analysis, public policy, and planning. Projections are predominantly derived using the cohort-component method (CCM), which has several beneficial attributes such as transparency, ease of operation and close linkages to demographic theory. However, a serious limitation is that the model quickly becomes intractable as the number of individual characteristics is increased (Van Imhoff and Post, 1998; Zagheni, 2015). Indeed, for each added characteristic, the complexity of the model increases exponentially. As population dynamics are inherently multifaceted, omitting important sources of heterogeneity in demographic behavior threatens the realism of the model (Cohen, 1995; Bélanger and Sabourin, 2017) and fewer questions of interest can be explored with less detailed output (Wilson and Rees, 2005).

In recent decades, the microsimulation method has become a serious contender to CCM, as the ability to handle complexity is one of its main selling points. Adding individual characteristics and interactions is straightforward. In the past, the primary barriers to using microsimulation were related to its demands on data and computational resources. With improvements in data availability and advances in estimation methodology and computational capacity, these limitations have been mitigated (Spielauer, 2011). Still, many demographers and projection practitioners have been hesitant to make the switch, and the CCM and microsimulation modeling literatures have mainly developed in parallel. A potentially key factor behind the limited uptake of microsimulations in demographic projection work has been the lack of existing guidance on how to recreate the cohort-component method within a microsimulation framework (Puga-Gonzalez et al., 2022).

In this paper, our objective is to start bridging the gap between the two literatures and provide direction on how to transition from a CCM to microsimulation projection, retaining the strengths of both approaches. The lack of documentation on model design choices and validation is a recurrent criticism in the microsimulation field (Li and O'Donoghue, 2013; Morrison, 2008). Following best-practice recommendations, we will clarify the underlying assumptions of the CCM, provide accurate descriptions of the microsimulation modeling choices, and validate the microsimulation model against a trusted benchmark CCM. Furthermore, we will show how quickly the microsimulation results converge toward the expected values of the CCM projection and how the more detailed output of the microsimulation methodology can be used to improve policy analysis. The microsimulation model code, written in Python, is published online and can easily be extended by other academics and practitioners to suit their needs.[1]

A second contribution of this paper is to add to the literature on multi-regional population projections. Despite being highly demanded, regional population projections have received scarce scholarly attention relative to national projections (Mazzuco and Keilman, 2020). Not surprisingly, there are relatively few studies employing dynamic spatial microsimulation models for such purposes (Wilson et al., 2022), despite the fact that multi-regional CCM projections tend to be difficult to manage because of the added spatial heterogeneity. We take as our point of departure the multi-regional CCM projection of Statistics Norway and reinterpret it into a dynamic spatial microsimulation projection on the full-count Norwegian population. Comparisons show that the microsimulation model recreates the results from the CCM, both for municipalities with large and small populations. This is not a common achievement even for comparisons of aggregate national projection results (Marois and Samir, 2021; Puga-Gonzalez et al., 2022).

Microsimulations are inherently random, as individuals are made subject to stochastic events based on demographic probabilities. Thus, the microsimulation method, unlike the CCM, provides not only the expectation of the projected population but the full distribution of outcomes. This distribution of realizations may be interpreted

[1] Code is available at `https://github.com/statisticsnorway/pop_sim`.

as a measure of (model-specific) uncertainty.[2] Little attention has been devoted to uncertainty in regional population projections (Wilson et al., 2022), although a recent study surveying users shows that such information is considered helpful (Wilson and Shalley, 2019). We discuss the advantages of results from microsimulations with respect to policy and planning purposes, and demonstrate that this particular uncertainty is most important for the younger population segments and is of higher relative magnitude for less populous areas.

In the next section, we discuss methods for regional population projections and their limitations. In Section 3, we describe the dynamic spatial microsimulation model and explain modeling choices. Results are presented in the fourth section, with discussion and conclusions provided in Section 5.

2 Multi-regional population projections

Regional population projections are of clear value to both the public and private sectors. They are used to inform policy-making, planning and administration, as well as academic research and market analysis. When utilized to gauge the demand for such things as future hospitals, schools, nursing homes, water and energy supply, roads and other local infrastructure, population projections assist in high-cost decisions that are often impossible to reverse in the short term. Efforts to improve projection models, for instance by increasing accuracy and detail, can therefore potentially contribute significant improvements to the efficiency of resource allocation.

Local population dynamics are complex and affected by a vast array of demographic, economic, geographic, and social factors. Despite the complicated interrelations, population projections are often conducted with simple methods that have undergone little development over the last decades, are easily calculated, and have low

[2]Uncertainty in projections may have many sources besides the randomness related to draws. For a discussion, see Lutz and Goldstein (2004).

data requirements (Wilson et al., 2022). Among these are the extrapolative methods that project population totals and the ratio methods that arrive at local future populations by dividing an aggregate projection using shares. Criticisms leveled at these approaches include the lack of strong theoretical basis given reliance on past trends (Cameron and Cochrane, 2017), no age-disaggregation of results, and that some of these methods require long time trends which are typically not available for subnational areas (Wilson, 2011). The simplified cohort-component method, which projects each of the cohorts an interval ahead using simple cohort change ratios, is also an option but concerns about prediction power have been raised (Wilson, 2016). The most popular of the available methods is the cohort-component method (CCM), where aggregate demographic behavior (components) is modelled explicitly. The microsimulation method is even more detailed on demographic behavior and projects at the level of the individual or household. Recent reviews of regional projection methods can be found in Wilson (2011) and Lomax et al. (2022).

2.1 The multi-regional cohort-component model

The cohort-component method (CCM) has long been the workhorse of population projections, sanctioned by academics, statistical agencies and international organizations like the United Nations and the World Bank (Burch, 2018). Being linked to the demographic balancing equation, it also represents a general theoretical model of population dynamics. The balancing equation is an identity which describes the determinants of population change. In the multi-regional case, it may take the following form:

$$Pop_{t+1,m} = Pop_{t,m} + Births_{t,m} - Deaths_{t,m} + NetMigration_{t,m} \tag{1}$$

where $Pop_{t+1,m}$ is population one period ahead, $t + 1$, in area m. The future population depends on population at the start of the period and the number of births, deaths and net migration between t and $t+1$. Net migration is often composed of both domestic (D) and international migration (I), and can be further decomposed into in-flows and out-flows:

$$\begin{aligned} NetMigration_{t,m} =& Immigration^{I}_{t,m} - Emigration^{I}_{t,m} \\ &+ InMigration^{D}_{t,m} - OutMigration^{D}_{t,m} \end{aligned} \tag{2}$$

The CCM utilizes that the balancing equation must hold true for each cohort, sex, and area. Basically, all that is required to project the local population one period into the future is a count of the population in each area by sex and age at beginning of time t and assumptions of demographic behavior. Period by period, one can expand the horizon by using the population group from the previous period and *project* it into the future using the appropriate assumptions.

Assumptions on demographic behavior are often in the form of rates. In our case, these would be specific to age, sex and municipality. For instance, the number of births may be given by:

$$Births_{t,m} = \sum_{x=15}^{49} Pop^{F}_{t,m}(x) \times FertilityRate_{t,m}(x) \tag{3}$$

where the number of births in year t in municipality m is given by the sum of births for the female population Pop^F in fertile age. In our case, fertile age is defined as 15 to 49 years of age, $x = [15, 49]$. The number of births in each age group x is given by the number of women in that group multiplied by the age-specific fertility rate. Similar expressions may be formulated for deaths and migration events.

2.2 The official regional population projections for Norway

Statistics Norway has conducted official population projections for Norway since the 1950s. The current multi-regional CCM model is defined for all 356 Norwegian municipalities.[3] These vary substantially in size. In 2022, the largest municipality was the capital Oslo with about 700,000 inhabitants and the smallest municipality was Utsira with less than 200 inhabitants. Most Norwegian municipalities are small with a median size of about 5000 inhabitants.

The model projects the number of persons based on three individual characteristics: municipality of residence, sex, and age. Demographic rates specific to the individual characteristics are derived using the empirical Bayes method to determine regional variation and combine them with expected time trends in demographic behavior (Leknes and Løkken, 2021; Thomas and Tømmerås, 2022).

Migration behavior is handled somewhat differently than fertility and mortality. Domestic migration is separated into two sequential processes: out-migration, followed by in-migration. First, the propensity to out-migrate on the basis of individual characteristics is estimated. Second, internal in-migrants and immigrants (from abroad) are allocated to municipalities using a migration matrix. The matrix defines the share of each group of given characteristics that flows into the municipalities each year. It is constructed on the basis of data for the last ten years, but with some adjustment to the base year in the first projection years. Further detail of the procedures is available in Leknes and Løkken (2022).

2.3 Limitations of the cohort-component method

The CCM approach is mathematically simple, fast to compute and well-established. However, the framework displays severe limitations related to heterogeneity. The combination of population characteristics determines the number of groups in CCM projections. So long as the number of groups are limited, the method works well. However, if the number of groups becomes large, either by adding new characteristics or increasing the level of detail, the CCM model quickly becomes mathematically

[3] The model is named BEFREG, which is an abbreviation of the Norwegian words for *population projections* and *regional*.

intractable. This issue is related to how additional characteristics are implemented. Indeed, additional characteristics are included multiplicatively in the model, such that the full set of interactions between characteristics makes up separate components of the system of interest (Van Imhoff and Post, 1998). Increasing the number of characteristics causes substantial expansion of input and result matrices. This problem is closely related to the "curse of dimensionality" in mathematics.

