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The Use of Household Welfare Functions to Estimate Equivalence Scales

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Preface

The present paper is almost identical to my cand. polit. thesis at the Department of Economics, University of Oslo. The thesis was written while I was employed by the Division of microeconometrics, Statistics Norway. Director of Research Jørgen Aasness has been my advisor. He introduced me to the fascinating subject of equivalence scales, and has also been extremely helpful during the process of writing this thesis. Without the hours he has devoted to discussing the topic with me, this thesis would not have been what it is today. I am also grateful to Lone Bakken, Erland Pettersen, Tor-Egil Ruud, Dag Einar Sommervoll, Knut Reidar Wangen, and Luca Zamparini, who all gave extremely valuable comments.

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Chapter 1

Introduction

In practical policy making, it is frequently necessary to make welfare comparisons between households. Since households have different needs, such comparisons cannot be based on income solely. For instance, it is clear that a two-person household normally needs more resources than a person living alone. One approach is to consider income per capita. This is probably better, but also unsatisfactory since there are returns to scale for a lot of goods, such as heating, and because different household members have different needs. A common method to compare the level of material well-being of two households is to scale the incomes by *equivalence scales.* An equivalence scale may be defined as the ratio between the income of two different types of households known to be at the same level of welfare. Instead of income one may also consider total consumer expenditure. We are only going to consider static models, and consequently, income and total consumer expenditure will be used as synonymous terms in the theoretical models.

To give any meaningful discussion of equivalence scales, it is necessary to identify when two households or two individuals are at the same level of welfare. The concept of welfare is rather vague. In the present work, we shall restrict attention to *material well-being*, that is, the well-being obtained from consumption. It is obvious that this is an extremely narrow view of human welfare. Nevertheless, it has a couple of advantages. First, it is relatively easy to operationalize, which makes it useful for empirical purposes. Furthermore, material well-being is interesting for a number of policy-making issues, such as determining transfers and taxation. The two concepts welfare and well-being will be used interchangeably. In the empirical part, we shall define a *household* as a group of people sharing the same dwelling and sharing at least one meal a day (cf. Statistics Norway 1996). However, in the theoretical part, the term "household" may be given a wider interpretation. In fact, most of the theory could for instance be applied to municipalities as well as households.

The use of equivalence scales to perform inter-household comparisons of welfare can at least be traced back to the 19$^{\text{th}}$ century. At that time, most scales were based on supposed calorific needs. A huge number of equivalence scales based on needs have been constructed subsequently, and they are still widely used (see Nelson (1993) and van Praag and Warnaar (1997) for overviews). Another common approach is to estimate equivalence scales from observed economic behaviour, mainly demand. It is well known that from observed demand behaviour satisfying certain properties, it is possible to derive a utility function that rationalizes it. Nonetheless, this utility function is not appropriate for inter-household comparisons of welfare without further assumptions. A number of approaches, which all entail additional assumptions on the utility function, have been suggested. Some of these are reviewed in Chapter 2.

In most of this literature, the household is modelled as a unitary decision-maker, that is, as if it were a single agent maximizing a utility function depending on its composition. This is clearly a very simplified picture of household decision making. Furthermore, only individuals are able to enjoy consumption, so it is somewhat ambiguous what we shall mean by a household utility function.

The concepts introduced above will be useful throughout this thesis. The topic of the thesis is to study to what extent a more explicit modelling of intra-household behaviour may permit the identification and estimation of equivalence scales. Particularly, this is studied within the framework of a household behaving as if it maximized a household welfare function, i.e. a function that aggregates the individual utilities of every household member.

Chapter 3 explores this approach from a theoretical point of view. First, the definition of equivalence scales is extended to this model. It is then shown that within this context, it is possible to identify equivalence scales if we are willing to make some additional assumptions. Furthermore, this approach may give insight into assumptions that normally remain implicit in conventional approaches to the problem. Especially, the household utility function contains a mixture of individual utility functions and intra-household distribution effects. This reduced form is useful in a positive study of behaviour, but for constructing equivalence scales, the individual utilities are of main interest.

After this theoretical discussion of equivalence scales, we make an attempt at estimating equivalence scales from Norwegian budget data. Chapter 4 shows how it is possible to derive a Linear Expenditure System from a model of a household maximizing a welfare function. Ordinary and generalized least squares are then applied to estimate this system. A problem with the LES is that when considering a single cross section with constant prices, some of the parameters are not identifiable from demand data alone. Particularly, some parameters which may be interpreted as necessary consumption are not identified. This problem is solved by considering particular groups of goods for which it is natural to assume that adults or children do not have any necessary consumption, such as babies' nappies and cigarettes. Chapter 5 then proceeds to discuss problems with these estimates, particularly problems of measurement error, outliers and omitted variables. A test of the identifying assumption is also presented.

Finally, Chapter 6 shows how it is possible to use the estimates from the preceding chapters to estimate equivalence scales. Although we make strong assumptions that are not testable from demand data, it is shown that we can obtain estimates belonging to some well-known classes of equivalence scales.

The appendix contains some proofs and additional estimation results, a complete classification of the grouping of goods, a list of notational conventions as well as lists of symbols and abbreviations.

Chapter 2

Previous contributions to the equivalence scale literature

2.1 Definitions and notation

A common assumption in consumer econometrics[1] is that the household behaves as if it maximizes a utility function

$$U : (\mathcal{Q}, \mathcal{Z}) \rightarrow \mathbb{R} \tag{2.1}$$

where $q \in \mathcal{Q}$ is a vector of quantities of goods and services chosen from the consumption set $\mathcal{Q} \subseteq \mathbb{R}^J$ and $z \in \mathcal{Z}$ is a vector of household characteristics. In the present work, we shall only consider demographic composition, defined as the number of agents belonging to each of K different demographic groups. Then z is a vector giving the number of household members in each group $1, \dots, K$, and $\mathcal{Z} \subseteq \mathbb{N}^K$ is the set of possible demographic compositions. It is outside the scope of this paper to discuss the general conditions for existence of a household utility function or a set of household indifference curves (see Samuelson (1956) for an early contribution). Furthermore, Chapter 3 shows that under relatively weak conditions, a household utility function may be interpreted as a reduced form of a household maximizing a Bergson-Samuelson welfare function. It will be assumed that every households have preferences and when necessary, utility, which may be described by the function U.

For a set of prices $p \in \mathbb{R}^J_+$, we denote the cost function associated with the utility function U by

$$C(p, \mathcal{U}, z) = \min_{q \in \mathcal{Q}} \{ p'q \,| U(q, z) \geq \mathcal{U} \}, \tag{2.2}$$

that is, the amount of money necessary for reaching utility level $\mathcal{U}$ given prices p. An equivalence scale may formally be defined as

$$L(p, \mathcal{U}, z, z_0) = \frac{C(p, \mathcal{U}, z)}{C(p, \mathcal{U}, z_0)} \tag{2.3}$$

where z_0 is the composition of the reference household, for instance a single adult. That is, $L(p, \mathcal{U}, z, z_0)$ is the ratio between the income required for a household with composition z to that of a household with composition z_0 required to attain utility level $\mathcal{U}$ given prices p.

2.2 Interpersonal comparisons of welfare

For relation (2.3) to make any sense, it is clear that we need to make utility comparisons between different households, that is, it has to be possible to tell whether two different households have

[1] Such as Deaton and Muellbauer (1980) or Pollak and Wales (1981).

the same material standard of living. It is useful to consider a classification of interpersonal comparability similar to the one developed by Roberts (1980) and Sen (1977). Consider H households, and index a typical household by h, which has a utility function which may be represented by U^h. Denote by ϕ^h a transformation of U^h and by $\vec{\phi}$ the H-vector of ϕ^h's. Then $\vec{\phi} \in \mathcal{T}$, where $\mathcal{T}$ is the H-dimensional functional space of transformations of utility functions. Denote by $\tau \subseteq \mathcal{T}$ the set of *invariant transformation* of the U^hs. That is, for every $\vec{\phi} \in \tau$ and every h, U^h and $\phi^h \circ U^h$ contains the same information on the state of household h.

Ordinal non-comparability (ONC) τ is simply the set of lists of independent, monotonically increasing, transformations. This is the assumption in standard consumer theory.

Cardinal non-comparability (CNC) τ is the set of lists of individual affine transformations, that is, each ϕ^h is an affine transformation, but it is not necessarily the same transformation for different households. This is the usual assumption for von Neuman-Morgenstern utility functions.

Ordinal level comparability (OLC) τ is the set of lists of *identical*, strictly monotonic, transformations. That is, every element of $\vec{\phi} \in \tau$ may depend on z^h, but not on h itself.

Cardinal full comparability (CFC) τ is the set of lists of identical, strictly positive, affine transformations.

Cardinal ratio-scales (CRS) τ is the set of lists of identical, strictly increasing, linear transformations, i.e. for all $\vec{\phi} \in \tau$, there is a $v > 0$ such that $\vec{\phi}$ is the vector of identical elements $x \to vx$. This assumption makes it possible to say that one household is twice as well off as another.

Complete cardinal comparability (CCC) [2] $\tau = \{(\mathrm{Id}, \dots, \mathrm{Id})\}$ where Id is the identity-operator, that is, no transformation at all is allowed on any U^h.

The levels may be classified as

$$\begin{aligned} \text{ONC} &\supset \text{CNC} \supset \text{CFC} \supset \text{CRS} \supset \text{CCC} \\ \text{ONC} &\supset \text{OLC} \supset \text{CFC} \supset \text{CRS} \supset \text{CCC} \end{aligned}$$

where $x \supset y$ means that y is a stronger concept than x, that is, y implies x. For equivalence scales to make any sense, we require that preferences are at least OLC (for further details, see Blackorby and Donaldson (1991)). Throughout the present work, we shall assume that household utility functions satisfy OLC.

A fundamental difficulty is what should be understood by the term "utility". In standard economic theory, utility refers to what could be labeled *preference utility.* This is conceptually different from *experience utility*, the hedonic quality of experiences such as pleasure (Kahneman and Varey 1991, 128). Another way of putting it is that two agents may have identical sets of indifference curves, but if they are at the same indifference curve, they don't necessarily get the same satisfaction from it (Fisher 1987). Although material well-being is most closely related to experience utility, only preference utility is (partially) recoverable from observed consumer behaviour. For further discussion, see Kahneman and Varey (1991). Furthermore, as argued by Scanlon (1991), interpersonal comparisons will inevitably involve some degree of value judgement. Nonetheless, for practical policy questions, interpersonal comparisons of well-being is necessary, and to quote Pollak (1991, 39), "the most convincing argument that [interpersonal] comparisons are possible is the frequency with which we make them".

[2] This concept is not considered by Roberts (1980) or Sen (1977).

2.3 Problems of identification of equivalence scales

A crucial question is how z is determined, that is, where children do come from? Looking at much of the economic literature, a popular answer seems to be that "storks bring them" (Deaton and Muellbauer 1980, 208). In real life, the parents take a more or less rational choice to get children, and in most developed societies it is relatively easy to avoid getting unwanted children, although the decision is normally irreversible. Hence if a couple choose to get x children given an income y, we can conclude that this particular couple prefers the combination of x children and income y to any other number of children and income y. This is the problem of conditional versus unconditional comparisons introduced by Pollak and Wales (1979). In the traditional estimation of equivalence scales, we consider preference relations of the type $R(z)$ where $qR(z)\,q'$ means that a household with composition z prefers the consumption bundle q to q', that is, $R(z)$ is conditional on the household composition z. An unconditional preference ordering is a preference relation R, such that $(q,z)\,R\,(q',z')$ means that a consumption bundle q and a household composition z is preferred to a consumption q' and a composition z'. Looking at expenditure data, only conditional preferences are recoverable. In a world where children are "brought by storks" the concept of (conditional) equivalence scales seems relatively clear. Under unconditional preference orderings, on the other hand, it is not clear what equivalence scales should mean, although they might be replaced by generalized cost of living indexes (Pollak 1991). Another difficulty is that unconditional preferences treats children almost as a consumer good, and it is easy to ignore the utility of the children (Bojer and Nelson 1999).

