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Documents

Some Results from the Literature on the Impact of Carbon Taxes on the Petroleum Wealth

1. Introduction[1]

In this paper we review parts of the literature on optimal extraction of exhaustible resources. The focus is on the impact of a global carbon tax on the petroleum wealth of fossil fuel producers, and we want to study the effects of taxation under different assumptions about the cost functions and market power of the fossil fuel producers. The resource wealth is defined as the discounted net revenue flow from extraction.

The survey is motivated by the development of a numerical model for the global markets of fossil fuels which is documented in Berg *et al.* (1996). Our starting point is to examine the impact of a global carbon tax on the petroleum wealth of fossil fuel producers under different market assumptions. Some of the main results from Berg *et al.* (1996) will be presented in the last section of the paper.

In the first part of the paper we review some of the theoretical results from the literature. We start by looking at the simple Hotelling model with constant unit costs of extraction and perfect competition. However, when the simple Hotelling model is extended to take account of rising marginal extraction costs, imperfect competition and the substitution in demand of different exhaustible resources, it becomes increasingly difficult to derive unique theoretical results. This fact motivates the development of numerical models such as the one in Berg *et al* (1996) to investigate the effects on the petroleum wealth of an international carbon tax on fossil fuel consumption.

Taxation of exhaustible resources, like fossil fuels, has been examined theoretically in e.g. Burness (1976) and Dasgupta and Heal (1979), who consider the traditional Hotelling model, and Heaps (1985) and Lasserre (1991), who also examine more extended models. In general they find that some tax systems, like a constant sales tax, may decrease the extraction rate, while other systems, like franchise taxes, may increase the rate. A franchise tax, a licence fee or a fixed property tax is independent of the rate of extraction (Lasserre (1991)). With a constant sales tax or unit tax, Dasgupta and Heal (1979) found that the tax is shared between producers and consumers during an initial interval. Furthermore, a constant profit tax, or eventually a constant ad valorem tax when extraction costs are zero, has no effect on the depletion path, and the tax is born solely by the producers.[2] As the depletion path in this case in unchanged, the tax will have no effect on carbon emissions. Sinclair (1992) argued that the optimal ad valorem carbon tax rate should be falling over time in order to reduce the depletion rate. However Sinclair's argument has been criticised in Ulph *et al.* (1991) who found that the issue of an optimal carbon tax is much more complex. The problem of finding an optimal carbon tax will not be dealt with here. Instead we concentrate on the effects of a constant carbon tax on fossil fuel consumption.

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[1] I want to thank Kjell Arne Brekke, Sverre Grepperud, Snorre Kverndokk, Knut Einar Rosendahl and Stein Hansen for comments on earlier drafts. This paper was financed by The Norwegian Research programme in Petroleum Economics and Politics.

[2] An ad valorem tax is put on the value of production, not on the production volume in physical units.

The remainder of the paper is organised as follows. We present the results in stages, starting in section 2.1 with the simple Hotelling model with constant unit costs of extraction and perfect competition. In sections 2.2 and 2.3 we extend the models to consider more complex cost functions, and finally we introduce imperfect competition in section 3. In the last section we report some results from two recent numerical studies of the effects of a global carbon tax on the petroleum wealth of fossil fuel produsers.

2. Perfect competition

2.1 Constant extraction costs

Before we look at the effects of taxation we review some basic results from the literature on optimal resource extraction. We start by looking at a simple Hotelling model for the oil market. Hotelling (1931) viewed the problem of how to extract a fixed stock of a natural resource from the vantage point of a social planner. He then showed that a competitive industry facing the same extraction costs and demand curve as the goverment, having perfect information about resource prices, will arrive at exactly the same extraction path for the resource, i.e., the efficient extraction path determined by each firm acting independently in the competitive industry will yield the socially optimal extraction path.

We look at the optimisation problem of a representative producer of the exhaustible resource in a competitive market.

$$(1) \quad \max_x V = \int_0^\infty [p(t) - c - v]x(t)e^{-rt}dt$$

$$(2) \quad \text{s.t.} \quad \dot{A}(t) = x(t)$$

$$A(t) \leq R$$

$$x(t) \geq 0$$

The representative producer maximises the discounted revenue flow (resource price minus the unit costs of extraction and the tax), subject to the stock constraint. R is the total amount of the resource.[3] A unique extraction path can be derived with the help of the stock constraint in (2) and a terminal condition, together with a specified demand function.[4]

[3] There is a list of symbols at the end of the paper. All variables are functions of time, but the time notation may sometimes be excluded for simplicity.

[4] We do not consider uncertainty in this paper. A resource would be extracted more slowly by a monopolist or a competitor, when the reserve base is not known with certainty, see Pindyck (1978).

We assume that demand is represented by an isoelastic demand function as in (3).

(3) $p(t) = x(t)^{-\varepsilon}$

Demand is assumed to be zero above a maximum price ($\overline{p}$). The existence of a maximum price or a backstop price can be interpreted as the existence of a perfect substitute for the exhaustible resource. This assumption implies that the entire resource stock will be exhausted in finite time (*T*). The final period of extraction is thus defined by the equation *A(T)=R,* which says that at time *T* the accumulated production is equal to the initial amount of the resource. After the price has reached the backstop price, the price, and thus the scarcity rent (in the case with constant unit costs), remain constant. Thus, there is no incentive for the producer to hold back reserves for production in the following periods as this would lead to forgone interest earnings. Also, the price will always reach the backstop level in the last period of production. Otherwise it would be optimal for the producer to keep reserves for production immediately after this period when the price jumps to the backstop price. The backstop price enters the technical problem as a terminal condition in (4).