An example of the seriousness of the dimensionality problem can be provided by comparing the number of population group cells in the baseline model relative to models with additional individual traits. The Norwegian multi-regional CCM projection includes age (0-119 years), sex (male and female), and region (356 municipalities). These characteristics lead to an aggregate table of $120 \times 2 \times 356 = 85,440$ cells. Suppose that parity (0, 1, ..., 3, 4 or more children), marital status (single, married/cohabiting, widowed, divorced) and immigrant status (native or from Eurasia, Africa or America) were also added. The number of cells would increase to $85,440 \times 5 \times 4 \times 4 = 6,835,200$. If we were to further include a modest number of socioeconomic characteristics, for instance education level (primary, high school, college/university), income quartile and housing situation (renter or owner), the number of cells reaches $6,835,200 \times 3 \times 4 \times 2 = 164,044,800$. It is clear that the complexity of the model increases exponentially with added heterogeneity, which results in greater incident of cells with few or no individuals.[4]

2.4 Microsimulation models

Microsimulation models are an especially promising alternative to the macro-class CCM projections. The microsimulation framework dates back to the 50s and 60s (Orcutt, 1957; Orcutt et al., 1961). A vast number of such models have been de-

[4]A closely related issue is that changes to the model using the CCM are costly to implement, making it difficult to develop or improve the model framework. Since the dimensions of the matrices in the model are fully determined by population characteristics, changing or developing these characteristics typically involves a full rewrite of the model code.

veloped since and applications are published in general interest, field, and method-specific journals.[5]

There exist two main types of microsimulation models, the dynamic and static variants. In dynamic models, transitions between demographic states are modelled explicitly, whereas in static models time has no effect on individual characteristics except when subject to re-weighting to impose "static aging" (Spielauer, 2011; Tanton, 2014). In this paper, we will focus on dynamic microsimulation models. Dynamic microsimulation models operate at the individual level and simulate life courses of the population of interest. The framework makes it possible to implement interactions between individuals and between their characteristics. It provides detailed output and is thus a powerful tool to gain knowledge on population dynamics and effects of policy changes.

Because of their structure, microsimulation models are good at handling complexity. Each individual is described by a vector of characteristics (C). The full population of individuals (N) can then be represented by a matrix with $N \times C$ cells. Since microsimulation models handle complexity well, they have of late been employed for the purpose of regional modeling. This literature is of a limited size, but notable examples include Ballas et al. (2005), Marois and Bélanger (2014, 2015), Wu et al. (2011), and Wu and Birkin (2013).

The results of a microsimulation model are inherently uncertain, as life events are generated randomly using appropriate base rates. This makes the model more informative than CCM by offering a complete picture of the possible outcomes and their distribution, helping to reveal some of the inherent uncertainty in projections.

[5] Statistics Norway already maintains several microsimulation models for taxation, labor supply, education, and social benefits (Hansen et al., 2008; Dagsvik et al., 2008; Andreassen et al., 2020).

3 A dynamic spatial microsimulation model for Norway

In constructing the dynamic spatial microsimulation model, we follow the official population projections for Norway that use the multi-regional CCM approach. This guides the modeling choices, including the selection of individual characteristics, the length of the time steps, the handling of aging, and the procedure for randomly drawing individual demographic behavior. In the following, we will elaborate on these choices. As emphasized by Puga-Gonzalez et al. (2022), microsimulations may be valuable for surfacing assumptions of CCM projections that frequently lay hidden for practitioners.

3.1 Individual characteristics and assumption

We use the same characteristics and assumptions as in Statistics Norway's 2022 publication of regional population projections. The state of individuals vary on the basis of sex (male or female), one-year age groups (0-119 years old), and municipality of residence (356 municipalities). The model inputs are the start-of-year population in 2022, collected from the population registry, as well as demographic rates and matrices describing the future behavior of the population for the period 2022-2050.[6]

3.2 Time, aging and life events

To recreate the CCM framework, we formulate a discrete time dynamic microsimulation model. Basically, this means that the model provides information on all individuals and their characteristics on January 1st in the projected years. This is the 'snapshot approach' in contrast to a model with continuous time (see also discussion in Zagheni (2015)). Using the 2022 population as our year of departure, we project the population until 2050.

[6]The start-of-year population of 2022 is identical to the end-of-year population in 2021. The model can therefore be reformulated based on that time stamp.

In contrast to the CCM projections, the microsimulation model keeps track of individual states and life histories by explicitly modeling each person's demographic events. To project the population one year into the future, several steps of state adjustment are conducted. First, the individuals are subject to deterministic aging. This means that all individuals get one year added to their age. Accordingly, there are no individuals left with age zero, i.e. those who will be born during the year are missing. This brings us to the second step, which is births.

Compared to the population projections where the number of children born is calculated by summing up the product of fertility rates and women in each age group, the microsimulation model draws whether a particular women will have a child or not. In practice, we draw from a Bernoulli distribution where the probability of the occurrence birth is p_i^B for individual i to determine whether or not the event occurs.[7] The newborn individuals (the 0-year-olds) are placed in their mother's municipality of residence, and the sex of each child is randomly drawn.[8]

An important distinction is that in a dynamic microsimulation model the order of the life events are typically sequential, whereas in the CCM they are simultaneously determined — the transition from one year to the next happens *at one scoop*. This is reflected in the next step where the rest of the life events — death, emigration and domestic out-migration — are jointly determined in a multinomial trial.

In practice, the probability of death (p_i^D), emigration[9] (p_i^E) and domestic out-migration (p_i^{DO}), determines four non-overlapping outcomes. Let u_i be individual i's realization of a uniformly distributed random variable, $u_i \sim U(0,1)$. The individual dies if $u_i < p_i^D$, emigrates if $p_i^D \leq u_i < p_i^E + p_i^D$, and moves to another municipality

[7] The demographic probabilities are the same for all individuals within the same year, municipality, sex and age-group, but for simplicity of notation we use the individual subscript i.

[8] There are naturally slightly more male than female births. The probability of being assigned male gender is 0.51369, i.e. the same as in the CCM projection.

[9] In the official CCM model (BEFREG), the emigration count is adjusted to equal that of a national emigraton assumptions projection model. An intermediate step in setting the emigration propensities is to calculate an adjustment factor and change the propensities accordingly. We implement a similar adjustment in the dynamic spatial microsimulation model such that the expected number of emigrants equals the national emigration assumptions.

if $p_i^D + p_i^E \leq u_i < p_i^D + p_i^E + p_i^{DO}$. If $p_i^D + p_i^E + p_i^{DO} \leq u_i$ then individual i is not affected by any events. Note that newborns are also subject to these events, while immigrants are not. Those who die or emigrate are removed from the population. Finally, the placement of internal migrants and immigrants are decided using the moving matrix. The resulting population from these steps (Pop_{t+1}) will serve as the base population for the next period $t+2$.

This procedure is followed until the target period of 2050 is reached, meaning we have one complete realization of the stochastic population development specified above. This step is then repeated many times until we have a sufficiently large[10] sample of simulated populations, which finally is used to calculate the expectation (mean) and distributional characteristics of the dynamic spatial microsimulation projection. A step-by-step account and comparison of the modeling procedure for the multi-regional CCM projection and the dynamic spatial microsimulation counterpart is provided in Appendix Table A1.

The simultaneous determination of the demographic events means individuals can only be subject to a single life event each year besides being born. This restriction is implicit in the CCM framework and needs to be implemented in the microsimulation framework to ensure that the two models produce equivalent results. However, such a restriction can quite easily be relaxed in the microsimulation framework.[11] We speculate that disregarding the simultaneous assignment of events by multinomial trial is the foremost candidate for explaining why previous attempts to recreate CCM results using a microsimulation model have been unsuccessful.

[10]In the model presented in this paper we use 1000 simulations, but the required number of simulations needed depends on the size of the groups of interest and the size of the probabilities.

[11]The microsimulation framework is very flexible with respect to the sequence of demographic events. Draw order can be random, time steps can be shorter (with appropriate adjustments of rates and probabilities), or conditional probabilities of competing risks can be fully specified.

4 Results

In the following, we demonstrate the equivalence of the results from the CCM projection and the microsimulation model. Furthermore, we investigate some technical aspects of the microsimulation model related to run time and how the results converge to the deterministic results with an increasing number of simulations. Finally, we discuss how the distribution of simulation results provides a measure of uncertainty that can be used to inform policy.

4.1 Equivalence

It is considered best practice to validate a new model against a trusted benchmark (Morrison, 2008). As we translate the multi-regional CCM projection into a dynamic spatial microsimulation counterpart, a natural comparison is established: the expected value of the simulations should equal the CCM results using the same assumptions.