There are approaches that consider the joint decision of consumption and family composition (Ferreira et al. 1998), but in the present work we are only going to consider short run behaviour where z is fixed and assumed to be exogenous. This is mainly because it permits us to give a clear *definition* of equivalence scales. Whether we are actually able to identify such scales is discussed below and in Chapter 3.

A well-known property from demand theory is that demand is unchanged by a monotonic transformation of the utility function (Mas-Colell et al. 1995). Consequently, using expenditure data, we are not able to distinguish between a household maximizing U and V where $U = F(z) \circ V$ where F is monotonically increasing in utility. On the other hand, for utility levels $\mathcal{V}$ and $\mathcal{U} = F(\mathcal{V}, z)$,

$$L(p,\mathcal{U},z,z_0) = \frac{C(p,\mathcal{U},z)}{C(p,\mathcal{U},z_0)} \neq \frac{C(p,F(\mathcal{V},z),z)}{C(p,F(\mathcal{V},z_0),z_0)}$$

since generally $F(\mathcal{V},z) \neq F(\mathcal{V},z_0)$ as long as F depends on z. That is, equivalence scales are not unique to a monotonic transformation of the utility function unless the transformation is independent of the demographic composition of the household.

Actually, the following proposition, quite close to the lemma in Blundell and Lewbel (1991, 52), holds.

Proposition 1 *Let $g : \mathbb{R}\times\mathcal{Z}^2 \to \mathbb{R}_+$ be a function with $g(\mathcal{U},z,z_0) > 0$ and $g(\mathcal{U},z_0,z_0) \equiv 1$ for all $(z,z_0) \in \mathcal{Z}^2$, and such that there is a function ϱ such that $\mathcal{U} \to g(\mathcal{U},z,z_0)\,\varrho(\mathcal{U})$ is increasing in $\mathcal{U}$ for all (z,z_0). Then for any demand function D that can be obtained from the maximization of a utility function, and for any price regime $p^0 \gg 0$, there is a unique cost function C such that the Marshallian demands arising from C are D and*

$$\frac{C(p^0,\mathcal{U},z)}{C(p^0,\mathcal{U},z^0)} = g(\mathcal{U},z,z_0).$$

Proof. See Appendix A.1

This proposition says that for almost any equivalence scale g and any demand system[3] D,

[3]Since we want the set of preference relations to generate D, we have to require that D is obtainable from the maximization of a utility function. This holds if D satisfies adding up, homogeneity of degree zero and Slutsky-symmetry (Mas-Colell et al. 1995, Ch. 3.H).

there is a utility function such that the demand system obtained from utility maximization is D, and the equivalence scale arising from these preferences is g.

Remark 1 *If g is monotonically increasing or decreasing in $\mathcal{U}$, we can choose $\varrho(\mathcal{U}) \equiv 1$ or $\varrho(\mathcal{U}) \equiv -1$ respectively to satisfy the conditions of the theorem.*

Remark 2 *If g is differentiable, and there exists an integrable function h such that $g_{\mathcal{U}}(\mathcal{U}, z, z_0) / g(\mathcal{U}, z, z_0) > h(\mathcal{U})$ for all $(\mathcal{U}, z, z_0)$, then we can choose $\varrho(\mathcal{U}) = e^{-\int h(\mathcal{U}) d\mathcal{U}}$. It follows that $\frac{\partial}{\partial \mathcal{U}} g(\mathcal{U}, z, z_0) \varrho(\mathcal{U}) = \left(\frac{g_{\mathcal{U}}}{g} - h(\mathcal{U})\right) g e^{-\int h(\mathcal{U}) d\mathcal{U}} > 0$.*

It might seem that this result implies that it is impossible to identify equivalence scales. Nevertheless, as we shall see below, identification is possible if we impose further restrictions on utility than the demand function D.

2.4 Traditional approaches

One widely used method of estimating equivalence scales is the so-called Engel's method[4]. In his study of Belgian budget data, Engel (1857) found a strong negative relationship between the budget share of food and income or standard of living, and concludes that

> *das Mass der Ausgaben für die Ernährung unter übrigens gleichen Umständen ein untrügliches Mass des materiellen Befinden einer Bevölkerung überhaupt ist* (Engel 1857, 29).

Engel's claim is that, ceteris paribus, the budget share for food is the best indicator for welfare. The ceteris paribus assumption has later on been relaxed, and the approach is based on the assumption that households irrespective of demographic composition are on the same level of welfare if their budget share for food is the same. We can then construct equivalence scales from the ratio between the incomes of households with different demographic composition and the same budget share for food[5].

We should probably look at averages of households belonging to a certain group for the approach to make sense. It is useful to decompose the identifying assumption into two separate assumptions: (a) For households with an equal demographic composition, the lower the budget share for food is, the better off is the household, and (b) if two households of different demographic composition have the same budget share for food, they are both equally well off (Deaton and Muellbauer 1986). Assumption (a) follows from Engel's law, which states that the budget share for food is decreasing in income, and which has found wide empirical support (Houthakker 1987). Consequently, this assumption should not be too controversial. Assumption (b), on the other hand, is more difficult. Nicholson (1976) mentions a counter-example: Babies will normally have a higher share of food in their consumption than adults. Consider now a couple that gets a child. If the couple is completely compensated, say by a child benefit, they are at the same level of well-being as before. However, because the baby has a high consumption of food, the household's budget share for food has increased. Furthermore, households with

[4] We shall distinguish between Engel's method and Engel's model. The latter is a special case of the Barten (1964) model where the scaling factor is the same for every good. That is, there is a function $m : \mathcal{Z} \to \mathbb{R}_+$ such that the household utility may be written as

$$U(q, z) = \bar{U}\left(\frac{1}{m(z)} q\right) \quad \text{for all} \quad (q, z) \in \mathcal{Q} \times \mathcal{Z}$$

for some utility function $\bar{U}$.

[5] Although this procedure is called "Engel's method", Engel did probably not use it himself. On the other hand, he used scales based on supposed calorific needs in his study of the cost of living (Engel 1895). There seems to be some confusion about this in parts of the equivalence scale literature.

children will probably spend more time at home rather than having meals at restaurants. If "food" is defined as food at home, this will also lead to a higher budget share for food although the welfare may be unaffected (Brekke and Aaberge 1999). Consequently, it may be argued that Engel scales have a tendency to overestimate equivalence scales. It should be pointed out that it is not necessarily true that young children have a budget share for food that is higher than that of adults; it may even be argued that the opposite is true. Still, if a new child in the house has a different budget share for food compared to the other household members, this will bias the estimates of equivalence scales. The direction of the bias is, on the other hand, unresolved. Nevertheless, it is possible to construct sets of preferences such that Engel's method gives a correct measure of welfare. Browning (1992) provides the general class of household expenditure functions satisfying this. As an example, consider Deaton and Muellbauer's (1980, Ch. 8.1) expenditure function, which takes the form

$$c\left(p,\mathcal{U},z\right)=m\left(z\right)c_{0}\left(p,\mathcal{U}\right). \tag{2.4}$$

This is the expenditure function arising from Engel's model, and is a special case of the IB structure (see Section 2.5), so the equivalence scales are identified. Particularly,

$$L^{E}\left(p,\mathcal{U},z,z_{0}\right)=\frac{m\left(z\right)}{m\left(z_{0}\right)}, \tag{2.5}$$

so the equivalence scales are constant across price- and utility-levels. Furthermore, the budget share of good i,

$$w_{i}=\frac{\partial\ln C\left(p,\mathcal{U},z\right)}{\partial\ln p_{i}}=\frac{\partial\ln C_{0}\left(p,\mathcal{U}\right)}{\partial\ln p_{i}}, \tag{2.6}$$

is independent of the demographic composition for a given utility level. Consequently, as long as w_i is decreasing in $\mathcal{U}$ (Engel's law), the budget share for food is an appropriate indicator for household welfare. Engel insisted on using the budget share for food as a measure of welfare. Expenditures on food will generally include some luxury goods, though. As a solution to this problem, it has been suggested to use the budget share for *necessities* rather than the budget share for food as a measure of welfare. The term *iso-prop* is normally used for this approach, whereas the term Engel's method is reserved for the use of the budget share for food. The problems mentioned above may also apply to the iso-prop procedure. Despite the criticisms presented above, Engel's method and iso-prop are widely used for estimating equivalence scales (Deaton and Muellbauer 1986; Gozalo 1997; Lancaster and Ray 1998; Livada et al. 1996; Murthi 1994; Røed Larsen and Aasness 1996).

Another traditional technique which still has wide popularity is the method originally suggested by Rothbarth (1943). He states that

> *How much additional income does a family of given size require to compensate it for the cost of upkeep of an additional child? We should expect the answer to depend on the standard of living of the parents, for there will be a broad correspondence between the standard of living attained by the parents and the standard of living of the child. The technique under consideration consists in taking 'excess income' as a criterion for the standard of living of the parents, 'excess income' being the residual after provision has been made for expenditure on rent, rates, state insurance, travel, income tax, food, fuel and clothing. The families are, provisionally, taken to be equally well off, if their excess income is equal* (Rothbarth 1943, 123).

The quote above tends to indicate that excess expenditure is expenditure on luxuries in general. When the "Rothbarth method" is used today, it is commonplace to identify excess expenditure with expenditure on "adult goods", that is, goods that are only consumed by adults (Nelson 1993, 478).

For this procedure to be correct, Gronau (1988) shows that the household utility function has to be separable in the children's and the adults' utilities. Assume that the parents and the children have utility functions U^A and $U^B : \mathcal{Q} \to \mathbb{R}$. Then the household utility function is separable if there exists a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$U(q) = f\left[U^A\left(q^A\right), U^B\left(q^B\right)\right],$$

where there is some allocation rule such that $q^A + q^B = q$ for all q. Browning (1992) derives conditions on the expenditure function for this procedure to be correct. In the terminology of Nelson (1992), we have to assume that preferences are *stable*, that is, U^A does not depend on the presence and number of children, and *separable*, that is, the presence of children has only income effects on the parents consumption.

If there is a normal good i that is only consumed by parents, households spending the same amount y_i on good i will have the same household utility. Hence if we observe a reference household with total expenditure y_0 and expenditure y_i on good i, and a household with composition z also spending y_i on good i, but with a total expenditure y, we know that the equivalence scale is

$$L^R(z, z_0) = \frac{y}{y_0}.$$

Typical adult goods include tobacco and alcohol. Cramer (cited in Deaton and Muellbauer 1986) claims that these goods are not well chosen since they have generally low Engel elasticities, so rich and poor households do not differ much in their consumption of these goods. On the other hand, Røed Larsen et al. (1997) finds that these elasticities are relatively high in a study of Norwegian data.

It is clear that L^R will depend on y_0 or the level of utility of the reference household, so it is not necessarily independent of base (cf. Section 2.5). For details on how it is possible to carry out an estimation of Rothbarth scales, see Gronau (1991).

It should also be noted that if we use tobacco and alcohol as the adult goods, this technique implies that household welfare is proportional to the consumption of these goods, which might seem paradoxical (Browning 1992, 1443).