(4) $p(T) = \overline{p}$

The current value Hamiltonian of the optimisation problem presented in (1) is

(5) $H = [p(t) - c - v]x(t) - \pi(t)x(t)$

We define the scarcity rent or the resource rent (π) as the negative of the shadow value of the optimisation problem, so that the rent is a non-negative number. The scarcity rent is thus the value of increasing the fixed stock of the resource with one unit

One can establish the existence of an optimal solution by resorting to *Theorem 15* in Seierstad and Sydsæter (1987), p. 237. We then assume that $x(t) \in U$, where U is a fixed, non-negative subset of R^1. U is closed and bounded.

The Hamiltonian is concave in (A, x). As a consequence the necessary first order conditions according to the maximum principle are also sufficient conditions for an optimal (interior) solution. The first order conditions are

(6a) $\frac{\partial H}{\partial x} = p - c - v - \pi = 0$

(6b) $\dot{\pi} - r\pi = \frac{\partial H}{\partial A} = 0$

From (6b) we see that the resource rent along the optimal path will grow at the rate of the interest rate.

(7) $\frac{\dot{\pi}}{\pi} = r$

Under the assumption of *zero extraction costs* ($c=0$) and no taxation ($v=0$), the scarcity rent of the resource equals the price on the resource. We get the standard Hotelling rule which says that in a competitive market for an exhaustible resource, the relative change in the price of the resource, i.e., the scarcity rent, must equal the interest rate. This is also called the fundamental principle of exhaustible resources.

$$(8) \quad \frac{\dot{p}}{p} = r$$

With *constant nonzero unit extraction costs* and no taxation the producer price is equal to the sum of the constant unit cost and the scarcity rent. The introduction of costs prolongs the extraction period as does the introduction of taxes since taxes and constant unit costs are introduced symmetrically into the optimisation problem.

With constant unit costs the relative change in the resource price will be less than the relative change in the scarcity rent. The price path will then be modified according to

$$(9) \quad \frac{\dot{p}}{p} = r\frac{\pi}{c+\pi}$$

The solution to the first order differential equation in (7) is the Hotelling rule that the resource rent is growing exponentially with rate r until the resource is depleted.

$$(10) \quad \pi(t) = \pi(0)e^{rt} \quad \text{for} \quad x>0$$

Combining (10) and (6a) we then get the optimal time path of the resource price

$$(11) \quad p(t) = c + v + \pi(0)e^{rt}$$

We substitute for the price from (11) into the demand function in (3) and solve for production to get the optimal production profile of the representative competitive producer

$$(12) \quad x(t) = \left[c + v + \pi(0)e^{rt}\right]^{-\frac{1}{\varepsilon}}$$

Equations (11) and (12) describe the interior solution to the optimisation problem. We are, however, interested in the effect of an increase in the carbon tax on the resource wealth of the producer. The wealth of the resource is defined as the present value of the resource rent times the extraction at each point in time. In the case of a finite amount of the resource and constant unit costs of extraction, the value of the optimal resource wealth at time t=0 is equal to the initial resource rent times the resource base.

$$(13) \quad V^* = \int_0^T \pi(t)x(t)e^{-rt}dt = \int_0^T \pi(0)x(t)dt = \pi(0)\int_0^T x(t)dt = \pi(0)R$$

To examine the effect on the resource wealth of introducing a constant carbon tax, one can therefore study the effect on the initial resource rent ($\pi(0)$).

We know that the integral of the extraction over the entire period of production must be equal to the initial amount of the resource.

$$(14)\quad \int_0^T \left[c+v+\pi_0 e^{rt}\right]^{-\frac{1}{\varepsilon}} dt = R$$

From the terminal condition in (4) and the optimal price path in (11) we have

$$(15)\quad \overline{p} = c+v+\pi_0 e^{rT}$$

Equations (14) and (15) determine the two unknowns; T and $\pi(0)$. Differentiation of (14) and (15) w.r.t. the carbon tax, v, will give us the effect of an increase in the carbon tax on the initial resource rent and hence on the petroleum wealth, see (13).

$$(16)\quad \frac{d\pi(0)}{dv} = -\frac{e^{-rT}\left\{\dfrac{x(T)}{r\pi(0)} + \dfrac{1}{\varepsilon}\displaystyle\int_0^T \left[c+v+\pi(0)e^{rt}\right]^{-\frac{1}{\varepsilon}-1} e^{rT} dt\right\}}{\dfrac{x(T)}{r\pi(0)} + \dfrac{1}{\varepsilon}\displaystyle\int_0^T \left[c+v+\pi(0)e^{rt}\right]^{-\frac{1}{\varepsilon}-1} e^{rt} dt} < 0$$

From (16) we see that the effect on the initial resource rent is negative, since the expressions in both the numerator and denominator are positive figures, but less than one in absolute value. Further, $d\pi(0)/dv$ is greater than e^{-rT} in absolute value since the term in the brackets in the numerator is greater than the denominator. Thus the petroleum wealth of a fossil fuel producer is reduced when the carbon tax is increased.