Table 1. Difference between average microsimulation results and the CCM results. Population size, births and deaths for selected years

	Population size			Births			Deaths		
Year	CCM result	Diff.	Percent diff.	CCM result	Diff.	Percent diff.	CCM result	Diff.	Percent diff.
2025	5,534,755.8	8.9	0.0002	54,509.1	0.6	0.0010	42,757.9	8.8	0.0207
2030	5,658,700.1	60.1	0.0011	56,959.8	-5.0	-0.0088	45,285.6	9.4	0.0208
2040	5,884,245.6	73.8	0.0013	59,600.3	14.4	0.0242	52,809.5	3.6	0.0068
2050	6,028,827.8	73.1	0.0012	56,104.6	10.9	0.0195	57,526.8	0.8	0.0015

Notes: The table shows projected population size, births, and deaths in 2025, 2030, 2040, and 2050 for Norway from the multi-regional CCM projections. It also displays the difference in (average) result between the CCM projections and the dynamic spatial microsimulation model.

Instead of running the model on a limited sample of the population, we use the full-count data from the population registry. This makes the precision of the projections higher, and we can disregard concerns related to the representativeness of the sample.[12]

Our results suggest that we are able to fully recreate the CCM mechanics with the microsimulation approach. With a 1000 simulations, the difference between the total population size from the CCM projections and microsimulation model is 0.001 percent in 2050 (see Table 1). The differences are marginal also when investigating the number of births and deaths, which deviate at most 0.02 percent (births in 2040). There seems to be no systematic relationship between the size of the error and time period, suggesting that errors do not accumulate. To put these results into context, we can make comparisons using studies with similar validation exercises. The only other contribution we find that validates a microsimulation model against CCM projections is Puga-Gonzalez et al. (2022). Using a 1%-sample, they create a microsimulation model for Norway and validate it against UN population projections. Their average (absolute) percentage differences over 50 year periods are in the magnitude of a hundred times higher.[13]

Figure 1 compares the projected population in 2050 by sex and age across the two models. The numbers are practically identical.[14] In Table 2, we proceed by comparing even more detailed results from the models. The table is constructed in two steps. First, we calculate percentage differences: $100 \times (\widehat{Pop^{Sim}} - Pop^{CC})/Pop^{CC}$. Second, we show descriptive statistics of this measure for different levels of aggregation and for selected years. Panel A shows results across municipalities. The average percentage difference is extremely small. It is 0.0015 in 2025 and reaches 0.0023 in 2050. Standard deviations vary from 0.0016 in 2025 to 0.0032 in 2050, indicating that errors are typically small. This is confirmed by the largest difference, which is a meager -0.36 percent. Appendix Figure B2 shows the spatial distribution of these

[12]Employing smaller representative samples is common in the microsimulation literature (Li and O'Donoghue, 2013).

[13]Ballas et al. (2005) make a multi-regional simulation model for Ireland. They validate the model using within-sample predictions.

[14]In Appendix Figure B1, we calculate the ratio of the results for the CCM projections relative to the microsimulation model. The ratios are closely centered around unity. There are two outliers where the differences amount to a humble one permille.

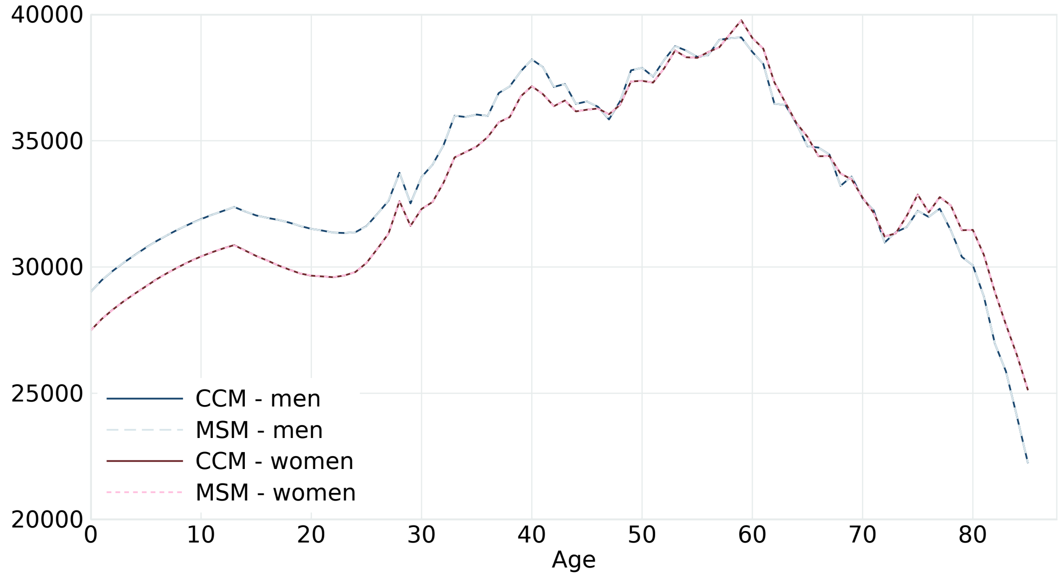


Figure 1. Projected population size by age and sex in 2050. Results from the cohort-component model (CCM) and the microsimulation model (MSM)

Notes: The figure shows the projected population size by age and sex in 2050 using CCM projections and the microsimulation approach. The age span is limited to 0-85 years of age, as the number of individuals in each group drops sharply for ages exceeding 85. For both genders, the differences across one-year age groups are indistinguishable by visual inspection.

percentage differences in 2050, indicating that there are no systematic geographical patterns in the discrepancy of the models.

In Panel B and C, we show the relative results for even smaller sub-populations utilizing sex and age characteristics of the projected population. The average percentage difference and standard deviation are quite similar to the ones in Panel A. The minimum and maximum values for the relative percentage differences increase somewhat, which is natural when examining cells with smaller populations. In Panel B, the highest absolute value for outlier percentage difference is 0.57 and the corresponding number in Panel C is 1.87. These deviations originate from smaller municipalities with few persons in certain cells (see Appendix Figure B5), and consequently small differences in results amount to higher differences in relative terms. It is worth mentioning that the outlier values of differences in (average) results are extremely small. These small deviations in results must be attributed to the stochastic nature of microsimulations, and would be reduced by expanding the number of

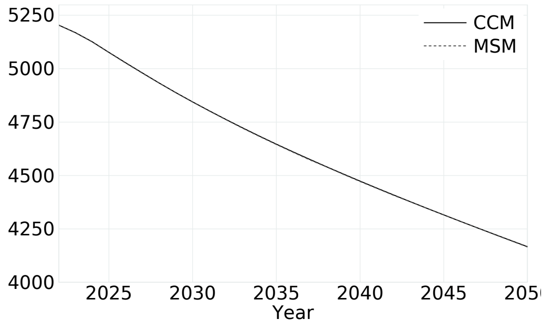
Table 2. Comparisons of the percentage differences in projected population size between the CCM and the microsimulation model. Different levels of aggregation and for selected years

	2025	2030	2040	2050
Panel A – Municipality (N=356)				
Mean	0.0015	0.0011	0.0045	0.0023
St.dev.	(0.0016)	(0.0023)	(0.0028)	(0.0032)
Min/max	[-0.14 / 0.12]	[-0.19 / 0.3]	[-0.18 / 0.33]	[-0.36 / 0.28]
Panel B – Municipality by sex (N=712)				
Mean	0.0015	0.0012	0.0047	0.0023
St.dev.	(0.0016)	(0.0022)	(0.0027)	(0.0031)
Min/max	[-0.28 / 0.19]	[-0.28 / 0.36]	[-0.29 / 0.33]	[-0.57 / 0.34]
Panel C – Municipality by sex and age group (N=2,848)				
Mean	0.0020	0.0015	0.0057	0.0035
St.dev.	(0.0019)	(0.0026)	(0.0029)	(0.0032)
Min/max	[-0.67 / 0.73]	[-0.94 / 0.89]	[-1.24 / 0.86]	[-1.22 / 1.87]

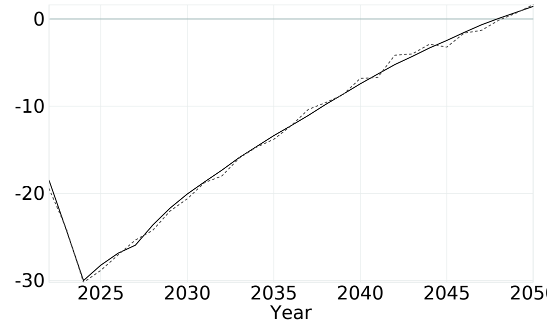
Notes: The table displays descriptive statistics for percentage differences in population size results between the CCM projections and the microsimulation model (1000 simulations) for different levels of aggregation and for the years 2025, 2030, 2040 and 2050. Means, standard deviations (in parentheses), and minimum and maximum values (in brackets) are presented. Panel A shows the differences across municipality, Panel B across municipality by sex, and Panel C across municipality by sex and five age groups: 0-19, 20-49, 50-79, 80-119.

simulations further.