2.5 Demand system approaches and independence of a base level of utility

One method that permits identification of equivalence scales and which also has the nice property that equivalence scales are mere numbers, and not functions, is the *independent of base* (IB) or *equivalence scales exactness* (ESE) concept introduced independently by Blackorby and Donaldson (1993) and Lewbel (1989)[6].

Definition 2 *An equivalence scale is* ***independent of base*** *if* $L(p, \mathcal{U}, z, z_0) = L^{IB}(p, z, z_0)$ *for all* $(p, \mathcal{U}, z, z_0) \in \mathbb{R}^N_+ \times \mathbb{R} \times \mathcal{Z}^2$.

To get equivalence scales that are IB, we have to make quite strong assumptions on the cost function (2.2).

Proposition 3 *A cost function C satisfies IB if and only if it can be written as*

$$C(p, \mathcal{U}, z) = m(p, z)\, C^0(p, \mathcal{U}) \tag{2.7}$$

for all utility levels $\mathcal{U}$ and prices p. Without loss of generality, we can choose m to be homogenous of degree zero in prices.

[6] Both contributions have working paper-versions from 1988.

Proof. See Lewbel (1989, 380f).

From (2.3) it follows immediately that

$$L^{IB}(p,z,z_0)=\frac{m(p,z)}{m(p,z_0)}. \tag{2.8}$$

A number of popular demand systems, such as AIDS and Translog, has (2.7) as special cases. Estimation of (2.8) then follows from standard techniques (Blundell and Lewbel 1991; Dickens et al. 1993; Lancaster and Ray 1998; Ray 1996).

The reason why the result in Proposition 1 does not apply, is that we have assumption (2.7) in addition to the demand system, that is,. we have more restrictions on utility than observed market behaviour.

The IB assumption is extremely convenient for estimation purposes, but it has a couple of disadvantages. Most important is probably that it is difficult to confirm it empirically. Given a demand system, we cannot verify whether the true cost function is of the form (2.7) or not, although we can empirically reject the given functional form (Blundell and Lewbel 1991, 50). To see this, assume that the utility function U satisfies IB and generates a demand function D. Then both U and $g(z)\circ U$, where $g(z)$ is a monotonically increasing transformation of U, will generate D, but the latter does not necessarily satisfy IB, so the equivalence scales associated with this preference structure is different from those obtained through the IB-assumption. Consequently, if we only observe D, it is not possible to conclude that preferences satisfy IB. On the other hand, it is not possible to obtain every possible demand structure from an IB utility function, so if we observe a demand behaviour that is incompatible with IB, we can reject it. To the best of my knowledge nobody has found restrictions on the *utility function* that are necessary or sufficient to generate IB, nor given any microfoundations for the concept[7]. Consequently, the intuition behind (2.7) remains relatively obscure.

A number of studies have found that IB is incompatible with observed demand behaviour (Blundell and Lewbel 1991; Lancaster and Ray 1998; Ray 1996). Pendakur (1999) argues that this is due to too strong restrictions on the functional form of the Engel curves. Using non-parametric Engel curves and parametric demographic effects, he cannot reject the assumption of independent of base cost functions. Still, this does not give direct support in favour of IB as argued above. There are few studies of the theoretical plausibility of the IB assumption. One is Lewbel (1991). He mainly discusses demand systems based on Barten-scales (Barten 1964), that is, systems based on a household utility function of the form

$$U(q,z)=U^0\left(\frac{q_1}{m_1(z)},\dots,\frac{q_J}{m_J(z)}\right) \tag{2.9}$$

for some functions $m_1,\dots,m_J$. For a utility function to satisfy both Barten scaling and IB, preferences are either homothetic, or the ms satisfy $B'm(z)=0$ for some $S\times J$ matrix B where $S\geq 1$. Although he shows that there are versions of AIDS and Translog that satisfies this, the restriction is rather strong.

Brekke and Aaberge (1999) shows that IB implies Hicks demand functions of the form

$$\tilde{q}(p,\mathcal{U},z)=m(p,z)\nabla_p C^0(p,\mathcal{U})+C^0(p,\mathcal{U})\nabla_p m(p,z). \tag{2.10}$$

Substituting from the expenditure function, the vector of budget shares becomes

$$w(p,y,z)=\omega(p,z)+w\left(p,\frac{y}{L^{IB}(p,z,z_0)},z_0\right), \tag{2.11}$$

that is, up to a constant ω that doesn't depend on income, the budget shares are equal for equivalent income across households. Consequently, IB assumes that at the same welfare level,

[7] Microfoundations here means a model of intra-household behaviour that generates IB, cf. Chapter 3.

households have the same budget share for different goods up to a constant. Hence it is closely related to the Engel approach, and most of the criticism raised against Engel's method applies here as well.

Another criticism against IB is that it is plausible that equivalence scales vary across utility levels. For instance, if all individuals require a certain minimum consumption whereas demographic effects are weaker on the consumption of luxuries, the scales may be decreasing. This corresponds to the findings in van Praag and van der Sar (1988). Conniffe (1992) also argues that equivalence scales vary for different utility levels from a theoretical point of view.

2.6 The Leyden approach

Instead of imposing further restrictions on utility- or cost-functions, it is possible to identify equivalence scales if we have more information than demand behaviour. This is the approach taken by Bernard van Praag and his followers (see e.g. van Praag and van der Sar 1988). These contributions, mainly done at the Leyden University, from which the name "Leyden approach" springs, are based on the so-called *income evaluation question* (IEQ). A sample of respondents are asked what they consider a bad income, a good income and so forth. They go on to assume that each of these states correspond to a utility level $\mathcal{U}_k$ which is identical for all respondents. This is then used to construct an expenditure function. From this, the derivation of equivalence scales is immediate from (2.3).

A crucial assumption for these scales to be correct, is that each respondent identifies the same subjective level of utility to the same question. This is far from obvious, but it is probably difficult to test. One reason is that it is unclear what we should mean by "same level of utility" as discussed in Section 2.2. Furthermore, it is uncertain whether a respondent is able to give a meaningful answer to such a question. They are probably able to tell whether their income is good or not, but if it is not good, it might be difficult to tell how much more they need to get a "good" income.

Furthermore, it is clear that persons with different incomes will give different answers to what they consider for instance a good income. In the construction of the cost function, this is normally taken into account. This may however give rise to inconsistencies as to the cost of children. Assume that large families are systematically worse off than persons living alone. It may then be the case that the larger families give a lower answer to the IEQ than the singles when scaling by the "true" equivalence scale. In this case it may be difficult to separate the effect of income and number of children, and may to some extent explain why equivalence scales estimated using the Leyden-methodology generally give lower estimates on the cost of children (see e.g. Buhmann et al. (1988) for a comparison).

The approach of using more information than revealed market behaviour is probably a good idea. Nonetheless, to profit fully from this, we need to assure good quality of the new data in the sense that it measures what we intend it to measure. It is far from obvious that the IEQ satisfies this.

Chapter 3

The Bergson-Samuelson welfare function and equivalence scales

The approaches in Chapter 2 were based on the existence of a household utility function. In this chapter, we are going to give a more explicit model of intra-household behaviour using Bergson-Samuelson welfare function. We shall see how this may be seen as a foundation for the household utility function. We will also extend the definition of equivalence scales to this setting and discuss to what extent this approach may give additional insight into the problem of estimating equivalence scales

3.1 The study of intra-household behaviour

In the preceding section, it was assumed that the household maximizes a "household utility function" depending on total household consumption. The notion of household utility function is quite unclear since households often consist of more than one individual. To get a clearer view of this concept, it is necessary to focus on what is actually going on inside a household.

In some societies, it might be a good approximation to reality to model a household as if it maximizes a particular agent's utility function (normally the husband's). This means that the household utility function is similar to this agent's utility function. There are at least two ways of interpreting intra-household distribution in this case. First, it might be that the head of household gives the other household members enough consumption goods to obtain a certain required utility level, and then spends the remaining resources on himself. This may be due to some limited degree of altruism or a set of social norms. Although normally rather unfair, this mechanism is Pareto optimal. Another interpretation is the one found in Bojer (1977). She assumes that some sort of social norm dictates that for each household member i and every good j, there is a number m_{ij} such that agent i gets a share m_{ij} of the household consumption of good j. Given this constraint, the head of household maximizes his or her utility. This distribution mechanism is generally inefficient. Even if everybody have identical preferences, the allocation may be inefficient if preferences are non-homothetic.

Another approach is to go ahead and model the whole intra-household decision process. This is a typical example where game theory is required, and both cooperative and non-cooperative approaches have been suggested (see Lundberg and Pollak (1996) for a survey of some of these works). The most successful approach is probably the one of Manser and Brown (1980) and McElroy and Horney (1981)[1]. They use different cooperative bargaining solutions, such as the Nash bargaining solution, to determine household decisions. There are some difficulties associated with bargaining models though. First, if there are more than two agents in the household, the core might be empty. Then there is no stable solution to the cooperative game

[1]See also Chiappori (1988), McElroy and Horney (1990), and Chiappori (1991) for a discussion of these contributions.

(Mas-Colell et al. 1995, Appendix 18.A). Furthermore, it is rather difficult to model the influence of children in such models. To assume that children participate in bargaining is probably a bit far fetched. Then we have to include the children's consumption in the parent's utility functions, which is probably not satisfactory either. On the other hand, bargaining models introduce a very useful concept, namely bargaining power. In the present work, we will not use bargaining models in the modelling process, but informal allusions to this class of models will be done when necessary.

An extension to the bargaining models is the general model of Bourguignon, Browning, Chiappori, and Lechene (Browning et al. 1994; Browning and Chiappori 1998). They base their work on the assumption that the intra-household distribution is efficient. From this assumption, they are able to draw a number of interesting conclusions. In the present context, both this approach and the bargaining approach is inappropriate because it is difficult to compare the welfare level of different households which is necessary for the construction of equivalence scales. Consequently, we are going to use a somewhat more stylized model. Still, we will try to go beyond the unitary model of the household behaving as if it maximized a single utility function.

3.2 The Bergson Samuelson welfare function

In the present work, we shall model the household as if it maximizes a *Bergson-Samuelson welfare function* (BSWF) (Bergson 1938; Samuelson 1947). This approach to household decisions can at least be traced back to Samuelson (1956). He notes that a "social utility function" does not exist except for some particular cases. Since most studies of demand have households as their units of study, this is problematic. He goes on to suggest the use of a BSWF to aggregate household preferences. Assume that a given household consists of N individuals indexed by i, and that each individual has a personal utility function $u^i : \mathcal{Q} \to \mathbb{R}$. Then a BSWF is a function $W : \mathbb{R}^N \to \mathbb{R}$ that maps the utility of each individual into a composite measure of welfare. For BSWFs to make sense, we will normally have to assume that individual utilities are CCC[2]. There are different ways of interpreting this household decision procedure. One might see this as some sort of bargaining procedure where everybody gets a share. For instance, if the outside opportunity is naught, a Cobb-Douglas BSWF might be seen as a generalized Nash product, so the solution in this case corresponds to the solution to a Nash bargaining problem. Another interpretation is that the u^i's are the agents individualistic preferences, and that W is the utility function of an altruistic head of household. This corresponds to the separable utility function required for the estimation of Rothbarth scales (Gronau 1991).