The effect on the period of extraction can be shown to be positive, so that an increase in the taxation will prolong the period of extraction.

$$(17)\quad \frac{dT}{dv} = -\frac{1}{r\pi(0)}\left[e^{-rT} + \frac{d\pi(0)}{dv}\right] > 0$$

Rosendahl (1996) models a global competitive fossil fuel market with constant unit costs of extraction. He specifies an exponential utility function where the marginal utility is bounded above. He shows that as the available amount of the resource, R, increases from zero to infinty, $d\pi(0)/dv$ increases monotonously from minus one to zero. Thus increasing R implies that the tax burden on the producers decreases.

2.2 Unit costs of extraction increasing in cumulative extraction

As we shall see, the specification of the cost function is crucial to the solution of the optimisation problem. We now assume that the unit costs of extraction are increasing in accumulated production, as for example in the specification below.

(18) $c(A) = \alpha e^{\eta A}$

η is here the convextity factor of the cost function. In this case the unit costs of extraction will increase over time provided there is a nonzero production level.

(19) $\dot{c} = \frac{dc}{dA}\dot{A} = \frac{dc}{dA}x = \eta c(A)x$

No fixed quantity is assumed for the total availability of the resource, but in line with Farzin (1992), only a limited total amount will be economically recoverable at a given price. This is due to the assumption $c''(A)>0$[5], which means that increasingly large quantities of the fossil fuels can be exploited only at increasing incremental costs, i.e. the unit cost is increasing and convex in A. It will therefore be optimal to extract only a finite amount of the resource since the price is bounded above by the existence of a backstop technology. In the previous section with constant unit costs and a finite resource stock, we had physical exhaustion of the resource, whereas we now have economical exhaustion of an infinite resource base.

The assumption of a backstop technology and an infinite resource base might be a more realistic approach than assuming a given resource stock. Even in the later stages of resource use, there is no really «fixed» reserve base to be extracted. Given the economic incentives, reserves can be maintained or increased through further exploration even though the physical returns to exploration activity decreases as «depletion» ensues. Pindyck (1978) therefore suggests it makes more sense to think of resources like oil as being «nonrenewable» rather than «exhaustible».[6] We do not, however, consider exploration activity in our model.

The optimisation problem of the producer can now be written as in (1), replacing the constant unit costs with the cost function in (18) and disregarding the finite resource stock constraint, $A(t) \leq R$.

Existence of an optimum in partial equilibrium can be shown in line with Farzin (1992) by the use of *Theorem 15* in Seierstad and Sydsæter (1987), as in the case with constant unit costs of extraction. The first order conditions will also in this case be sufficient conditions for an optimal solution to the maximisation problem of the representative producer.

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[5] It is sufficient to assume that $\lim_{A\to\infty} c(A) = \infty$ and $c'(A)>0$.

[6] Since production costs rise as reserves decline, producers must simultaneously determine the optimal levels of exploratory activity and production -resulting in an optimal reserve level- that balance revenues with exploration costs, production costs, and the «user cost» of depletion (or the scarcity rent). Under perfect competition the price rises more slowly than in the case of production without exploration. The pattern of optimal exploratory activity depends on initial reserve levels and on rates of depletion. If the initial reserve endowment is small, the price profile will be U-shaped rather than steadily increasing as in the Hotelling model and its variants. This helps explain the fact that the real prices on many nonrenwable resources have fallen over the years. In later stages of resource use, or throughout if the initial reserve is large, price will increase over time as in the Hotelling model. However the introduction of exploratory activity has the effect of reducing the rate of increase of the price. Thus observed rates of growth of resource rents below market interest rates need not be indicative of monopoly power, see Pindyck (1978).

In the case with physical exhaustion the resource rent was exponentially rising over the entire production period. The presence of Ricardian characteristics mitigates Hotelling's prediction on a rent rising at the rate of interest.[7] With increasing marginal costs, the resource rent will not increase exponentially along the optimal path of the competetive producer. The resource rent is growing at a slower rate because of the increasing costs of production as the best resources are depleted first and one has to take account of the reduction in future extraction costs as a result of storing the marginal unit of stock and hence keeping down accumulated production. From the first order condition we now have

$$(20)\quad \dot{\pi} - r\pi = -\frac{\partial c}{\partial A}$$

Depending on how $\frac{\partial c}{\partial A}$ changes over time, the resource rent can increase or decrease monotonically over time, remain constant or more generally change nonmonotonically over time.[8] The resource rent will in this case approach zero as time goes to infinity. The fact that the resource rent can be falling along an optimal path, means that we can have production after the price has reached the constant backstop price. The relevant terminal condition is now that production stops when the unit cost of extraction equals the backstop price minus taxes.

$$(21)\quad \bar{p} = c(\bar{A}) + v$$

The resource is therefore economically exhausted when the resource rent equals zero.

The effect of taxation in this model of optimal extraction can be derived in the same way as in the case with constant unit extraction costs. However, in this case it is more difficult to solve for the optimal price and production paths and it is hence more difficult to derive theoretical results on the effects of an increase in the carbon tax on the petroleum wealth. However, both the monopolist and the competitor will be more «conservationist», that is, prices will be initially higher, but grow less rapidly relative to the case of constant extraction costs, see Pindyck (1978).