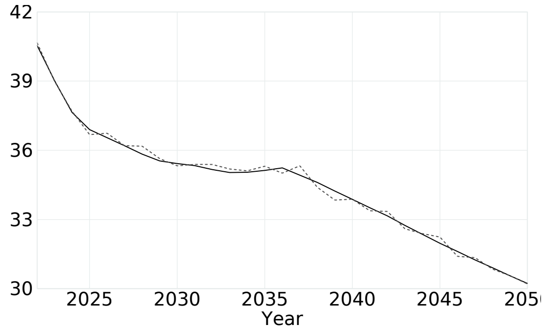
In models such as these, differences in opposite directions cancel out. For instance, if we overshoot both the number of births and deaths the population size may be unaffected. For brevity, we select the median-sized municipality and conduct comparisons of results for population size and demographic events: births, deaths, and net migration (see Figure 2). The results for the two models are for all practical reasons identical, easing any concerns about misspecifications in the microsimulation model. The corresponding results for the largest and smallest municipality in Norway are displayed in the appendix, providing further evidence for the conclusion (see Figures B3 and B4).



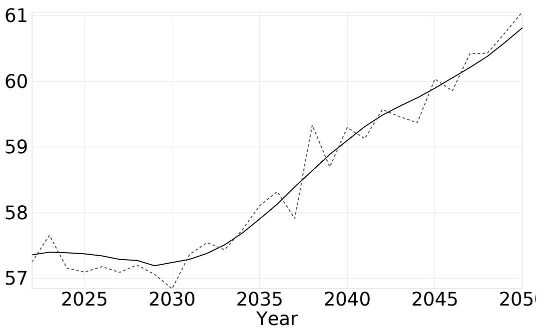
(a) Population



(b) Net migration



(c) Births



(d) Deaths

Figure 2. Projected population size and demographic events for the median sized municipality. Comparing CCM and microsimulation results

Notes: The figure compares the results from the CCM projections and the microsimulation model for a median-sized Norwegian municipality. Panel (a) displays the comparison for population size, whereas panels (b), (c), and (d) show the comparison for net migration, births, and deaths, respectively. Net migration is the sum of international and internal net migration.

4.2 Convergence

The deterministic CCM projections provide, for each set of assumptions, one result for each population group of interest. The simulation model, on the other hand, simulate multiple realizations using draws. A prudent question is, how many draws are necessary to get stable results? That is, how many simulations are needed to obtain average results that correspond to the CCM projections?

Despite their many benefits, microsimulation models are more computationally demanding and have larger storage demands compared to CCM projections. The running of complex simulations is also time-consuming. Thus, there are cost and time benefits from limiting the number of simulations. With results from both the

CCM projections and the corresponding microsimulation model, we have the ingredients needed to shed new light on the issue and assist others in forming priors for the necessary number of simulations.

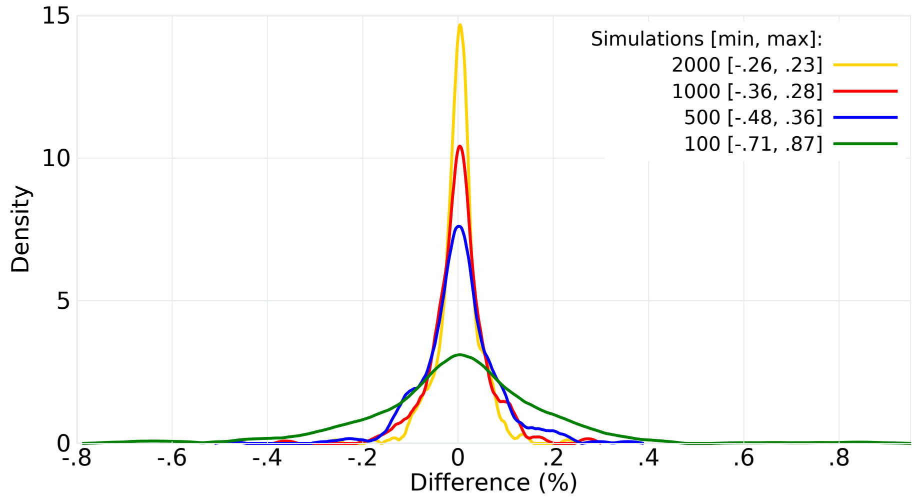


Figure 3. Distribution of percentage differences between the two models in municipality population size in 2050. Results by number of simulations

Notes: The figure shows the distribution of percentage differences between the CCM and microsimulation model in projected municipal population size. It displays the distributions resulting from 100, 500, 1000, and 2000 simulations.

Figure 3 displays percentage differences in municipality population size between the CCM projections and microsimulation model with different numbers of simulations. As can be seen, the deviations lessen with an increased number of simulations. The distributions become increasingly clustered around zero and the tails become thinner and shorter. For instance, the maximum absolute difference is 0.87 percent with only 100 simulations, whereas this value drops to 0.26 percent with 2000 simulations. The results suggest that relatively few simulations are necessary if total municipal population size is of interest. When investigating smaller sub-populations, e.g. children of childcare age (1-5 years in Norway), a larger number of simulations is necessary to obtain the same level of precision (see Figures B5 and B6). Put simply, projection errors tend to increase as population size decreases (Wilson et al., 2018).

The main results presented in this paper consist of 1000 individual simulation runs and the computational requirements are relatively large in terms of processor time, memory and storage. The model runs in approximately 24 hours on 40 CPU cores and demands about 800Gb of RAM (20Gb per core).[15] Results for each individual simulation, such as total population size, number of deaths, births, in- and out-migrants for each age, gender and region group, are temporarily stored in order to calculate distributional statistics. In total, this requires roughly about 20Tb of storage space.

4.3 Uncertainty and policy-implications

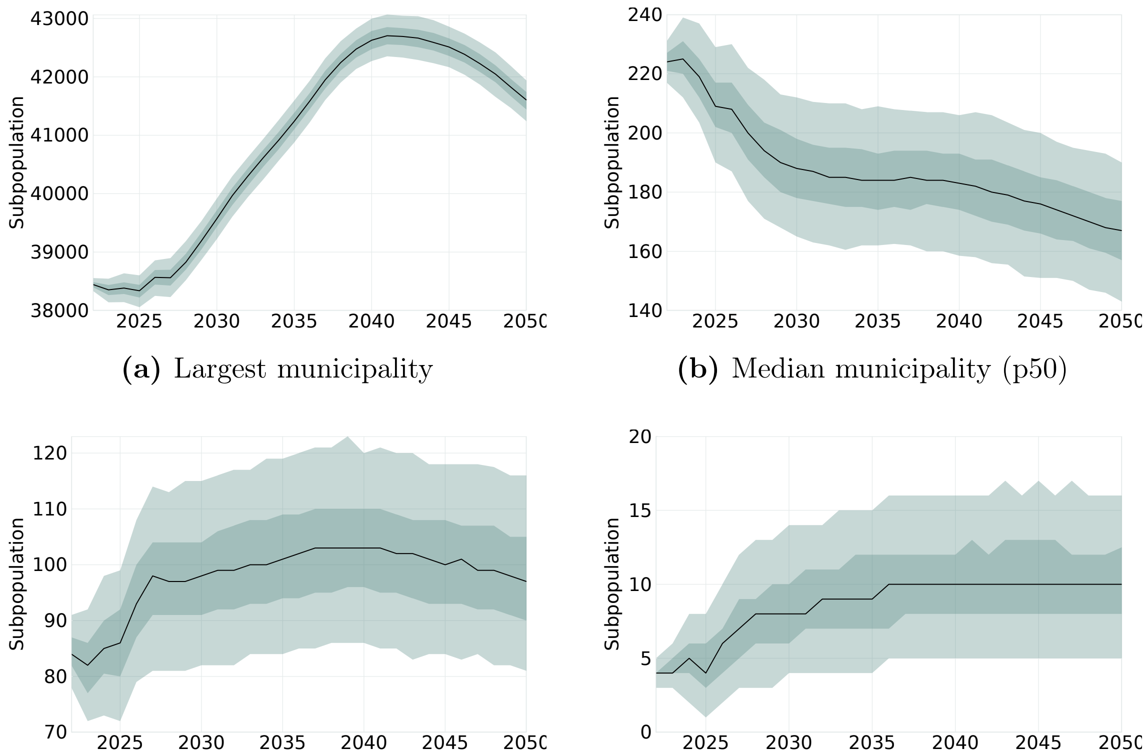
The classical cohort-component approach projects the population deterministically conditional on the demographic rates. In other words, the population development follows a single path for each group in the model.[16] Microsimulation models are probabilistic in nature, as the events in the model happen randomly according to the distributional assumptions made. Each simulated population path represents just one potential outcome of the random process. By aggregating all simulated paths, we obtain a sample of realizations conditional on the model assumptions. This sample can be used to quantify the uncertainty of the underlying random process, such as by computing the 50th or 90th percentile prediction intervals for a given year.

To illustrate the usefulness of such measures of uncertainty, we examine the results for a population group that are interesting from a policy and planning perspective — children of childcare age (1-5 years old). In Norway, all children have the right to childcare services after their first birthday, and local governments are legally obliged to ensure provision. Projections for this group are therefore used to inform scaling decisions for childcare services and investments.