Before discussing equivalence scales in the present setting, it is necessary to introduce a few new concepts. We are only going to consider welfare functions that satisfy the Paretian property:

Definition 4 *A BSWF is said to satisfy the* ***Paretian property*** *(PP) if, for any two vectors of utility levels* u *and* $\mathrm{u}^\dagger$, $\mathrm{u} \geq \mathrm{u}^\dagger$ *implies* $W(\mathrm{u}) \geq W(\mathrm{u}^\dagger)$. *Furthermore, a BSWF satisfies the* ***strict Paretian property*** *(SPP) if* $W(\mathrm{u}) > W(\mathrm{u}^\dagger)$ *whenever* $\mathrm{u} \geq \mathrm{u}^\dagger$ *and there is at least one* $\mathrm{u}_i > \mathrm{u}_i^\dagger$.

We will sometimes need SPP, but this is too strong for a number of useful welfare functions, such as the Rawlsian welfare function (Rawls 1971). In general, a BSWF may give different weight to different agents. A particular case is when all agents are given the same weight, that is, that the identity of the agent is irrelevant for her weight in household welfare.

Definition 5 *A BSWF satisfies* ***anonymity*** *(AN) if for any vector of utility levels* u *and any permutation of* u, $\mathrm{u}^\dagger$, *we have* $W(\mathrm{u}) = W(\mathrm{u}^\dagger)$.

[2] It is possible to impose restrictions on W such that individual utilities only have to satisfy CFC (Mas-Colell et al. 1995, Ch. 22.D), but we are not going to follow that approach.

We shall generally assume that when AN holds, two agents with identical utility functions receive the same consumption bundle. A sufficient (but not necessary) condition is that every individual utility function and the BSWF are concave.

Finally, it will sometimes be necessary to normalize the value of the welfare function. One convenient concept which leads to a normalization is the concept of *agreement* of Aczél and Roberts (1989).

Definition 6 *A BSWF is said to satisfy* ***agreement*** *(AG) if for every* $\mathcal{U} \in \mathbb{R}$ *we have* $W(\mathcal{U}\iota) = \mathcal{U}$.

In this expression, ι denotes a vector of ones. AG simply means that if everybody is at the same level of utility, then the household should also have that level of utility. The concept implies that individuals do not derive any increased welfare from being together. This is certainly not true for most people, but since the focus is on material well-being, the assumption is less severe than it may seem.

As seen in Chapter 2, it is sometimes useful to describe the households as if it maximizes a utility function of the form $U(q;z)$. Assume that all agents in demographic group $k \in (1, \ldots, K)$ share the same utility function u^k. For a household with composition z which has a total consumption bundle q, the welfare maximization problem is

$$\max_{q_i \in \mathcal{Q}} W\left(\left\{u^i(q_i)\right\}\right) \text{ subject to } \sum_{i=1}^{N} q_i \leq q. \tag{3.1}$$

Repeating this exercise for all q and z, we can construct the function U. As long as all the u^i's are strictly quasi-concave and $\mathcal{Q}$ is convex, this yields a unique solution (Mas-Colell et al. 1995, Theorem M.K.4).

Lemma 7 *When* $U(q;z)$ *is the value function associated with the problem (3.1),* U *is continuous in* q *when* W *and* u^i *are continuous for all* i*. Furthermore, if* W *and* u^i *are quasi-concave for all* i*, then* U *is also quasi-concave in* q.

Proof. See Appendix A.2.

Consequently, under relatively weak conditions, we don't make any big mistake by modelling a household which in reality behaves as if it maximizes a BSWF as if it maximized a household utility function. The assumption of welfare maximization will however put restrictions on how a household utility function can depend upon the demographic composition.

3.3 Construction of equivalence scales

Before discussing equivalence scales in the present setting, it is useful to introduce the counterpart to some standard concepts from duality theory. Some of these definitions may also be found in Pollak (1981). For a household maximizing a BSWF, we may define the *household expenditure function*, which gives the cost of reaching welfare level $\mathcal{W}$ for a household with demographic composition z facing prices p, as

$$C^*(p, \mathcal{W}, z) = \min_{q_i \in \mathcal{Q}} \left\{ \sum_{i=1}^{N} p' q_i \middle| W^z\left(\left\{u^i(q_i)\right\}_{i=1}^{N}\right) \geq \mathcal{W} \right\}. \tag{3.2}$$

Furthermore, its inverse with regard to $\mathcal{W}$, the *indirect welfare function*, is defined by

$$V^*(p, y, z) = \max_{q_i \in \mathcal{Q}} \left\{ W^z\left(\left\{u^i(q_i)\right\}_{i=1}^{N}\right) \middle| \sum_{i=1}^{N} p' q_i \leq y \right\} \tag{3.3}$$

where W^z is a BSWF for a household with composition z.

Following the definition in Chapter 2, a natural way of defining an equivalence scale is

$$L(p, \mathcal{W}, z, z_0) = \frac{C^*(p, \mathcal{W}, z)}{C^*(p, \mathcal{W}, z_o)} \tag{3.4}$$

where z_0 denotes the composition of the reference household.

The crucial question is to what extent it is possible to identify equivalence scales. It is quite clear that without any restrictions on the individual utility functions and the welfare functions, a converse to Proposition 1 still holds. Assume that the u^i's are known. If some welfare function W satisfies observed demand, then $f(z) \circ W$, where f is some monotonic transformation of W that depends on z, also rationalizes this demand system. Since W and $f(z) \circ W$ do not necessarily generate the same equivalence scales, we cannot generally identify equivalence scales. The problem is obviously even worse if $\{u^i\}$ is unknown.

This section will focus on a particular household with some composition $z \in \mathcal{Z}$ and N members. Let Θ be the N-dimensional functional space of vectors of individual CCC utility functions u^i. Furthermore, denote the space of BSWFs by Φ. We can now define a *demand-generating function* $G : \Theta \times \Phi \rightarrow \mathcal{D}$ where $\mathcal{D}$ is the space of demand functions $D : \mathbb{R}_+^J \times \mathbb{R}_+ \rightarrow \mathbb{R}_+^J$ obtainable from the maximization of a BSWF, that is, for any $\{u^i\} \in \Theta$ and any $W \in \Phi$, we have $G(\{u^i\}, W) = D$ where

$$D(p, y) = \sum_i \arg\max_{q^i} \left\{ W\left(\left\{u^i\left(q^i\right)\right\}\right) \middle| \sum_i p' q^i \leq y \right\}. \tag{3.5}$$

That is, G gives the demand function associated to a set of utility functions and a welfare function.

In this section we shall abstract from public goods. Under relatively weak conditions, household welfare maximization then corresponds to a decentralized solution, that is

$$D(p, y) = \sum_i \arg\max_q \left\{ u^i(q) \,| p' q \leq \lambda^i(p, y)\, y \right\} \tag{3.6}$$

where the vector of λ^is, $\lambda = \arg\max_\lambda \left\{ W\left(\left\{V^i\left(p, \lambda^i y\right)\right\}\right) | \sum_i \lambda^i = 1 \right\}$ and V^i is individual i's indirect utility function. This condition is at least assured for individual demand satisfying the conditions of the second fundamental theorem of welfare economics (Mas-Colell et al. 1995, Ch. 16.D), and will be assumed to hold in what follows.

Now the following result holds:

Lemma 8 *Let $W^1, W^2 \in \Phi$ be two welfare functions. If for every $\bar{u} \in \Theta$ we have $G(\bar{u}, W^1) = G(\bar{u}, W^2)$, then there is a monotonic transformation f such that $W^1 = f \circ W^2$.*

Proof. See Appendix A.3.

From the knowledge of $\bar{u} \in \Theta$, we can deduce individual demand functions D^i. We then get J equations

$$\sum_i D^i\left(p, \lambda^i(p, y)\, y\right) = D(p, y) \tag{3.7}$$

which may be solved for λ^i, agent i's share of the total household income. The existence of a solution is assured by the assumption that demand may be obtained from the maximization of a BSWF and that household behaviour may be seen as a decentralized process. Solving for the first $N-1$ goods, the implicit function theorem (Simon and Blume 1994, Theorem 15.7) assures the local existence of the λ^i-functions if

$$\begin{vmatrix} yq_1^{1\prime}\left(\lambda^1 y\right) & \cdots & yq_1^{N\prime}\left(\lambda^N y\right) \\ \vdots & \ddots & \vdots \\ yq_N^{1\prime}\left(\lambda^1 y\right) & \cdots & yq_N^{N\prime}\left(\lambda^N y\right) \\ 1 & \cdots & 1 \end{vmatrix} \neq 0. \tag{3.8}$$

This will hold as long as the Engel curves are non-parallel in an open set containing the point considered.

The main difficulty is that there may be multiple solutions. Lemma 8 assures the uniqueness of the BSWF if we can observe D for any utility function in Θ. Normally we are only able to observe D for one point in Θ, and then there may be multiple BSWFs yielding the same demand behaviour. Consequently, to identify the BSWF, we need either to observe demand for multiple points in Θ, or to have conditions on $\{u^i\}$ that guarantees the uniqueness of λ. One necessary (but probably not sufficient) condition for the latter is that condition (3.8) holds for every open set in $\mathbb{R}_+^J \times \mathbb{R}_+$ (possibly after a relabeling of goods). This may be seen as a converse to Gorman's (1953) celebrated result. He proved that the income distribution in an economy does not matter for aggregate consumption if and only if the Engel curves of all the consumers are linear and parallel. If this holds, we cannot identify the amount of money spent on each agent, and hence it is impossible to deduce the BSWF.

These results may be used to identify equivalence scales.

Lemma 9 *Let $\{u^i\} \in \Theta$ be a vector of individual CCC utility functions and $D \in \mathcal{D}$ be an observed demand function. If there is a unique function λ that solves (3.7), then there is at most one AG $W \in \Phi$ such that $G\left(\{u^i\}, W\right) = D$.*

Proof. See Appendix A.4.

Corollary 10 *If the assumptions of the Lemma hold for all households $z \in \mathcal{Z}$ and there is a unique solution to (3.7), then equivalence scales are identifiable from D.*

The proof is immediate since a unique solution to (3.7) guarantees that all BSWFs satisfying D are transformations of each other, and by the lemma, there is only one welfare function that is also AG.

Unfortunately, this result relies heavily on the assumption that individual utility functions are CCC and known. It is well known that if a consumer's demand is given by a (known) demand function $D(p, y)$ that is homogenous of degree zero, satisfies adding up and has a symmetric Slutsky matrix, we can derive the expenditure function, and hence the utility function (Mas-Colell et al. 1995, 3.H). Nevertheless, this utility function is only identified up to a monotonic transformation.

There is one way of resolving this problem. If we assume that households follow some sort of normalization rule, it might be possible to identify CCC individual utilities. One normalization is to assume that utility is money metric[3] subject to some base level of prices $p_0 \gg 0$. If we observe that an individual i has a market behaviour which is consistent with the maximization of a utility function $\tilde{u}$, we define her money metric utility as

$$u(q) = \min_{q_0} \left\{ p_o' q_0 \left| \tilde{u}(q_0) \geq \tilde{u}(q) \right. \right\}, \tag{3.9}$$

that is, $u(q)$ is the cost of reaching the same indifference curve as q given prices p_0. It is clear that we can define u for any set of indifference curves, and u is not invariant to any transformations apart from the identity, so u is CCC. Furthermore, for a given set of indifference curves and a p_0, the corresponding money metric utility function is unique. Define the vector ι^k as the K-vector $(0, \ldots, 0, 1, 0, \ldots, 0)$ where the 1 is in the k^{th} position. Then the following proposition holds:

[3] This was suggested to me by Jørgen Aasness.