Differentiation of the terminal condition in (21) w.r.t. the tax yields

$$(22)\quad \frac{d\bar{A}}{dv} = -\frac{1}{\frac{\partial c}{\partial A}} < 0$$

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[7] Lasserre (1991) contrasts the Hotelling and the Ricardian view of optimal resource extraction. To Hotelling (1931), exhaustibility and non-renewablilty are the important characteristics to emphasize. In the Ricardian view, the resource base is heterogeneous; supply sources differ in quality. Exhaustibility is not really an issue, although depletion may be manifest in a drop in quality of supply sources. Heterogeneity of reserves can be accounted for by the assumption that unit costs are increasing in accumulated production. In a modified Hotelling model with increasing unit costs and a given initial amount of the resource, some rents will be Ricardian rents, not scarcity rents in the sense of Hotelling. Here we assume increasing unit costs and an infinite amount of the resource initially, and strictly speaking all rents in the model presented here are Ricardian rents. In this paper we refer to the rent in the model with increasing costs as the resource rent.

[8] This generalised Hotelling rule appears to have been first derived by Kay and Mirrlees (1975) in a general model with fixed resource stock.

Since $C(A)$ is an increasing function of accumulated production, the total extracted amount must be less after taxes are imposed.

2.3 Technological change

We now modify the cost function in (18) and assume that the unit extraction costs are decreasing in time to reflect technological progress.

$$(23)\quad c(A,t) = \alpha e^{\eta A - \tau t}$$

τ is the rate of technological change in production of the resource. The unit cost changes over time according to

$$(24)\quad \dot{c} = \frac{\partial c}{\partial A}\dot{A} + \frac{\partial c}{\partial t} = [\eta x - \tau]c$$

With sufficiently low production, the unit extraction cost is falling over time, i.e. when $x<\tau/\eta$.

The optimisation problem of the producer is the same as in (1), only now we replace the unit costs with the cost function in (23). Existence of an optimal solution can be proved following Farzin (1992), and the first order conditions from Pontryagin's maximum principle are the same as in the previous optimisation problems.

To have a finite time when one switches to the backstop technology in this model, one must have a falling backstop price. In the case with a constant backstop price the extraction profile will not reach zero production in finite time. The reason is that if the unit cost reaches the backstop price and production stops, after some time with zero production, the technological progress will reduce the unit cost of fossil fuel production and hence make production possible with a positive scarcity rent. The solution in this case implies that the price path will remain constant equal to the backstop price after some time T^* and production will from then on follow a steady state extraction path given by $x^* = \frac{\tau}{\eta}$ where the unit costs are constant over time. A falling backstop price might be a more realistic assumption as the costs of producing the non-polluting energy substitute decrease over time because of technological progress. We therefore assume that the backstop price develops over time according to (25).

$$(25)\quad \bar{p} = \kappa e^{-\omega t}$$

With a falling backstop price the optimal steady state production is zero as long as the relative reduction in the backstop price is greater than the parameter for technological change in the cost function, that is if $\omega \geq \tau$.

The effects of carbon taxation are even less clear in the case with technlogical change. We can not solve the first order conditions to get explicit expressions for the optimal price and extraction paths of the representative producer. Instead we look at the implications of taxation that can be observed from the terminal condition.

In the case with a *constant backstop price* the steady state extraction path is independent of a constant carbon tax, although the rate of extraction until the steady state is reached will be affected.

With a *falling backstop price* the unit costs reaches the backstop price minus the tax at time $t=T$ and production ceases. T is defined by the equation

(26) $\kappa e^{-\omega T} - \nu = \alpha e^{-\tau T} e^{\eta A(T)}$

The effect of an increase in the carbon tax on the total extracted amount of the resource is not uniquely determined from this equation as was the case with no technological change. This is because two endogeneous variables; the time T when production switches to the backstop technology, and the accumulated production A, enter the expression in (26) and cannot both be determined by this single equation.

Thus, under more realistic assumptions about the cost functions where the unit costs are increasing in cumulative extraction and falling over time according to technological progress, it is difficult to derive unique theoretical results. This fact motivates the development of numerical models. In section 4.2 we report results from a numerical model where the cost function in (23) is applied.

3. Imperfect competition

So far the dynamics of the resource rent has been examined under competitive conditions. As in static theory the market structure in a dynamic environment is also important. Perfect competition is seldom a realistic assumption in markets for exhaustible resources. We will first look at the pure monopoly case. Then we will discuss a Nash-Cournot model with a cartel and a competitive fringe. In this section we will resume the assumption of constant unit costs of extraction to keep the models simple and to concentrate on the effects of imperfect competition.

3.1 Monopoly

«Natural monopoly» is of particular importance in the energy sector, and in some markets the extraction of fossil fuels can best be described as a monopoly. A monopolist that controls the entire stock of a nonrenewable natural resource will act to maximise the discounted net revenue flow from extraction. The monopolist will, however, take account of the fact that he is facing a falling demand schedule. The focus of an optimising monopolist is therefore on marginal revenue rather than on the price as in the competitive case. However compared to the static model, in an intertemporal model there is limited scope for the monopolist to exercise his monopoly power, see Stiglitz (1976). When a monopoly restricts supply in any given period, it raises the reserve stock it will hold during subsequent periods, which amounts to increasing its supply in those periods. In a sense the resource monopoly competes itself over time.