Figure 4 displays the projected size of the selected group (sample average) and the

[15] The server used to run the simulations is a HPE DL560 Gen10 with four Intel Xeon Platinum 8170 processors (96 cores) and 1.6Tb RAM.

[16] Uncertainty in macro models can also be derived, but is rarely done because of the difficulty associated with doing such complex calculations. See for instance Lee (1998), Keilman (1990) and Dunstan and Ball (2016) for discussions.



(a) Largest municipality

(b) Median municipality (p50)

(c) Small municipality (p25)

(d) Smallest municipality

Figure 4. Uncertainty of preschool population in selected municipalities
Notes: This figure shows the projection uncertainty in the population group aged 1–5 years in four different municipalities. The black line represents the mean population while the shaded areas represent the 50 percent (inner/darker) and 90 percent (outer/lighter) projection intervals. The municipalities are selected to show uncertainty across a range of population sizes, ranging from the largest municipality (Oslo) in panel (*a*) with a population of 699,827 to the smallest municipality (Utsira) in panel (*d*) with a population of 188 at the start of 2022. In addition, we included the 50th percentile municipality in terms of population size (Årdal) in panel (*b*) and the 25th percentile municipality (Karlsøy) in panel (*c*) with total populations of 5 204 and 2 129 inhabitants, respectively.

corresponding sample distribution of simulations for four municipalities: the most populous municipality, the median-sized municipality (p50), a small-sized municipality (p25), and the least populous municipality. We see that the absolute uncertainties are positively correlated with the population sizes. On the other hand, the uncertainty relative to population size is negatively correlated with the population sizes. For instance, for the smallest municipality, the 90th percent prediction interval in 2035 is roughly [4, 15], while the average projection is around 9. The length of the same prediction interval is 120 percent of the average projection. In comparison, for the largest municipality, the corresponding prediction interval is less than one percent of the average projection. Figure B8 shows how the relative uncertainty is inversely proportional to the population size of the municipality. This follows from the law of large numbers. In particular, the relative uncertainty of the simulation results, which is measured by the coefficient of variation (ratio of the standard deviation and mean), decreases as the product of the number of simulations and individuals increases. Hence, the mean population size from the microsimulations is close to the expected value (i.e. CCM projections) for regions with a larger population. However, for smaller group sizes in sparsely populated municipalities, the relative dispersion of the projected population size is larger, and the relative uncertainty of the projection is also influenced by factors such as the projection horizon, event probabilities, and the nature of the population (i.e., synthetic or not).

How should we interpret the uncertainty provided by the prediction intervals? It is important to recognize that the intervals quantify uncertainty conditional on the assumptions on demographic behavior being correct. Although this measure does not incorporate all uncertainty aspects, it provides more information than the deterministic development path and thus enables policy-makers and planners to make better decisions.[17]

Let us take the small-sized municipality as an example. In 2022, it is expected to provide childcare to a population of about 85 children. The average result from the projection indicates that the number of children will increase to about a hundred around 2030. We do however have more information as we have projected the population using microsimulations. In 2035, the 90 percent prediction interval spans from the no-change scenario of 85 to about 120 children. Taking into consideration that there is a 50 percent likelihood (given assumptions) that the number of children exceeds 100 but is not likely to go beyond 120 may facilitate better decisions and

[17]Alternative sources of uncertainty include measurement error in the estimation of local demographic probabilities and uncertainty about the components trends. See Lutz and Goldstein (2004) for an in-depth discussion. It is possible to incorporate these types of uncertainty into the microsimulation framework, but that is not the focus of this paper.

contingency plans related to building use and employment strategies in the childcare sector. Given that the ratio of children to childcare facilities is just above 20 in this municipality in 2021, an extra 35 children would potentially require a sizeable adjustment of capacity.

Similar evaluations may be conducted for other policy-relevant population subgroups, for instance children in school age, or the elderly, perhaps with healthcare provision in mind. We find that the prediction interval is generally larger for younger populations compared to older populations (see Figure B7). The reason for this is that the younger population has higher migration propensities and is affected by the fertility assumptions (i.e. for those not yet born).

5 Discussion and concluding remarks

Regional population projections are a highly valued tool that assists users in planning decisions and policy formation. It is for instance used for scaling public services and investments in schooling, health care, and infrastructure to meet future demand. As decisions based on population projections tend to be costly, the models that are accurate in predictions and provide sufficient detail are most helpful.

The projection model using the cohort-component method has its merits, but displays serious challenges when it comes to handling heterogeneity in population characteristics and behavior. This issue is especially pressing when it comes to multi-regional projections that are hard to handle because of the additional geographical component. The model becomes intractable with further expansions. We address this shortcoming by examining a different model class known for its favorable properties related to incorporating complexity — the dynamic spatial microsimulation model.

We take as our point of departure the official multi-regional CCM projections of Norway and translate the model into a microsimulation counterpart. This approach has several advantages. First, it demonstrates that it is possible to directly transform CCM projections into a microsimulation model without making concessions with respect to the quality of the projection. In other words, this approach preserves the favorable properties of the CCM and adds on the advantages of the microsimulation framework. Second, in the process, we shed light on CCM assumptions that are often hidden for microsimulation practitioners. Of principal importance is the simultaneous setup of the CCM projection.

We demonstrate equivalence of the results from the two models and examine convergence properties over the number of simulations. The number of simulations necessary depends on the statistic of interest. Few simulations are necessary to obtain sufficient precision for aggregate numbers, while a higher number of simulations are necessary if one desires to precisely estimate smaller population groups. In our application, the results indicate that 1000 simulations are sufficient to obtain near equivalent outcomes for small groups separated by sex, age and municipality of residence.

Very little attention has been devoted to estimating uncertainty for regional population projections, even though the uncertainty tends to be higher for small areas where the need for quantifying uncertainty is the greatest (Wilson et al., 2022). Users report that they are generally interested in such information (Wilson and Shalley, 2019). The microsimulation framework provides information on certain aspects of uncertainty through the extra information provided by multiple simulated population development paths. We demonstrate the usefulness of the distribution of simulations for small area planning and policy. A possible avenue for development of dynamic spatial microsimulation models is to include additional aspects of uncertainty — for instance related uncertainty in the estimated demographic behavior.

Dynamic spatial microsimulation models have some requirements that differ from the multi-regional CCM projections. They are typically computationally demanding and require larger amounts of data storage. Another issue is data requirements.

The advances in computer technology and data availability have made these issues less serious (Spielauer, 2011). Nevertheless, inputs for spatial models may be especially challenging because time series tend to be short due to boundary changes. Developing methods for handling data and estimation issues specific to the spatial setting is a promising avenue for research.

Comparison of the two methods for population projections and clarification of the implicit and explicit assumptions may facilitate academic discussion for demographers interested in dynamic spatial microsimulation, which in turn may progress the field and increase uptake among practitioners too. The work may potentially lessen the development costs and the costs related to acquiring necessary expertise, which have been pointed out as barriers for more extensive use of microsimulation models (Harding, 2007). In future work, we plan to go beyond exploring equivalence of the two models and add on additional characteristics of the population. Consequently, we will be able to utilize new knowledge on the determinants of demographic behavior in the model, which is promising with respect to model realism and information richness.

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Appendix

A Step-by-step model descriptions

In both models, individuals/groups have the following characteristics: 2 genders, 120 age-groups, 365 municipalities.

In both the cohort-component model and the microsimulation model, the demographic events are defined as follows:

- All demographic event rates are specific to age, sex, municipality and year, and calculated based on end-of-year age.
- Fertility rates are only defined for females between the ages of 15 and 49.
- Migration rates (internal and external) are only defined for groups below 70 years of age.
- Mortality rates are defined for all groups and are set to a constant 0.5 from age 108 and onward.
- Fertility rates, mortality rates, internal migration rates (matrix), total immigration and expected emigration numbers are predetermined inputs for both models.