Proposition 11 *Let $D^z \in \mathcal{D}$ be a set of demand functions for every $z \in \mathcal{Z}$ that are obtainable from the maximization of an AG welfare function. Assume that each household is known to maximize an AG welfare function, that every individual utility function is money metric with regard to some price level $p_0 \gg 0$, that there is a unique solution to (3.7), and $\iota^k \in \mathcal{Z}$ for every $k \leq K$. Then there is a unique set of equivalence scales.*

Proof. See Appendix A.5.

This proposition shows that if we are willing to make strong assumptions, it is possible to identify equivalence scales. It is difficult to test most of these restrictions, such as AG and money metric utility, so an estimate of equivalence scales based on these assumptions will have to rely on belief in the assumptions. As argued above, the AG assumption is probably not too problematic since we are comparing material well-being. The assumption of money metric utility, on the other hand, is more problematic. First of all, it is not clear how p_0 is determined, or how we should estimate (or postulate) it. Secondly, it is probably difficult to come up with a good explanation why households choose this particular form of utility functions to consider. The assumption of $\iota^k \in \mathcal{Z}$ for all k is also quite restrictive. If for instance "children" is one of the demographic groups, it implies that we should be able to observe households consisting of a single child and no adults. It should be emphasized that this is a sufficient condition, and not necessarily a necessary assumption. There might for instance be conditions under which it is possible to obtain u^2 if we observe a household $z = (1, 1, 0, \ldots, 0)$ when u^1 is known.

The discussion above assumed that all the agents have non-parallel Engel curves. It is shown in Chapter 6 that if all the agents have similar utility of money-functions and linear parallel Engel curves, it is not possible to identify the welfare function, but it is still possible to identify equivalence scales.

3.4 Returns to scale in household consumption

Equivalence scales would not be particularly useful for the model given in Section 3.3. In that model, all consumption goods are private goods, so a household is just a number of individuals sharing the same income. In the real world, there are a number of gains from living in the same household. One is obviously that humans generally enjoy living together. Although this effect is important, we shall ignore it. Probably more important for the construction of equivalence scales, is the presence of returns to scale. Several persons may share a number of goods, and the cost of such goods as housing does not increase linearly in the number of household members.

To model returns to scale in consumption, we shall employ a trick from Browning and Chiappori (1998). Assume that for all goods, some of the consumption may be enjoyed as a private good, and some as a purely public good. Browning and Chiappori use telephone expenditures as an example. A fraction of the expenditure is the subscription fee everybody has to pay. This is a purely public good for the household. The phone calls are on the other hand purely private goods. This division into a purely public and a purely private part is obviously more dubious in some other cases, but it is a useful tool in modelling returns to scale.

The household is able to distinguish between the public and the private goods, but this is not possible for the econometrician. Let $q_i \in \mathcal{Q}$ denote agent i's vector of private consumption and $q^p \in \mathcal{Q}$ the vector of public goods. Her preferences are now denoted by a utility function $u^i : \mathcal{Q}^2 \to \mathbb{R}$. A household with a total consumption bundle q now seeks to solve the problem

$$\max_{q_i, q^p} W\left(\left\{u^i\left(q_i, q^p\right)\right\}\right) \text{ subject to } q^p + \sum_{i=1}^{N} q_i \leq q. \tag{3.10}$$

Unfortunately, introducing returns to scale imposes further difficulties in identifying equivalence scales. The proof of Proposition 11 relied on the econometrician being able to identify individual utility functions. With the knowledge of an agent's indifference map, and under the assumption of money metric utility, this was shown to be possible. When we introduce returns to scale using the procedure above, we are not able to identify the agent's indifference maps any more. This is because we are not able to observe q_i and q^p separately, so we are only able to construct an indifference map for $q_i + q^p$, which is not sufficient to identify the individuals preferences. It may in some cases be possible to obtain individual utility functions by observing households consisting of one and two members of some given group, but I have not been able to find under what assumptions this is possible. Nevertheless, if we were able to identify the individual's preferences, the weaknesses of this approach described in Section 3.3 still apply.

3.5 The Pangloss-problem of welfare functions

There is one difficulty that becomes clearer when considering BSWFs , although the problem is probably equally important when using household utility functions. When we construct equivalence scales, we want to see whether two different households are at the same "welfare level". Since a household may consist of more than one individual, it is necessary to gather the utilities of each individual to some aggregate description of household welfare, that is, we need a welfare function to make comparisons between households. In the approach outlined above, it was assumed that households maximize a welfare function. With no further discussion, it was argued that if we could identify this welfare function, we could also identify equivalence scales if we knew the individual utility functions. The unanswered question is now: Why should a social planner, trying to calculate equivalence scales, use the same welfare function to aggregate the individual utilities as the household is maximizing? As pointed out by Muellbauer (quoted in Pollak 1981), this approach is certainly "Panglossian"[4], in the sense that it is assumed that the welfare function used by the households is also the best welfare function to use for a social planner. Hopefully, constructing equivalence scales based on observed consumer behaviour is less ridiculous than dr. Pangloss's philosophy. Yet, it may be raised serious doubt as to the validity of the approach. If one agent is extremely influential in a household's decision making process, the welfare function the household maximizes may give a lot of weight on that agent's utility. Hence, the estimated BSWF reflects the intra-household distribution of power. A social planner, on the other hand, would probably wish to treat all the family members more or less equally. Basing equivalence scales on observed BSWFs may then give strongly misleading results. It should be remarked that basing calculations on observed "household utility functions", which is commonplace in much of the literature surveyed in Chapter 2, does not solve any problem since intra-household distribution may affect this utility function in the same way as it affects the BSWF. In reality, the use of reduced form utility functions will rather obscure the problem (which is certainly also within the Panglossian tradition). Careful use of an estimated BSWF may, on the other hand, give information as to how resources are allocated within the household.

Unfortunately, it is probably virtually impossible to construct equivalence scales that takes the above argument seriously, since it implies that increasing a households wealth not necessarily leads to an increased level of welfare from the social planners point of view. Yet, there is probably a relationship between individual indifference curves and utility level, so studying individual consumption, we may get some measure of the individuals utility level. Using the same welfare aggregator as the household, on the other hand, is probably more doubtful.

One approach that to a large extent escapes this criticism is the one considered in Chapter 6. There it is shown that if the utility of money is sufficiently similar between agents, the structure of the BSWF does not matter for household decisions as long as they are AG and AN.

[4]From the character dr. Pangloss in Voltaire's *Candide*, whose doctrine was that "everything is for the best in the best of worlds", or to quote him: *"Il est démontré (...) que les choses ne peuvent être autrement: car, tout étant fait pour une fin, tout est nécessairement pour la meilleure fin. Remarquez bien que les nez ont été fais pour porter des lunettes, aussi avons-nous des lunettes. Les jambes sont visiblement instituées pour être chaussées, et nous avons des chausses. (...) [P]ar conséquent, ceux qui ont avancé que tout est bien ont dit une sottise; il faillait dire que tout est au mieux"* (Voltaire 1990, 26f).

Consequently, the equivalence scales will remain the same whether the social planner uses the same welfare function as the household maximizes or another function. This is similar to Pollak's (1981) concept of the "independent society" whereas the more general situation described above corresponds to his "maximizing society". There may, on the other hand, be cases where the social planner wish to put more weight on some agents than others. In that case, this approach will also be subject to the Pangloss criticism. Furthermore, is we want to study intra-household distribution, the assumption of anonymity (AN) may be inappropriate since it to a large extent assumes equality in distribution among household members.

3.6 An evaluation of the performance of the BSWF-approach

At this stage, it may be useful to make a preliminary evaluation of the success of the Bergson-Samuelson welfare function in modelling household demand behaviour. There are probably two main classes of competing approaches, the "household utility function" approach, an the approaches giving a more explicit account of the intra-household decision mechanisms.

The main advantage of the BSWF compared to a reduced form utility function is that it gives a better explanation of intra-household allocation mechanisms, and also gives some structure on how household composition influences household demand. The empirical predictions of the BSWF-approach and maximization of a household utility function are to a large extent similar, since the welfare maximization problem may be rewritten as a utility maximization problem. Consequently, both approaches leads to demand functions with the usual conditions of adding up, homogeneity of degree zero, and a symmetric and negative semi-definite Slutsky matrix[5]. Furthermore, since the household maximizes a common welfare function, income will be pooled, i.e., the person earning the income should not matter for consumption. However, since the BSWF-approach gives a clearer account of the relationship between household behaviour and the characteristics of the individual household members, it is preferable to the reduced form utility function in a number of cases.

The constraints on the Slutsky matrix and income pooling are testable hypotheses. Browning and Chiappori (1998) performs a test of Slutsky symmetry using a QUAIDS demand system on Canadian data. They find that their data are not compatible with Slutsky symmetry, but with a Slutsky matrix which is the sum of a symmetric matrix and a matrix of rank one, which is consistent with their theory of household behaviour. Lewbel (1995), on the other hand, cannot reject Slutsky symmetry for a number of goods using non-parametric approaches on British data, so it is difficult to give a clear-cut conclusion. Despite a number of econometric problems[6], the income pooling hypothesis seems to be strongly rejected. For instance Lundberg et al. (1997) use a change in the UK child benefit where transfers were changed from a reduction in taxes for the income earner to a transfer paid directly to the mother, as a natural experiment, and find that the demand for children's and women's clothing change significantly after the reform.

Some of the richer models of intra-household behaviour give rise to demand systems that are consistent with these econometric findings, so in this sense, they are superior to the BSWF approach. However, it is relatively difficult to transform these models into empirical specifications that are estimable by traditional approaches. Consequently, the Bergson-Samuelson welfare function is probably a useful way of modelling household behaviour for empirical purposes.

[5] The Slutsky matrix is well defined using the household expenditure function defined above.

[6] Such as the endogeneity of incomes due to the household allocation mechanism.

Chapter 4

Estimation of a LES demand system

Before trying to estimate equivalence scales, we shall make a detour by first estimating a complete linear demand system. The estimated parameters are interesting in themselves, and in Chapter 6, these estimates will be used to estimate equivalence scales.

4.1 A simple demand system

Let y denote a household's total consumer expenditure and y_j the expenditure on good j. The class of demand systems that generate linear Engel curves is

$$y_j = m_j(p,z) + \beta_j(p,z)\left(y - \sum_{l=1}^{J} m_l(p,z)\right) \tag{4.1}$$

for some functions m_j and β_j. Any behaviour where the budget constraint holds with equality implies that $\sum_j \beta_j(p,z) \equiv 1$. There is a whole range of individual utility functions generating demand functions within this class. Generally, the indirect utility function has to satisfy the Gorman (1961) *polar form* $V(p,y) = \eta(p) + \pi(p)y$ for two function η and π (see also Gorman (1995) for a discussion of separable *utility functions* yielding linear Engel curves). In this empirical investigation we shall consider a simple demand system that will generate a linear expenditure system (LES). The LES is normally restricted to systems where the m's and β's do not depend on prices. Since we are considering cross-section data where every household faces the same vector of prices, we would not be able to identify the effect of prices. Consequently, a model that generates demand functions with price-independent parameters has been chosen. The empirical results may be given a wider interpretation, though.