The monopolist must satisfy conditions for an intertemporal profit maximum that are very similar to the Hotelling rule and the terminal conditions that were established for the competitive firm. In the case with constant unit costs of extraction, the optimisation problem of the monopolist can be written

(27) $\max_{x} V = \int_{0}^{\infty} [p(x) - c - v]x(t)e^{-rt} dt$

where the producer is subject to the same restrictions as in the competitive case, see (2). The difference is now that the monopolist is facing a downward sloping demand curve, given by the demand function in (3), and will take account of this relation when choosing the optimal extraction path over the time horizon.

The current value Hamiltonian in the monoply case is

(28) $H = [p(x) - c - v]x(t) - \pi(t)x(t)$

With a convex demand function the requirements for the *Theorem 15* in Seierstad and Sydsæter (1978) to prove existence of an optimal solution will no longer necessarily be met. We can therefore no longer prove the existence of an optimal solution. Neither will the Hamiltonian be concave in (A, x), and the necessary first order conditions according to Pontryagin are not sufficient conditions for an optimal solution as in the case of perfect competition. Instead we assume the existence of an optimal solution. The necessary first order conditions are now

(29a) $\frac{\partial H}{\partial x} = \frac{\partial p}{\partial x} x + p - c - v - \pi = 0$

(29b) $\dot{\pi} - r\pi = \frac{\partial H}{\partial A} = 0$

(29a) can be rearranged to yield

(30) $MR - c - v - \pi = 0$ where $MR = p + \frac{dp}{dx} x = p[1 - \varepsilon]$

MR is defined as the marginal revenue of the monopolist. (ε is the inverse elasticity of demand in the demand function in (3).)

From the first order condition (29b) we see that the scarcity rent must increase at the rate of the interest. In the special case with zero taxation and *zero costs* of production, the marginal revenue increases at the rate of the interest rate.

(31) $\frac{\dot{MR}}{MR} = \frac{\dot{\pi}}{\pi} = r$

From (30) it follows that if $\frac{d\varepsilon}{dt}=0$, i.e., if the elasticity of demand is constant, the price rises at the same rate as the marginal revenue, i.e., at the rate of the interest. Therefore, with constant elasticity of demand and no extraction costs, if the monopolist and the competitive producer face the same demand schedule and have the same initial reserves, then the price and output are identical under monopoly and perfect competition. Both stop producing when the price of the resource reaches the backstop price at time *T*. Since the price and extraction paths will be identical, the petroleum wealth of the producer is also the same under monopoly and perfect competition. However, for the monopolist the total rent is split into a scarcity rent (π) and a monopoly rent (εp) which is due to the fact that the monopolist can restrict demand and influence the price. The competitive producer in the simple Hotelling model receives only scarcity rent.

The introduction of (constant) costs of production reinforces the general presumption that monopoly ownership of a resource stock results in excessive conservation relative to the competitive case. The initial price of the extracted resource is higher under monopoly than under a competitive extraction programme, and the relative price increases more slowly, see Dasgupta and Heal (1979).

The optimal price and extraction paths of the monopolist can be found by solving the first order conditions in the same way as under perfect competition. From (29a), (29b) and (3) we get

$$(32)\quad p(t)=\frac{1}{1-\varepsilon}\left[c+v+\pi(0)e^{rt}\right]$$

$$(33)\quad x(t)=\theta\left[c+v+\pi(0)e^{rt}\right]^{-\frac{1}{\varepsilon}} \quad \text{where} \quad \theta=\left[\frac{1}{1-\varepsilon}\right]^{-\frac{1}{\varepsilon}}$$

Due to the assumption of a backstop price the resource will be depleted in finite time, *T*. We then have the following two equations to determine *T* and π(0)

$$(34)\quad \int_0^T \theta\left[c+v+\pi(0)e^{rt}\right]^{-\frac{1}{\varepsilon}}dt=R$$

$$(35)\quad \bar{p}=\frac{1}{1-\varepsilon}\left[c+v+\pi(0)e^{rT}\right]$$

The resource wealth of the monopolist is defined as the discounted value of the rent times extraction at each point of time. The rent is now the sum of the scarcity rent and the monopoly rent.

$$(36)\quad V^*=\int_0^T\left[\pi(t)+\varepsilon p(t)\right]x(t)e^{-rt}dt$$

By rearranging (36) we arrive at the following expression

$$(37)\quad V^*=\frac{\pi(0)}{1-\varepsilon}R+\frac{\varepsilon}{1-\varepsilon}\left[c+v\right]\int_0^T x(t)e^{-rt}dt$$

The effect on the petroleum wealth of an increase in the carbon tax can be found in the same manner as earlier; by total differentiation of (34), (35) and (37) w.r.t. v. However in this case the mathematical expressions are more complicated. To simplify the expressions we derive the results in the case of a *linear demand* schedule.

$$(38)\quad p(t) = \overline{p} - bx(t)$$

This linear demand schedule implies that when the price reaches the backstop price at time T production ceases so that we have $x(T)=0$. When we substitute for the linear demand function in (38) into the optimimisation problem of the monopolist in (27), the resulting first order conditions can be written as follows.

$$(39a)\quad x(t) = \frac{1}{2b}\left[\overline{p} - c - v - \pi(t)\right]$$

$$(39b)\quad \pi(t) = \pi(0)e^{rt}$$

From (39a) and (38) we arrive at the optimal price path in the monopoly situation with a linear demand schedule.

$$(40)\quad p(t) = \frac{1}{2}\left[\overline{p} + c + v + \pi(0)e^{rt}\right]$$

To derive at the expression for the resource wealth in this case we substitute for the linear demand function in the definition of the resource wealth and use (39a) which can be written as $\overline{p} - bx(t) - c - v = \pi(t) + bx(t)$. The optimal resource wealth of the monopolist with linear demand is shown to be

$$(41)\quad V^* = \pi(0)R + b\int_0^T x(t)^2 e^{-rt} dt$$

The resource wealth of the monopolist consists of two components as does the total rent he receives from resource extraction. Thus, the first term on the right hand side of the equation is the discounted value of the scarcity rent, while the second term is the discounted value of the monopoly rent.