The following table summarizes and compares the order of operations between the two models:

Table A1. Order-of-operations

Step	Cohort-component:	Microsimulation:
0	The CCM dataset has one cell for each combination of age, sex and municipality. This cell contains the number of individuals in that group. Except for the start year, this will be a decimal number.	The microsimulation dataset has one observation for each individual, with variables containing a unique person identifier, age, sex and municipality.
1	The end-of-year population of the previous year is used as the start-of-year population of the current year.	The end-of-year population of the previous year is used as the start-of-year population of the current year.
2	The age of the population is advanced by 1 to reflect the end-of-year age of the individuals.	The age of each individual is advanced by 1 to reflect the end-of-year age of the individuals.
3	The age-specific fertility rates are multiplied with the number of individuals in each cell of females, producing the expected number of newborns throughout the current year. 51.369% of newborns belong to the male group, while the rest are assigned to the female group. These are added as 0-year-olds (end-of-year age) to the now updated population of the municipality.	For each female we randomly draw from a Bernoulli distribution the event of a birth. The probability of a birth is determined by the fertility rate. Similarly, sex is assigned by random draw where the probability of being male is 0.51369. All births are added to the dataset as new individuals with a unique identifier, (end-of-year) age 0 and assigned the same municipality as the mother.
4	We scale all emigration rates, so that the expected number of emigrants is equal to the predetermined target emigration.	We scale all emigration rates, so that the expected number of emigrants is equal to the predetermined target emigration.
5	The mutually exclusive events of internal migration (out-moves), emigration and death are calculated by multiplying the rates of each event with the corresponding group size. We get the expected number of each event for each cell.	The mutually exclusive events of internal migration (out-moves), emigration and death are drawn simultaneously for each individual from a multinomial trial/distribution. An individual can either move, emigrate, die, or do nothing.
6	The groups of out-movers, as well as the predetermined number of immigrants, are distributed across municipalities according to the moving matrix, conditional on age, sex and municipality. This gives the number of in-movers and immigrants for each cell.	For individuals that move, and for the predetermined number of immigrants, we randomly draw a destination municipality conditional on the probability distribution in the moving matrix.
7	The number of out-movers, emigrants and dead are subtracted from each population cell (age/sex/municipality). The number of immigrants and in-movers are added to each population cell. This is now the current end-of-year population.	Individuals who die or emigrate are removed from the data. Movers and immigrants have their municipality variable updated to reflect their assigned destination. This is now the current end-of-year population.
8	To project the population for an additional year: go back to step 1	To simulate the population for an additional year: go back to step 1

B Additional figures

In this section, we show additional figures referenced in the main text.

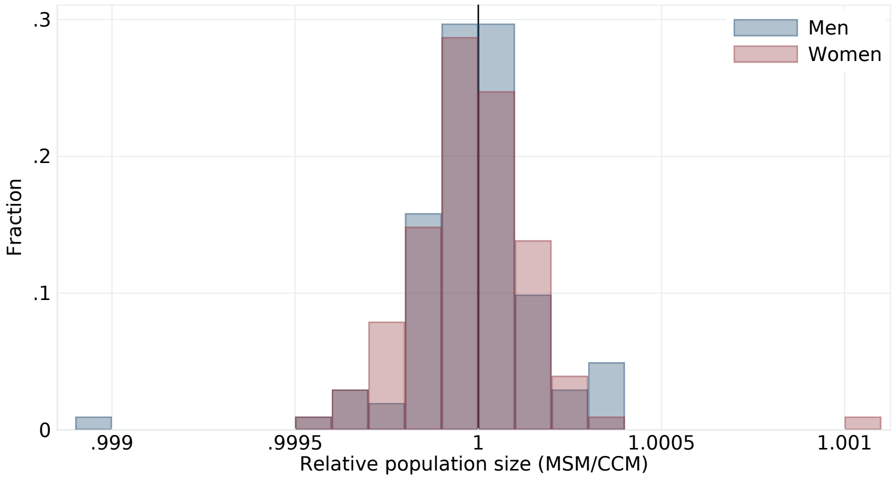


Figure B1. Comparing the results from the cohort-component model and the microsimulation model. Ratios of projected population size by age and sex in 2050.

Notes: The figure shows the ratio of projected population size by age and sex in 2050 from the microsimulation model relative to the cohort-component model. The age span is limited to 0-100 years of age. There are very few persons exceeding this age according to the projections.

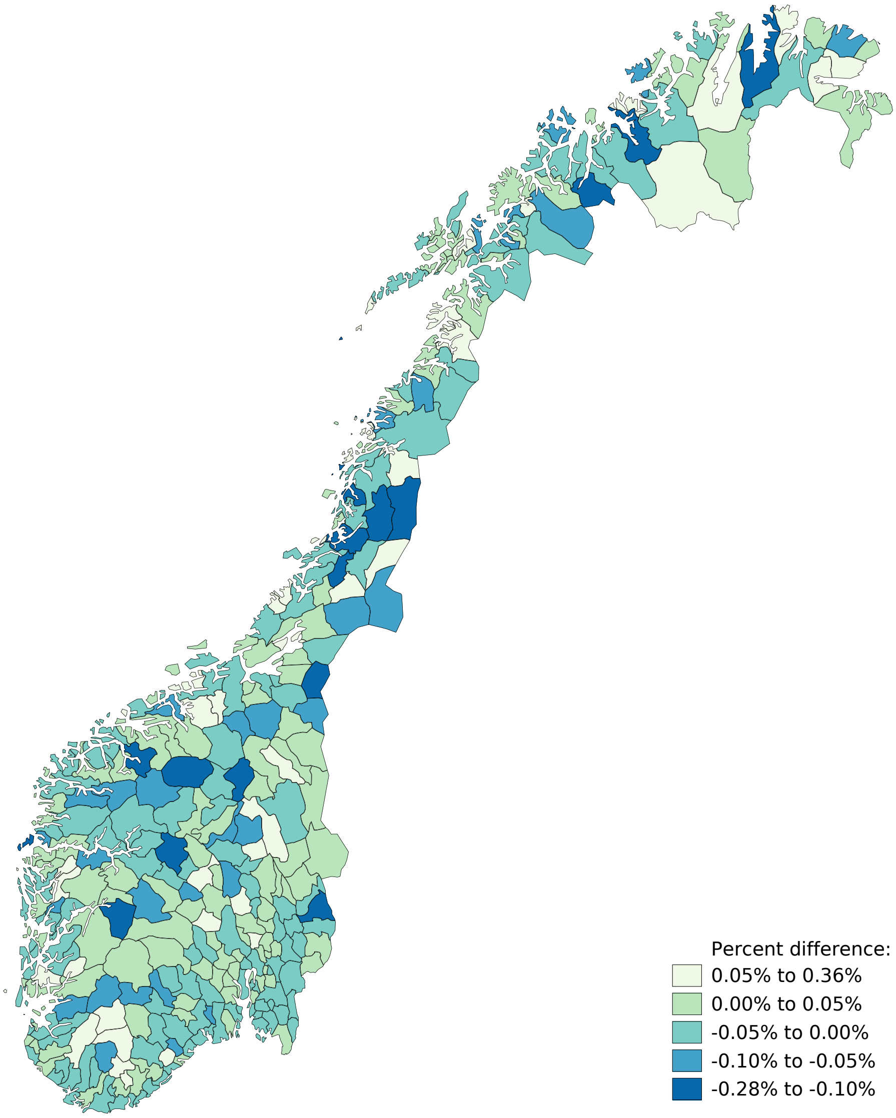
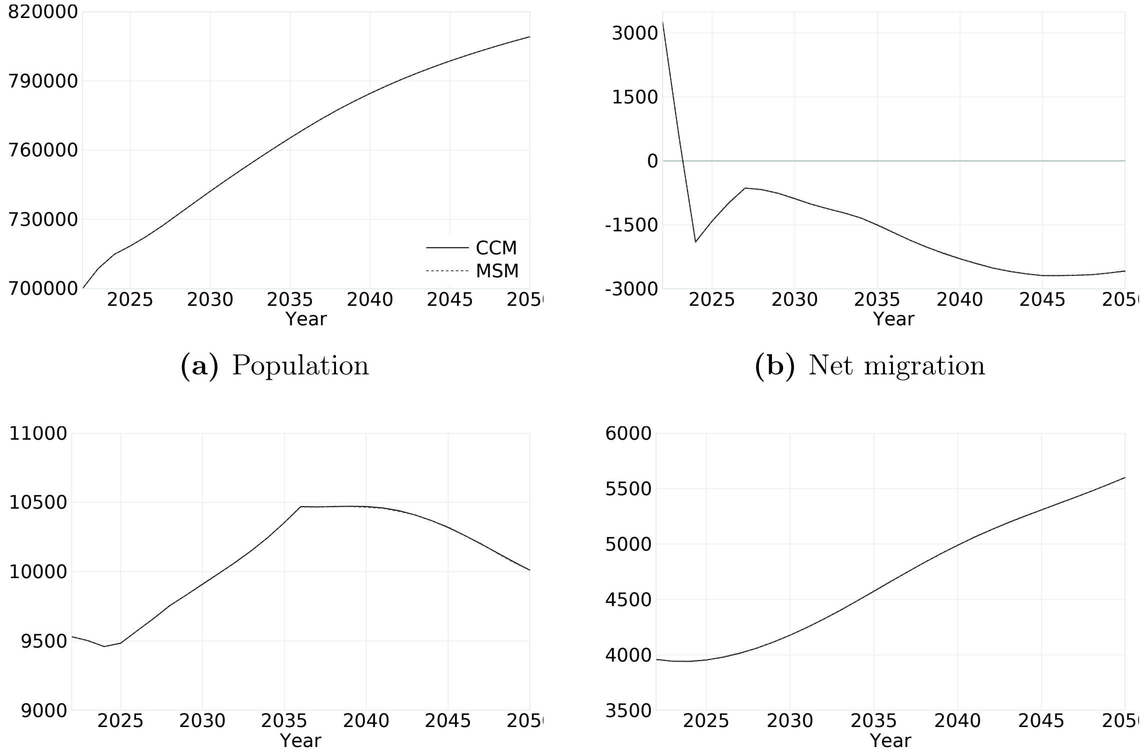


Figure B2. Municipality map representing the percentage difference in the projected municipality population of the two models in 2050

Notes: The figure shows the percentage difference in the projected municipality population between the cohort-component model and the microsimulation model in 2050. As the map shows, there are no broad geographical patterns in the accuracy of the microsimulation model.