Assume that each household behaves as if it maximizes a concave AG AN Bergson-Samuelson welfare function W as discussed in Chapter 3. Each household member consumes privately a vector of goods q_i and also has access to a vector of public goods q^p. We shall assume that she has a Stone-Geary utility function

$$u_i(q_i, q^p) = \left[\prod_{j=1}^{J} (q_{ij} - \mu_{ij})^{\gamma_j}\right]\left[\prod_{j=1}^{J} \left(q_j^p - \mu_j^p\right)^{\gamma_j^p}\right]. \tag{4.2}$$

An important simplifying factor in (4.2) is that the γ's and μ_j^p's are the same for every household member. The μ_j's may be interpreted as minimum quantities of different goods. We shall denote the vectors $(\mu_{i1}, \dots, \mu_{iJ})'$ and $\left(\mu_1^p, \dots, \mu_J^p\right)'$ by μ_i and μ^p respectively. All the utility functions yield non-satiation, so the budget constraint $p'\left(q^p + \sum_i q_i\right) \leq y$ will hold with equality for all households maximizing a PP welfare function. Maximizing household welfare subject to the budget constraint and the non-negativity constraints $q^p \geq 0$ and $q_i \geq 0$ yields the FOCs

$$
\begin{array}{ll}
\gamma_j \left[\prod_{j=1}^J \left(q_j^p - \mu_j^p\right)^{\gamma_j^p}\right] \frac{\left[\prod_{j=1}^J (q_{ij}-\mu_{ij})^{\gamma_j}\right]}{q_{ij}-\mu_{ij}} \frac{\partial W}{\partial u^i} \leq \xi p_j & \\
 & \text{for all } 1 \leq i \leq N,\ 1 \leq j \leq J \\
\gamma_j^p \left[\sum_{i=1}^N \left(\prod_{j=1}^J \left(q_{ij} - \mu_{ij}\right)^{\gamma_j}\right)\right] \frac{\left[\prod_{j=1}^J (q_j^p-\mu_j^p)^{\gamma_j^p}\right]^\lambda}{q_j^p-\mu_j^p} \frac{\partial W}{\partial u^i} \leq \xi p_j &
\end{array}
\tag{4.3}
$$

where ξ is the Lagrange multiplier. Since the utility of money-curve is the same for every individual, everyone will stay at the same level of utility, and consequently, $\frac{\partial W}{\partial u^i}$ is the same for everyone. We shall abstract from the possibility of getting corner solutions, so the FOCs hold with equality. Inserting from the budget constraint, the solutions are

$$
\begin{array}{l}
q_{ij} = \mu_{ij} + \frac{\gamma_j}{\gamma+\gamma^p} \frac{y-p'(\mu^p+\sum_i \mu_i)}{np_j} \\
q_j^p = \mu_j^p + \frac{\gamma_j^p}{\gamma+\gamma^p} \frac{y-p'(\mu^p+\sum_i \mu_i)}{p_j}
\end{array}
\tag{4.4}
$$

where $\gamma = \sum_j \gamma_j$ and $\gamma^p = \sum_j \gamma_j^p$. Define $\beta_j = \gamma_j + \gamma_j^p$ for all j. Summing over all agents, we get that the household's aggregate demand for good j is given by

$$
q_j = \mu_j^p + \sum_i \mu_{ij} + \frac{\beta_j}{\beta} \frac{y - p'\left(\mu^p + \sum_i \mu_i\right)}{p_j}
\tag{4.5}
$$

where $\beta = \sum_j \beta_j$. We recognize (4.5) as a linear expenditure system. In the remainder of the chapter, it will be useful to study the *expenditure* on good j instead of the *quantity* of good j. By definition, $y_j \equiv p_j q_j$ and $m_{ij} = p_j \mu_{ij}$, so (4.5) becomes

$$
y_j = m_j^p + \sum_{i=1}^N m_{ij} + \frac{\beta_j}{\beta} \left[y - \sum_{l=1}^J \left(m_l^p + \sum_i m_{il}\right)\right].
\tag{4.6}
$$

4.2 Econometric model

It is clear that β is not identifiable, so at the time being, we shall normalize it to unity. This assumption will be relaxed in Chapter 6. We have a sample of H households indexed by h. Each household has a demographic composition $\tilde{z}_h \in \mathcal{Z}$ and a total consumer expenditure y_h. Instead of using $\tilde{z}_h$, it is useful to define $z_h \equiv \begin{pmatrix} 1 & \vdots & \tilde{z}_h' \end{pmatrix}'$. Furthermore, define $m_j = \begin{pmatrix} m_j^p & m_{1j} & \cdots & m_{Kj} \end{pmatrix}'$.[1] Then household h has a demand for good j given by

$$
y_{jh} = m_j' z_h + \beta_j \left(y_h - \sum_{l=1}^J m_l' z_h\right).
\tag{4.7}
$$

We can rewrite (4.7) as

$$
y_{jh} = a_j' z_h + \beta_j y_h
\tag{4.8}
$$

where a_j is a $K+1$-vector of demographic effects. This system has the advantage that it may be estimated by ordinary least squares (OLS). Unfortunately, as pointed out by Muellbauer (1974) among others, the m_j's are not identifiable from the a_j's. To see this, define $\mathbf{y}_h = \begin{pmatrix} y_{1h} & \cdots & y_{Jh} \end{pmatrix}'$,

$$
A = \begin{pmatrix} a_1' \\ \vdots \\ a_J' \end{pmatrix} \text{ and } M = \begin{pmatrix} m_1' \\ \vdots \\ m_J' \end{pmatrix}.
$$

[1]To make the exposition clearer, we shall refer to the first element of z_h and m_j as the zeroth element, so e.g. that $z_{hk} = \tilde{z}_{hk}$.

Now we may rewrite (4.7) as

$$\mathbf{y}_h = Mz_h + \beta \left(y - \iota' M z_h\right) \tag{4.9}$$

and (4.8) as

$$\mathbf{y}_h = Az_h + \beta y. \tag{4.10}$$

Since these expressions have to be equal for all $z_h \in \mathcal{Q}$, we need

$$A = \left(I - \beta\iota'\right) M, \tag{4.11}$$

where I is the identity matrix. Since $\iota'\beta = 1$ due to adding up, each row in the matrix $(I - \beta\iota')$ will sum to 0, so $(I - \beta\iota')$ is singular. Hence it is not possible to identify M from the knowledge of A alone. One possibility is to look at different cross-sections, and use the variation in prices to identify the parameters. A difficulty with this procedure is that the price data are likely to contain measurement error, and good instruments are scarce. Consequently, an alternative procedure will be used in the present work.

If $\beta_j > 0$ for all j, then the rank of $(I - \beta\iota')$ is $J - 1$. Consequently, if we can find a j_k such that $m_{kj_k} = \bar{m}_k$ where $\bar{m}_k$ is a known number (usually zero) for each demographic group k and a similar condition for m_j^p, we can identify the other parameters of the matrix M. This means that we have to impose a priori restrictions on preferences. But, with the right grouping of goods, such restrictions may be rather plausible. For instance, it is likely that adults do not have necessary quantities of babies' nappies, and children do not (hopefully) have any necessary consumption of tobacco[2].

Equation (4.11) may be written as

$$a_{kj} = m_{kj} - \beta_j \sum_{l=1}^{J} m_{kl}, \tag{4.12}$$

so if we know that $m_{kj_k} = 0$, we get that

$$\sum_{l=1}^{J} m_{kl} = -\frac{a_{kj_k}}{\beta_{j_k}} \tag{4.13}$$

for each k. Consequently we have

$$m_{kj} = a_{kj} - \beta_j \frac{a_{kj_k}}{\beta_{j_k}} \tag{4.14}$$

for all j and k, which is an expression for m_{kj} which is a function of known parameters only.

To perform an econometric analysis, we will need a stochastic model. Assume now that household h has a demand for good j given by

$$y_{jh} = m_j' z_h + \beta_j \left(y_h - \sum_{l=1}^{J} m_l' z_h \right) + \varepsilon_{jh} \tag{4.15}$$

where ε_{jh} is a stochastic variable, which may be interpreted as differences in taste or simply errors of measurement. It is clear that this specification may lead to some expenditures being

[2] Is is obviously possible to raise objections to any such restrictions. There are adults who takes pleasure in wearing babies' nappies, and adolecents who smoke surely exists. Nevertheless, the m's represent *necessary* consumption and not any consumption. Futhermore, a number of such deviations from "ordinary" behaviour is likely to not be reported in the data, and consequently does not represent any difficulty.

negative. The problem could be solved by using tobit-models, but is ignored for simplicity. Due to adding up,

$$\sum_j \varepsilon_{jh} = 0. \tag{4.16}$$

Define the vector $\varepsilon_h = \begin{pmatrix} \varepsilon_{1h} & \cdots & \varepsilon_{Jh} \end{pmatrix}$ and the covariance matrix $\Sigma \equiv E\varepsilon_h\varepsilon_h'$, and assume that this is a finite matrix. From (4.16) it follows that Σ is singular. We shall assume that tastes and measurement errors are independent between households, i.e. $E\varepsilon_{jh}\varepsilon_{j'h'} = 0$ for all j, j' and $h \neq h'$. We could assume that ε_h satisfies some given parametric distribution. A common assumption is that $\varepsilon_h \sim N(0, \Sigma)$. In this case we could use maximum likelihood, and get asymptotically efficient estimates. Notwithstanding, the assumption of a particular distribution is difficult to justify, so in the present work we shall only assume that the distribution belongs to the family of distributions having finite first and second moments.

Initially, we shall assume that the stochastic error is identically distributed among households when the model is in expenditure form. It is known that the system in expenditure form often shows signs of heteroskedasticity. Biørn (1995, 153ff) and Pollak and Wales (1992, 16f) argues in favour of assuming that the errors are iid in the model in budget shares-form to reduce this problem. This is considered in Section 4.6.

For ordinary estimation procedures to work properly, we need ε_h and (y_h, z_h') to be independent of each other. If some of the regressors are measured with error, this measurement error will translate to the error term to create correlation between the regressor measured with error and ε_h. This is considered in Chapter 5. Omitted variables may also give rise to correlation between the regressors and the error term if the regressors are correlated with the omitted variables. The effect of considering some variables that are rather common in demand analysis, such as residential region and type, are considered in Chapter 5. There are at least two variables, education and social class, that seems to be omitted from most standard demand analyses, and that are likely to influence the demand pattern. The two are probably related to each other. The influence of social class on consumer behaviour is thoroughly discussed and documented by among others Bourdieu (1979), and social class is probably also correlated with income[3]. Education may also affect consumption, for instance through better knowledge of nutrition. Since education on average does affect income (see e.g. Card 1995), education and consumer expenditure are also likely to be correlated. Education and social class will however probably have the largest impact on the choice between close substitutes, such as between hamburgers and salad, rather than between aggregate groups of goods, such as food and clothing. Since we are going to work on highly aggregated groups of goods, this means that this problem is probably going to be less severe.

The non-linear model (4.15) might be estimated by non-linear SUR. In the present case, this

[3]It may be objected that social class only affects consumer behaviour through income, but this is probably not the whole truth. Newly rich persons will for instance probably behave differently from persons belonging to families that have belonged to the upper classes for a long time.

is equivalent to transforming it to a linear system[4]

$$\begin{aligned} y_{jh} &= a_j' z_h + \beta_j y_h + \varepsilon_{jh} \\ \varepsilon_h &\sim \text{iid } (0, \Sigma) . \end{aligned} \tag{4.17}$$

Since the regressors are the same in every equation, SUR is equivalent to OLS (Harvey 1990, 68). The OLS estimators $\hat{a}_j$ and $\hat{\beta}_j$ are then BLUE, and also consistent estimators of a_j and β_j (Greene 1997). To calculate the m_{kj}'s, we can use (4.14). Since this is a continuous function of the parameters (for $\beta_{kj_k} > 0$), Slutsky's theorem gives

$$\hat{m}_{kj} = \hat{a}_{kj} - \hat{\beta}_j \frac{\hat{a}_{kj_k}}{\hat{\beta}_{j_k}} \xrightarrow{p} a_{kj} - \beta_j \frac{a_{kj_k}}{\beta_{j_k}} = m_{kj} \tag{4.18}$$

as long as $m_{kj_k} = 0$. An expression for the asymptotic covariance matrix of the β's and the m's is easily derived using the delta-method. An approach based on bootstrapping is also found in Section 4.6.