We calculate the effects of taxation on the petroleum wealth as before. The equations (34) and (35) with a linear demand are modified as follows.

$$(42)\quad \int_0^T \frac{1}{2b}\left[\overline{p} - c - v - \pi(0)e^{rt}\right]dt = R$$

$$(43)\quad p(T) = \overline{p} \quad \Leftrightarrow \quad \pi(0) = e^{-rt}\left[\overline{p} - c - v\right]$$

We differentiate (41), (42) and (43) w.r.t. v. The impact of an increase in the carbon tax on the petroleum wealth of a monoplist is then shown to be

(44) $\frac{dV^*}{dv} = -\int_0^T x(t)e^{-rt}dt$

We see that the petroleum wealth of the monopolist is reduced by an increase in the carbon tax as in the competitive case. And in the special case with a linear demand schedule, the resource wealth of the monopolist is reduced by the discounted value of the tax increase.

3.2 Nash-Cournot duopoly

In the previous section we discussed the case of pure monopoly. Although one may experience a monopoly situation in certain regional markets, it is not a realistic assumption for e.g. the world oil market. The oil market can more reasonably be modelled as an international market with a cartel, corresponding to OPEC, and a competitive fringe on the supply side. While the fringe always considers the oil price as given, the cartel regards the price as a function of its supply.

Salant (1976) was the first to model this situation as a dynamic Cournot duopoly between a cartelized group of identical firms or countries on one hand and a fringe of identical firms or countries on the other hand. All firms compete on a common market where demand chokes off when the price exceeds a maximum price which can be interpreted as a backstop price.

By a Nash-Cournot game we mean that both the fringe producers and the cartel take the behaviour of all other producers as given when deciding their own production profile. The competitive fringe takes the price as given and adjusts production accordingly; the cartelized sector sets the price and supplies whatever quantity is requested to meet demand, given the quantity supplied by the fringe. Both actors play simultaneously. Any situation where each sector takes as given the optimal choice of the other and where neither can, under that assumption, increase its profits by altering its own strategy, is called a Nash-Cournot equilibrium.

We will look at the open loop Nash-Cournot equilibrium concept. In an open loop equilibrium, (in contrast to a feedback equilibrium), the strategies are determined at the outset of the game when the players only have initial state information. The strategies only depend on time, not on observations about the state of the system. This implies that the period of commitment is equal to the entire planning period, i.e., that the cartel can commit itself to the price path chosen at the initial time of optimisation. Such commitment technology assumes that the institutional framework is one of perfect future markets with no recontracting, or equivalently one of binding contracts, see Ulph (1982). If this is not the case the only reason to suppose that players will not deviate from their announced open-loop strategies is that these strategies are dynamically consistent. Dynamic consistency means that players have no incentive to change their strategies along the optimal path if they could observe the current state of the system. It can be shown that open loop Nash equilibria are dynamically consistent. The problem of time inconsistency does not appear in a world of perfect competition, or monopoly, but can appear in duopoly/oligopoly models. The strategy of a follower must always be dynamically consistent since it takes the strategy of other players as given and is hence left solving a standard dynamic optimisation problem. In monopoly, of course, the monopolist is the sole player and is likewise facing a standard dynamic optimisation problem. In open loop Nash-Cournot equilibrium all players are followers and thus the strategies are dynamically consistent.

In a Stackelberg equilibrium the cartel recognises that the fringe's output path will be a function of the price path set by the cartel and takes this reaction into consideration when choosing its optimal price path. It may be argued that the Stackelberg model is more realistic in the oil market, i.e., that OPEC will in fact take into account the effect its actions have on the competitive fringe. However the open loop Stackelberg equlibrium is in many cases dynamically inconsistent. The appropriate solution concept then involves feedback Stackelberg strategies, which are dynamically consistent. However these strategies are hard to compute. Ulph (1982) conludes that since Nash strategies make naive behavioural assumptions, while for some parameter values the Stackelberg open-loop strategy will be dynamically inconsistent, the solution concept should be endogeneous to the model since one cannot establish a priori the superiority of one solution concept over another.

The open loop Nash equilibrium follows from solving jointly the first order optimality conditions for the dynamic optimisation problems of the two players. This results in time dependent strategies contingent on the initial conditions of the stock variables.

As we have seen above for the competitive producer and the monopolist, the maximisation of the present value of their profits, given their reserve constraints, imposes restrictions on the price trajectory over time. In the case with *zero costs* and no taxation the price must rise at the rate of discount under perfect competition, while under monopoly it is the marginal revenue that must rise at the rate of discount. (See equations (8) and (31).) While, in the previous discussion, these dynamic constraints were developed and presented independently under alternative market structure assumptions, they must now hold simultaneously, as long as both the cartel and the fringe are in production. For the cartel, marginal revenue is now defined on the residual demand instead of the entire demand schedule as under monopoly.