(a) Population

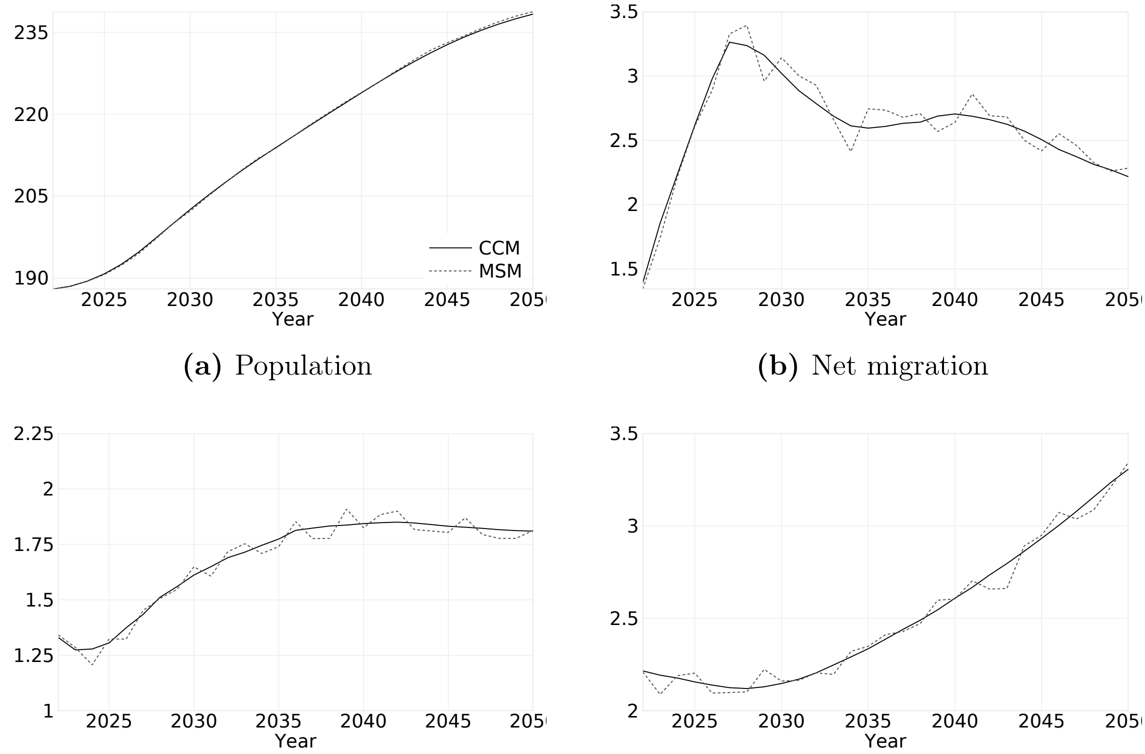
(b) Net migration

(c) Births

(d) Deaths

Figure B3. Projected population size and demographic events for the largest Norwegian municipality, Oslo

Notes: The figure compares the projection results from the CCM model and the microsimulation model for Oslo, the largest Norwegian municipality with a population size of 699 827 at the start of 2022. Panel (a) shows the comparison of population size, whereas panels (b), (c), and (d) show the comparison for net migration, births, and deaths, respectively. Net migration is the sum of international and internal net migration.



(a) Population

(b) Net migration

(c) Births

(d) Deaths

Figure B4. Projected population size and demographic events for the smallest Norwegian municipality, Utsira

Notes: The figure compares the projection results from the CCM model and the microsimulation model for Utsira, the smallest Norwegian municipality with a population size of 188 at the start of 2022. Panel (a) displays the comparison for population size, whereas panels (b), (c), and (d) show the comparison for net migration, births, and deaths, respectively. Net migration is the sum of international and internal net migration.

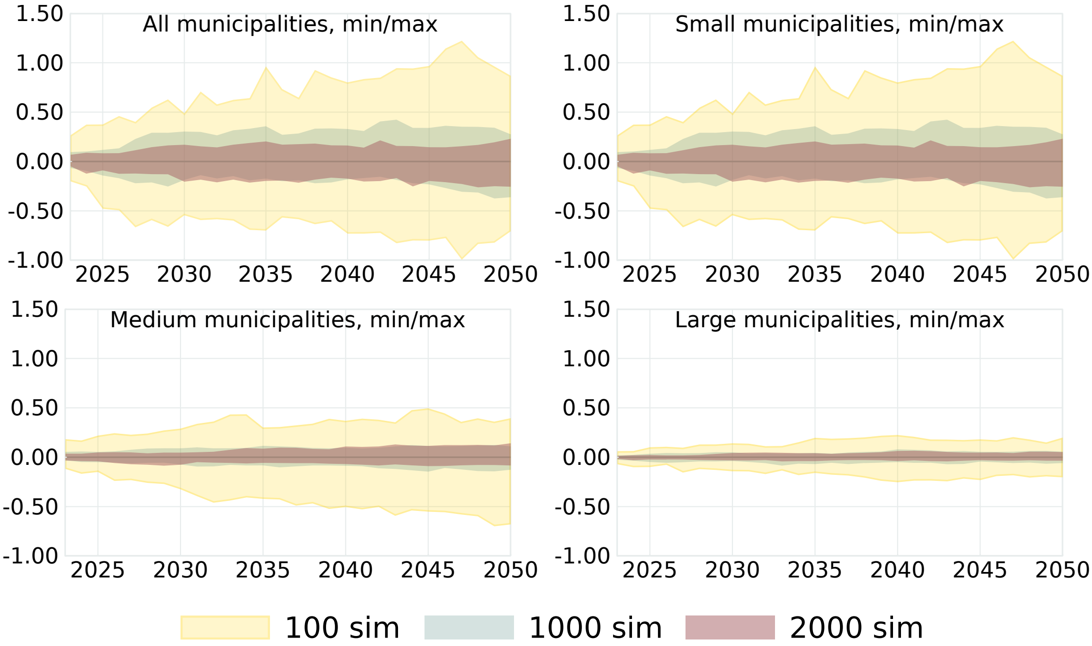


Figure B5. Minimum and maximum percentage differences between the two models in municipality population by number of simulations, over years 2023-2050

Notes: The figure shows the upper and lower bound of the percentage difference between the two models across years. In the first panel (top-left), the yellow (outer) area shows the range between the smallest and largest percentage difference between the CCM model and microsimulation using 100 simulation runs for all 356 municipalities. The green area shows the range using 1,000 simulation runs and the red (inner) area shows the range using 2,000 simulations. The second panel (top-right) shows the same thing for a subset of small municipalities with a population of fewer than 2,500 inhabitants (approx. 1/3). The third panel (bottom-left) shows the ranges for the medium-sized third of municipalities with a population between 2,500 and 10,000 inhabitants. Finally, the fourth panel (bottom-right) shows the range for a subset containing the largest third of Norwegian municipalities with a population of over 10,000 inhabitants. The main point of this figure is that more simulations ensure a tighter bound on the relative differences between the two models. Smaller municipalities (or population subgroups) benefit relatively more from additional simulations than larger municipalities. With sufficient simulations, the difference between the two model results does not accumulate over time.