4.3 The data

To estimate (4.17), we use data from the 1994 Norwegian Survey of Consumer Expenditure (Statistics Norway 1996). In this data set, we have expenditure data for 1339 households, where each household report their expenditures of 793 different goods during two weeks. Two households were deleted from the sample because they had negative total consumer expenditure[5] since the theory presented in Section 4.1 assumes a non-negative consumer expenditure, so the sample consists of $H = 1337$ households. With the exception of certain durable goods for which we have data for the whole year, all reported expenditures are multiplied by 26 to get approximate annual figures. We shall restrict the demographic composition of households to the number of children below 16 and the number of adults, denoted by z_1 and z_2 respectively. We are going to consider the demand for four groups of goods: child goods, adult goods, "neutral" goods and other goods. These groups are designed to avoid that adults have any "necessary" consumption of child goods, children have any "necessary" consumption of adults goods, and that there are returns to scale in the "necessary" consumption of "neutral" goods. Details of the classification are given in Appendix C. Some descriptive statistics of the data are given in Table 4.1.

4.4 OLS estimation results

Regressing expenditure on each consumption group on total consumption expenditure and demographic composition using OLS, we get the results reproduced in Table 4.2. The estimated necessary expenditures seem to be rather high and the estimated standard errors are also quite

[4]Since the regressors are the same in every equation, the linear SUR model is equivalent to OLS. Gallant (1975) shows that when the functional form is the same in every equation of a non-linear SUR model, it reduces to non-linear least squares (NLS). The error term in the linear model is

$$\varepsilon_{jh} = y_{jh} - a_j' z_h + \beta_j y_h$$

and in the non-linear model it is

$$\varepsilon_{jh} = y_{jh} - \left(m_j' - \beta_j \sum_l m_l' \right) z_h + \beta_j y_h .$$

Since there is a one to one relationship between the m's and the β's and the a's and β's given the restriction on the m's, minimizing the residual sum of sqares in the two cases is equivalent.

[5]Sales of durables is reported as negative consumption which explains how it is possible to get negative total consumer expenditure in the data.

Table 4.1: Some descriptive statistics

Variable	Minimum	Maximum	Mean	Std Dev	Skewness	Kurtosis
Number of children	0	7	.984	1.13	.985	.654
Number of adults	1	7	2.26	.867	.993	1.54
Total consumer expenditure	0	2371322.52	328463.09	217895.94	2.01	8.88
Expenditure on child goods	0	116812.00	9193.75	16418.55	2.35	6.55
Expenditure on adult goods	0	170293.00	20991.31	20173.14	2.03	6.27
Expenditure on neutral goods	0	255840.26	17349.06	20281.64	5.88	52.53
Expenditure on other goods	-12932.35	2257373.38	280928.98	200781.59	2.28	11.07

Table 4.2: Estimation of demand system by OLS. Standard errors in parenthesis.

	Estimates					Implied values		
Group	a_{0j}	a_{1j}	a_{2j}	β_j	R^2	m_{0j}	m_{1j}	m_{2j}
Child goods	2739.48	7080.33	-2569.74	.0161	.34	4833.01	7585.50	0
	(1110.44)	(333.44)	(445.86)	(.00181)		(1541.27)	(383.10)	(0)
Adult goods	-1067.92	-1028.01	5450.40	.0328	.23	3192.35	0	10679.72
	(1473.399	(442.42)	(591.59)	(.002409		(2492.75)	(0)	(1075.77)
Neutral goods	-3097.61	1206.38	5064.21	.0238	.15	0	1953.84	8866.41
	(1548.38)	(464.94)	(621.70)	(.00252)		(0)	(539.43)	(936.55)
Other goods	1426.06	-7258.70	-7944.86	.927	.98	121938.12	21821.13	139979.16
	(2431.55)	(730.14)	(976.31)	(.00396)		(60814,97)	(12674.76)	(26269.74)
Sum	0	0	0	1		129963.48	31360.48	159525.29
						(63905.27)	(13159.12)	(27849.01)

high. In Figure 4.1, the OLS residuals are plotted against the regressors. For the demographic effect, there does not seem to be any major problems. On the other hand, there are signs of considerable heteroskedasticity with regard to total consumer expenditure, which is a well-known feature of estimates of Engel curves in expenditure form.

4.5 Estimation using generalized least squares

Apart from causing inconsistent estimates of the covariance matrix, heteroskedasticity also makes OLS inefficient. If the true covariance matrices for each household Σ_h were known, generalized least squares (GLS) would be efficient. Since we don't know these matrices, we have two options. Either we can keep OLS, which is still consistent, and calculate the covariance matrix of the estimates using some consistent procedure such as White's (1980) estimator, or we can use feasible GLS (FGLS). Following Aasness and Nyquist (1983) we shall assume that the true model is

$$\begin{aligned} y_{jh} &= a_j' z_h + \beta_j y_h + \varepsilon_{jh} \\ \varepsilon_h &\sim \text{iid } (0, \Sigma_h) \\ \Sigma_h &= y_h^{\kappa} \Sigma. \end{aligned} \tag{4.19}$$

If $\kappa = 0$ we get model (4.17), whereas a model with identically distributed errors in the share equation corresponds to $\kappa = 2$. Since OLS is consistent, the residuals satisfy

$$\hat{\varepsilon}_{jh}^{OLS} = y_{jh} - \hat{a}_j' z_h - \hat{\beta}_j y_h \xrightarrow{p} \varepsilon_{jh}. \tag{4.20}$$

We can then use $\hat{\varepsilon}_{jh}^{OLS}$ to estimate κ since $E\left(\hat{\varepsilon}_h^{OLS}\right)\left(\hat{\varepsilon}_h^{OLS}\right)' \xrightarrow{p} y_h^{\kappa}\Sigma$. We could in principle estimate $J(J+1)/2$ different κ's, one for each independent element of Σ. Nevertheless, this would lead to a rather important loss of degrees of freedom, and the FGLS estimation would easily get rather cumbersome. Consequently, κ is assumed to be identical for every element of Σ. To simplify even further, only the diagonal elements of Σ will be used for estimation. Using the estimated residuals from the OLS, we get the system

$$\ln \hat{\varepsilon}_{jh}^2 = \ln \sigma_{jj}^2 + \kappa \ln y_h + u_{jh}. \tag{4.21}$$

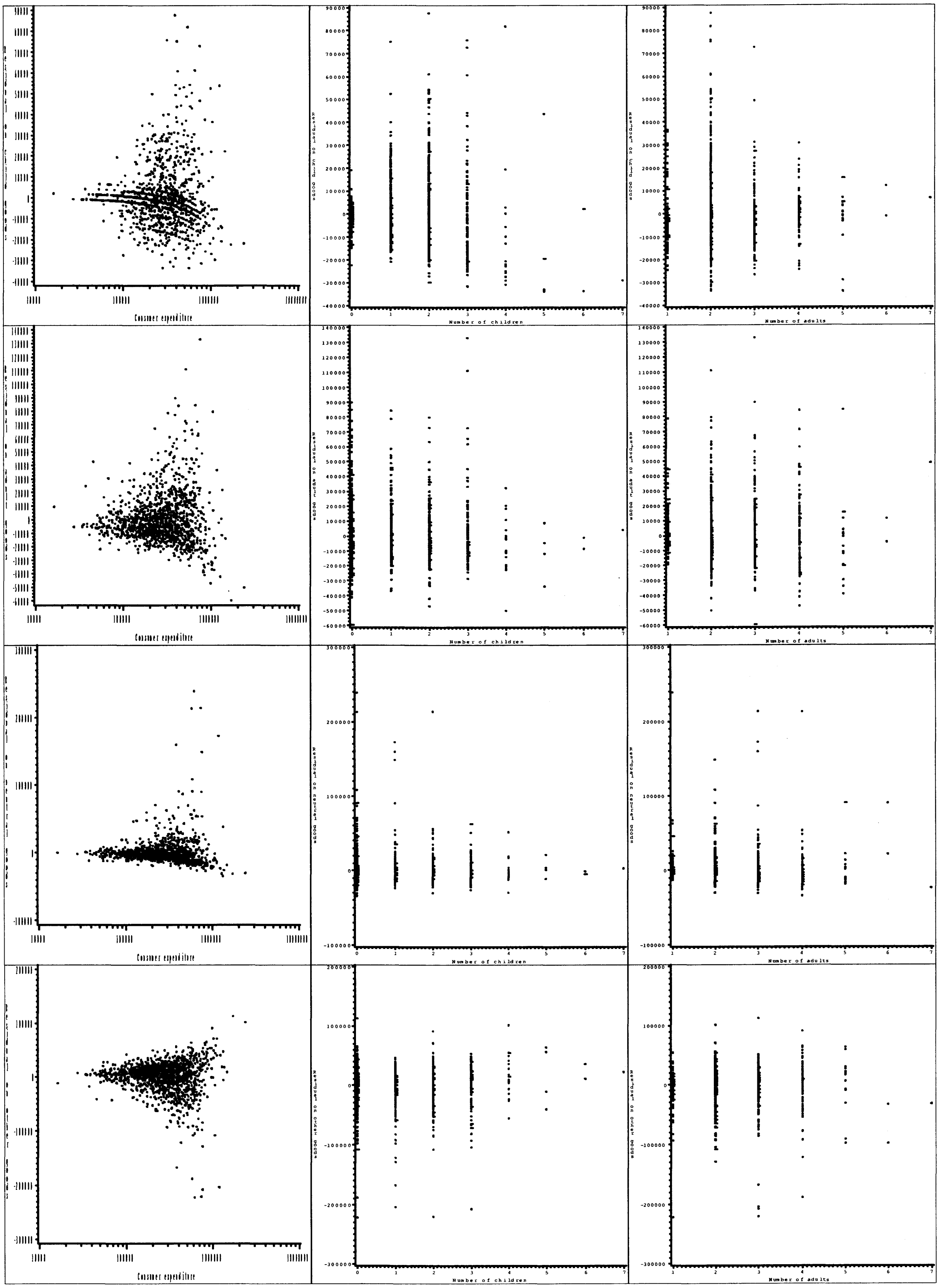

Figure 4.1: OLS residuals plotted against the regressors for the four equations (logarithmic scale for total consumer expenditure).