Some features of the equilibrium price path are known. With the existence of a backstop technology we have that $p(t) \leq \overline{p}$ for all t. Further, p(t) can never display a discontinuous increase at any point of time, as we assume the players have rational expectations. The price must not rise faster than the rate of discount, (in the absence of costs and taxes), even after the competitive fringe has exhausted its reserves. More precisely; if the fringe stops producing at time $\tilde{T}$, then we must have $p(t) \leq e^{r(t-\tilde{T})} p(\tilde{T}) \quad for \quad t \geq \tilde{T}$. However, this does not exclude that the price can rise faster than the rate of the interest in any interval $[t_1, t_2]$ where $t_1, t_2 > \tilde{T}$. The intuition behind this restriction is that the fringe, along the equilibrium path, must not be in a position to wish it had kept reserves instead. The presence of a competitive fringe thus imposes severe constraints on the monopoly power of the cartel. It is therefore in the interest of the cartel to set a low price initially so as to encourage rapid depletion of the fringe stock. With a low price initially the demand is high, and given that the price will rise no faster than the interest rate, the fringe has no incentive to keep reserves for later periods. But since the presence of the fringe restricts how fast the price can rise in the future, the cartel stands to lose profits in the short and medium run if its sets the initial price too low. The optimal initial price from the cartel's point of view is a balance between these two considerations, see Dasgupta and Heal (1979).

The equilibrium will consist, in general, of three phases; one in which only the fringe produces; one in which the cartel produces alone; and one where there is simultaneous production, see Ulph (1982). There are a number of cases that can occur, depending on the specific assumptions made about cost functions of the two groups of producers and about the demand schedule.

When the Nash-Cournot duopoly model of resource depletion was first analysed by Salant (1976), his focus was not on technological, but behavioural differences. Salant (1976) therefore assumed identical cost functions of the two groups of producers. In the simple case with zero marginal costs of extraction he shows that the sole possible pattern of equilibrium behaviour is one where the cartel and the fringe produce simultaneously in an initial phase, and the cartel produces alone in a final phase. No matter how small its supplies compared to those of the competitive industry as a whole, the cartel restricts its sales so as to take over the market after the fringe has exhausted its reserves. A sufficient condition for this result is that the consumer demand curve have a point of unit elasticity ($\underline{p}$) and that elasticity along the curve increase strictly with price. This condition is satisfied for all linear or concave demand curves and many convex demand curves.

If demand is isoleastic (and there are no costs of production), the cartel and the fringe will produce together over the entire period until their resources are exhausted at the time when the price reaches the backstop level.

In Salant's model the industry reserves last longer than under competitive exploitation, but exhaustion occurs earlier than under monopoly. The value of the firms in the cartel is raised by the exercise of market power, but not as much as under full monopoly. The formation of the cartel raises the discounted sum of profits, i.e., the resource wealth, of the competitive fringe at even greater percentage than that of the cartel, since the fringe enjoys the benefits from a higher price, due to the market power of the cartel, without having to restrict its supply.

Salant's model can be generalised to firms with rising, although identical, marginal costs. It retains the prediction that the cartel's market share increases over time until, in a final phase, it controls the totality of the market.

Ulph (1982) studies a similar model to Salant (1976), but adds the assumption that the fringe and the cartel may have different, but constant marginal costs of production. He shows that the conclusion in the Salant model, that the cartel always keeps some reserves to produce alone in a last phase, may not hold in the case when the costs of the cartel and the fringe differ. When the cartel has a substantial cost advantage over the fringe, i.e., the costs of production is much lower for the cartel than for the fringe producers, and initial cartel reserves are sufficiently abundant, the last phase to occur will be one where only the high cost fringe depletes. The cartel produces alone in an initial phase. When the price reaches a certain level, it becomes optimal for the fringe to enter the market, and in a second phase the cartel and the fringe produce simultaneously with the fringe increasing its market share. When the cartel exhausts its reserves, the fringe produces alone in a final phase.

If on the other hand the fringe has a cost advantage over the cartel and the fringe also has significant reserves, it is now the cartel that will produce alone in the last phase.

The particular equilibrium outcome will depend on the parameter values of the model, and Ulph (1982) presents the full set of solutions for Nash and Stackelberg equilibria in terms of the various cost conditions.

We have not been able to derive unique theoretical results on the effects of taxation on the petroleum wealth in the duopoly model. We will therefore in the Nash-Cournot case refer to some numerical results presented in the next section.

4. Numerical models-some results from recent studies

We have seen that even small modifications of the Hotelling model make it difficult to derive theoretical results about the impacts of taxation on the petroleum wealth. In this section we report some numerical results from two recent studies of optimal extraction of an exhaustible resource. First we briefly present the numerical results of Rosendahl (1996), then we look at the results from the more extended model of the global fossil fuel markets in Berg *et al.* (1996).