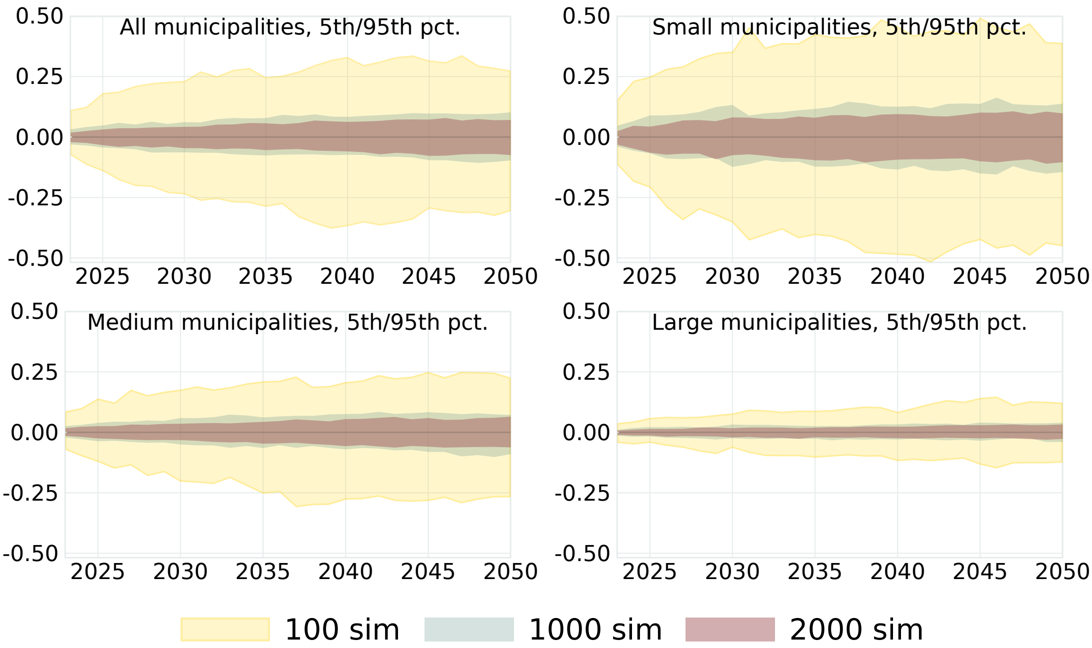
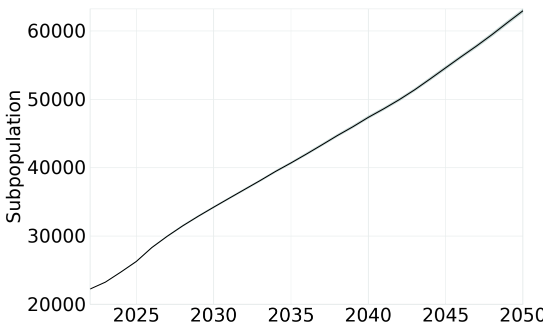
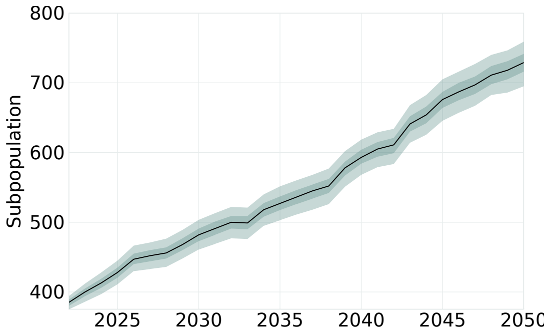


Figure B6. 90 percent prediction interval of percentage differences between the two models in municipality population by number of simulations, over years 2023-2050.

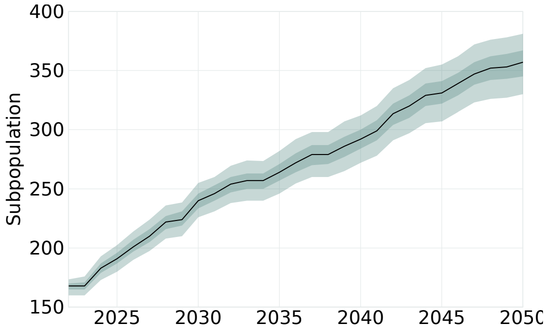
Notes: The figure shows the 90 percent prediction interval of the percentage difference between the two models across years. This is the range between the 5th and 95th percentile. In the first panel (top-left), the yellow (outer) area shows the range between the smallest and largest percentage difference between the CCM model and microsimulation using 100 simulation runs for all 356 municipalities. The green area shows the range using 1,000 simulation runs and the red (inner) area shows the range using 2,000 simulations. The second panel (top-right) shows the same thing for a subset of small municipalities with a population of fewer than 2,500 inhabitants (approx. 1/3). The third panel (bottom-left) shows the ranges for the medium-sized third of municipalities with a population between 2,500 and 10,000 inhabitants. Finally, the fourth panel (bottom-right) shows the range for a subset containing the largest third of Norwegian municipalities with a population of over 10,000 inhabitants. The main point of this figure is to show that the 90 percent prediction interval is much tighter than the minimum/maximum coverage demonstrated in Figure B5, and the 90 percent bounds from 1,000 simulations is relatively closer to the 2,000 simulation bounds than in the minimum/maximum case. Similar to the minimum/maximum case, the difference between the two model results does seem to stabilize at some point and does not keep increasing over time.



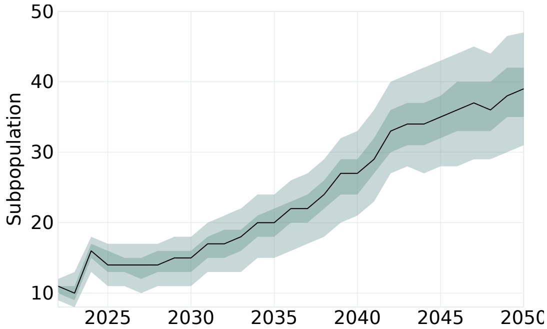
(a) Largest municipality



(b) Median municipality (p50)

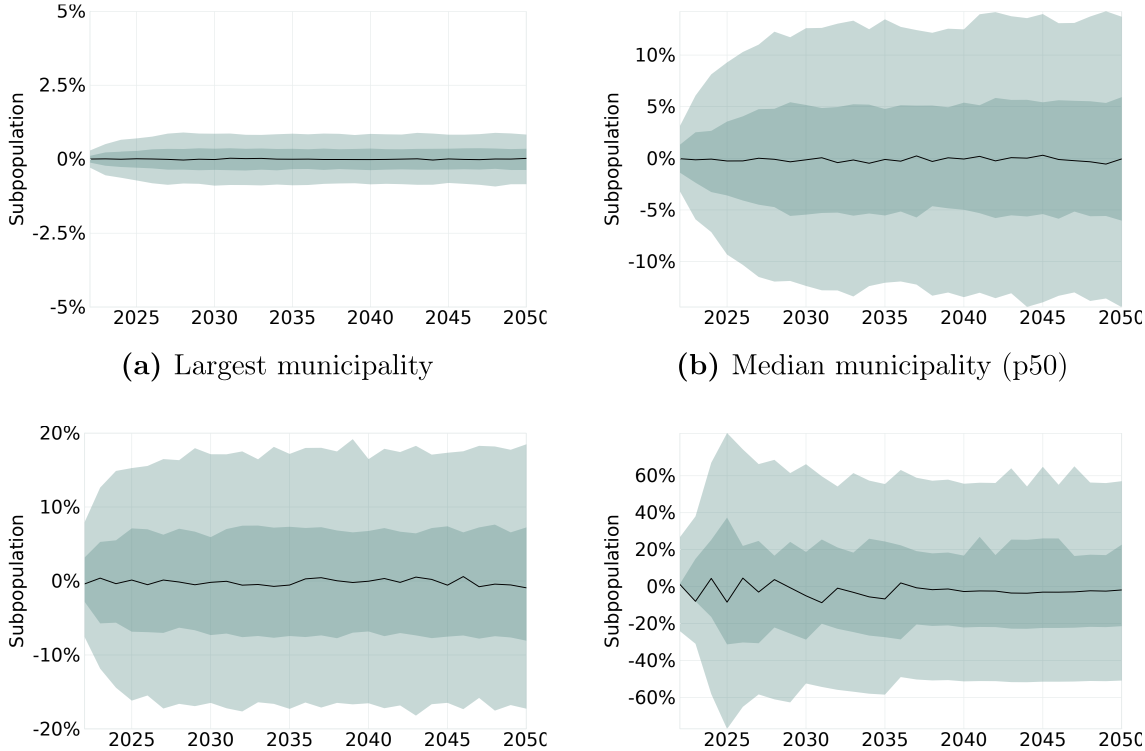


(c) Small municipality (p25)



(d) Smallest municipality

Figure B7. Uncertainty of the size of elderly populations in selected municipalities
Notes: The figures above show the projection uncertainty in the population group aged 80 or older in four different municipalities. The black line represents the mean population while the shaded areas represent the 50 percent (inner/darker) and 90 percent (outer/lighter) projection interval. The municipalities are selected to show uncertainty across a range of population sizes, ranging from the largest municipality (Oslo) in panel (a(with a population of 699 827 to the smallest municipality (Utsira) in panel (d) with a population of 188 at the start of 2022. In addition, we included the 50th percentile municipality in terms of population size (Årdal) in panel (b) and the 25th percentile municipality (Karlsøy) in panel (c) with total populations of 5 204 and 2 129 inhabitants, respectively.



(a) Largest municipality

(b) Median municipality (p50)

(c) Small municipality (p25)

(d) Smallest municipality

Figure B8. Normalized uncertainty of preschool population in selected municipalities

Notes: The figures above show the normalized projection uncertainty in the population group aged 1–5 in four different municipalities. By normalized, we mean all numbers from the microsimulation projection are divided by the expected projected size of the subpopulation from the CCM projections. The black line represents the mean normalized population, while the shaded areas represent the 50 percent (inner/darker) and 90 percent (outer/lighter) projection interval. The municipalities are selected to show normalized uncertainty across a range of population sizes, ranging from the largest municipality (Oslo) in panel *a* with a population of 699 827 to the smallest municipality (Utsira) in panel *d* with a population of 188 at the start of 2022. In addition, we included the 50th percentile municipality in terms of population size (Årdal) in panel *b* and the 25th percentile municipality (Karlsøy) in panel *c* with total populations of 5 204 and 2 129 inhabitants, respectively. This figure clearly demonstrates that the size of the subpopulation is inversely linked to the relative spread of the simulations, meaning that the uncertainty is larger for small populations.