4.1 Rosendahl (1996)

Rosendahl (1996) presents one of the first numerical analyses on the impacts of CO_2 taxes on the petroleum wealth within an intertemporal energy model. He studies a simple, dynamic model of a competitive fossil fuel market, assuming constant unit costs of extraction and a limited initial amount of the resource. He concentrates on a single fossil fuel, oil, and studies the consequences for the oil wealth of an average oil producer and for Norway of three different international carbon taxes. According to his study, an international carbon tax of $10/barrel of oil may reduce the petroleum wealth of the average oil producer by 33-42 per cent. The Norwegian petroleum wealth may decrease even more than this, by 47-68 per cent, due to higher unit costs, i.e., lower initial resource rent.

So far we have only considered one market. Demand for oil in the models above will however depend on the price on the competing fuels like natural gas and coal. The price path of a fossil fuel is then not only determined by the extraction profile of the same resource, but is also depending on the extraction path of the substitute. To take account of this dependency, we should include all the fossil fuel markets in the model. [9] In such a model the imposed carbon tax can differ between the energy resources according to their carbon content. Coal will then receive a larger tax than oil, whose tax will again be higher than the tax on gas, which is the cleanest of the three fossil fuels.

4.2 Berg, Kverndokk and Rosendahl (1996)

This survey is motivated by the development of a dynamic multiregional model for the three fossil fuels oil, gas and coal, documented Berg *et al.* (1996). The study follows up the work by Rosendahl (1996) and it analyses the impacts on the petroleum wealth of fossil fuel producers of introducing an international carbon tax. The analysis is made within an intertemporal general equilibrium model for the global energy markets. While Rosendahl only studies one fossil fuel, Berg *et al.* (1996) model the markets for natural gas, oil and coal. In addition they extend the Rosendahl study by introducing market power and extraction costs as functions of accumulated production and technological change. In particular, they are interested in the impacts on the oil wealth of average oil producers of a global carbon tax, under different assumptions about the market power of OPEC. Here we will quite briefly present the results for the oil market only.

Berg *et al.* (1996) present an intertemporal global general equilibrium model for the fossil fuel markets. In contrast to simple Hotelling models, which often are characterised as unrealistic, the model includes several important aspects, such as cost functions increasing with cumulative production and decreasing with technological change, and market power in the oil market. No fixed quantity is assumed for the availability of the resources, however, the unit costs of extraction for oil and natural gas are increasing in accumulated production. The world is divided into three demand regions: OECD-Europe, Rest-OECD and Non-OECD. Berg *et al.* (1996) assume that gas is produced and traded within competitive markets corresponding to the demand regions. The international coal market is also modelled as a competitive market, but due to the huge reserves of coal, it is not modelled intertemporally.

Two different models corresponding to different assumptions about the market power of OPEC in the international oil market are studied. In the *cartel model* the oil market is modelled as an international market with a cartel (corresponding to OPEC) and a competitive fringe on the supply side. To determine optimal production under the cartel assumption, the producers are modelled as a Nash-Cournot duopoly. In the *competitive model* there is perfect competition in the oil market

[9] To our knowledge Manne and Rutherford (1994) present the first dynamic multiregional model of the optimal extraction of more than one fossil fuel. They employ a five-region general equilibrium model to examine three issues related to carbon emission restrictions. First, they investigate the possible impact of such limits upon future oil prices. Second, they analyse the problem of «leakage» which could arise if the OECD countries were to adopt unilateral limits upon carbon emissions. And third, they quantify some of the gains from trade in carbon emission rights. However, gains and losses in the Manne and Rutherford study do not refer to the effects on the petroleum wealth of oil and gas producers which are the main focus here. Rather, Manne and Rutherford (1994) calculate the effect on GDP and macroeconomic consumption over time.

where there are two kinds of firms; low cost producers (OPEC) and high cost producers (Non-OPEC).

The results indicate that OPEC behaviour is crucial when analysing the impacts on petroleum wealth of carbon tax. When OPEC acts as a *cartel*, the crude oil price is almost unchanged initially by a global carbon tax. Moreover, over the first 40 years the tax burden is born mainly by the consumers, as OPEC reduces its production in order to maintain a high price level. This implies that the oil wealth of the fringe is reduced by merely 8 per cent, whereas OPEC's wealth is reduced by 23 per cent. With *perfect competition* in the oil market, low cost producers (OPEC countries) increase their production significantly in the beginning of the time horizon, which lowers the oil price and makes oil production unprofitable in the first period for high cost producers (Non-OPEC). In this case, the oil wealth of Non-OPEC will be reduced significantly, with more than 70 per cent, compared to the reference case in the cartel model. In addition to this, a tax will have a much greater impact on the Non-OPEC oil wealth in the competitive model compared to the cartel model, as nobody will act to maintain the price under perfect competition. The Non-OPEC wealth will now be reduced by 39 per cent, while OPEC wealth is reduced by 25 per cent as a result of introducing a carbon tax on fossil fuels.

List of symbols

p	consumer price of the resource
$\bar{p}$	backstop price
v	carbon tax
V	resource wealth
c	unit cost of extraction
x	production of the exhasutible resource
A	accumulated production
r	discount rate
R	given initial amount of the resource
T	last period of production
π	resource rent
η	convexity parameter in the cost function
τ	rate of technological change in production of the exhaustible resource
ω	rate of technological change in production of the backstop technology
ε	the absolute value of the inverse elasticity of demand
κ	initial backstop price
MR	marginal revenue for the monopolist producer
α	initial unit costs
b	constant in the linear demand function

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ISSN 0805-9411

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