We assume that $u_h = \begin{pmatrix} u_{1h} & \cdots & u_{Jh} \end{pmatrix}' \sim \text{iid}(0, \Lambda)$ for some finite positive definite matrix Λ. As long as ε_h has finite fourth moments, Λ will be finite. To impose the restriction that the κ is the same in every regression, we use a constrained SUR estimator described by Harvey (1990, 69f). Denote by ζ the unconstrained estimate of (4.21), i.e. $\zeta = \begin{pmatrix} \ln \sigma_{11}^2 & \kappa_1 & \cdots & \ln \sigma_{JJ}^2 & \kappa_J \end{pmatrix}'$, and define the matrix

$$R = \begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \end{pmatrix}.$$

Then the restriction $\kappa_1 = \kappa_2 = \ldots = \kappa_J$ is equivalent to $R\zeta = 0$. The constrained estimator is then

$$\zeta^\dagger = \zeta - \left(\Lambda \otimes X'X^{-1}\right) R' \left[R \left(\Lambda \otimes X'X^{-1}\right) R'\right]^{-1} R\zeta$$

where $X = \begin{pmatrix} 1 & y_h \end{pmatrix}_h$ and $\otimes$ denotes the Kronecker product. Since Λ is unknown, it has to be replaced by an empirical estimate by a procedure similar to the one used in ordinary SUR estimation. We then get an estimate of κ, $\hat{\kappa} = 1.32\,(.0524)$. It is seen that the hypothesis of no heteroskedasticity ($\kappa = 0$) is rejected against $\kappa \neq 0$ with a p-value of $3.94 \cdot 10^{-115}$. Identically distributed errors on a the budget shares corresponds to $\kappa = 2$. The hypothesis of $\kappa = 2$ is also rejected with a p-value of $3.68 \cdot 10^{-36}$.

When heteroskedasticity takes the assumed form, it is easily solved by weighting the observations. If we now define

$$\begin{aligned} y_{jh}^* &= \frac{y_{jh}}{y_h^{\kappa/2}} \\ y_h^* &= y_h^{1-\kappa/2} \\ z_h^* &= \frac{z_h}{y_h^{\kappa/2}}, \end{aligned}$$

we get that (4.19) is equivalent to

$$\begin{aligned} y_{jh}^* &= a_j' z_h^* + \beta_j y_h^* + \nu_{jh} \\ \nu_h &= \frac{1}{y_h^{\kappa/2}} \varepsilon_h \sim \text{iid } (0, \Sigma), \end{aligned} \tag{4.22}$$

i.e. we have a homoskedastic model where OLS is efficient on the new variables. Because the true κ is unknown, we use the estimate $\hat{\kappa}$ discussed above, which satisfies $\hat{\kappa} \xrightarrow{P} \kappa$. Since also $\hat{\varepsilon}_h \xrightarrow{L} \varepsilon_h$, it follows from Cramér's theorem[6] that

$$\frac{1}{y_h^{\hat{\kappa}/2}} \hat{\varepsilon}_h \xrightarrow{L} \nu_h. \tag{4.23}$$

This means that FGLS will be asymptotically more efficient than OLS since it is asymptotically equivalent to GLS. Using the estimate of κ from (4.21) in the model (4.22), we get the FGLS estimates reported in Table 4.3.

The new residuals are plotted against total consumer expenditure in Figure 4.2. Although there may still be some signs of heteroskedasticity, the overall picture is more favorable than above. The estimates also look more reasonable.

[6] This is the name given by Wills (1998). Lehmann (1999) calls this theorem Slutsky's theorem. It states that for two sequences of random variables x_n and y_n such that $x_n \xrightarrow{P} a$ and $y_n \xrightarrow{L} y$, we have that $y_n / x_n \xrightarrow{L} y/a$.

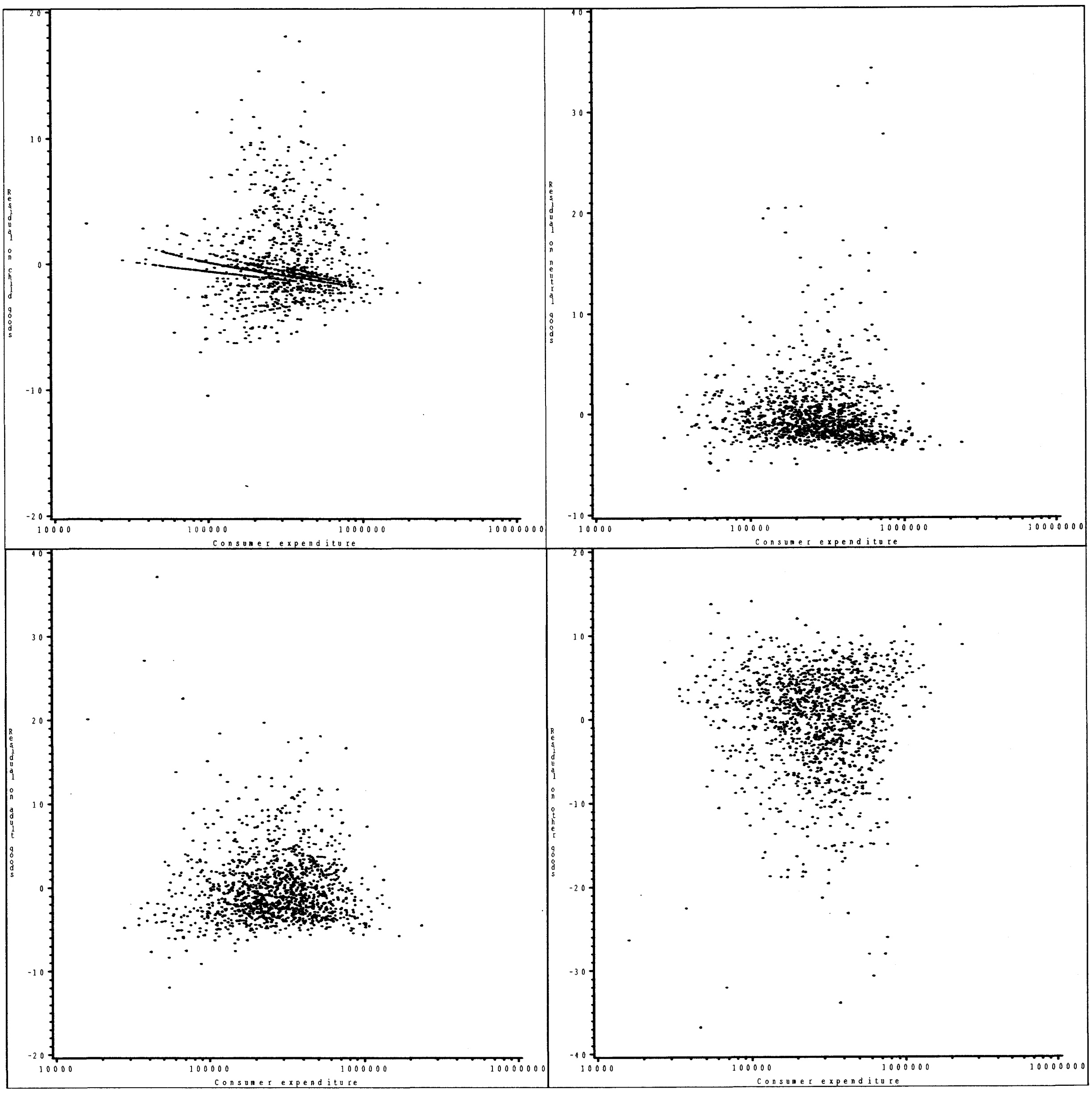

Figure 4.2: Residuals from FGLS plotted against the regressors for each of the four equations (logarithmic scale for total consumer expenditure).

Table 4.3: Estimation of demand system by FGLS. Standard errors in parenthesis.

	Estimates					Implied values		
Group	a_{0j}	a_{1j}	a_{2j}	β_j	R^2	m_{0j}	m_{1j}	m_{2j}
Child goods	658.20	5416.14	-1477.95	.0200	.44	1352.70	6156.92	0
	(651.62)	(274.78)	(319.26)	(.00206)		(825.30)	(302.13)	(0)
Adult goods	-1012.68	-1683.43	3905.46	.0455	.59	565.59	0	7264.12
	(919.75)	(387.85)	(450.63)	(.00291)		(1394.16)	(0)	(807.10)
Neutral goods	-1129.47	901.00	3094.76	.0326	.58	0	2105.72	5498.34
	(775.09)	(326.85)	(379.76)	(.00245)		(0)	(397.20)	(628.10)
Other goods	1483.95	-4633.72	-5522.28	.902	.99	32752	28718.62	61020.00
	(1357.24)	(572.34)	(664.98)	(.00429)		(22055.55)	(7575.52)	(13908.71)
Sum	0	0	0	1		34671.22	36981.27	73782.47
						(23596.66)	(7977.37)	(15097.69)

4.6 OLS versus feasible GLS

Although the FGLS estimates reported above seem to be reasonable, some difficulties may arise. We know that FGLS is asymptotically consistent, but its small sample properties are not very well known. Harvey (1990) argues that within a time series setting, OLS may on some occasions be more efficient than FGLS in small samples. This probably also extends to the heteroskedastic case. One way to get an idea of how this may be in the present model is to perform a bootstrap[7]. This will also allow us to get an alternative estimate of the standard errors of the estimated parameters. We shall assume that (4.19) is the true data generation process and that the true value of κ is 1.32. We then proceeds as follows:

1. Run FGLS as described above and obtain the residuals.
2. For each household h, draw with replacement a new vector of residuals $(\varepsilon^*_{1h}, \dots \varepsilon^*_{Jh})$ from the residuals from step 1.
3. Construct new variables for expenditure on each good through $y^*_{jh} = z'_h \hat{a}^{FGLS}_{jh} + \beta^{FGLS}_j y_h + y_h^{\kappa/2} \varepsilon^*_j$, where the original regressors are unchanged.
4. With this new data set calculate OLS, a new estimate of κ and a new FGLS estimate, as well as the values for the m's for both OLS and FGLS.
5. Repeat this procedure a large number of times.

In the present case, 10000 replications were made. The average and standard deviation of each estimate is presented in Tables 4.4 and 4.5 together with the corresponding true values of the parameters. The table also includes the average bias of the estimates, the bias as a proportion of the true value, the mean square error (MSE) and the MSE and bias in OLS relative to that of FGLS.

Both OLS and FGLS both produces estimates with a low bias. The variance is on the other hand generally lower for FGLS than for OLS, and the former is in most cases nearly twice as efficient as the latter measured by the MSE. The estimated asymptotic standard errors are also close to the bootstrapped standard errors. As seen in Table 4.6, the average estimate of κ is very close to the true value with an average bias of 0.01 or 0.76%, so the estimator of κ presented above seems to perform well. The same table shows results from a so-called "wild bootstrap" as well. The wild bootstrap is a class of procedures to reproduce heteroskedastic data without specifying a functional form. Here we have used Wu's procedure as described in Cribari-Neto

[7]See Davison and Hinkley (1997) or Efron and Tibshirani (1993) for a general introduction to the bootstrap and Horowitz (1997) or Jeong and Maddala (1993) for its application to econometrics.

Table 4.4: Results from bootstrapping the OLS estimates.

Variable	True value	Mean	Std Dev	Avg. bias	Bias proportion	MSE	$\frac{MSE^{OLS}}{MSE^{GLS}}$	$\frac{Bias^{OLS}}{Bias^{GLS}}$
a_{10}	658.2	651.46	1085.4	-6.74	0.0102	1178138.59	2.79	0.597
a_{11}	5416.14	5414.42	364.01	-1.72	0.000318	132505.14	1.71	1.65
a_{12}	-1477.95	-1480.01	497.15	-2.06	0.00139	247160.65	2.44	0.644
β_1	0.2	0.0201	0.00340	-0.180	0.900	0.03246	0.999	0.999
a_{20}	-1012.68	-1008.85	1529.09	3.83	0.00378	2338130.90	2.81	0.748
a_{21}	-1683.43	-1684.75	503.42	-1.32	0.000784	253430.47	1.69	1.42
a_{22}	3905.46	3898.22	694.10	-7.24	0.00185	481829.66	2.36	7.24
β_2	0.0455	0.0456	0.00471	0.000119	0.00262	0.0000221	2.60	2.04
a_{30}	-1129.47	-1105.59	1284.71	23.88	0.0211	1651050.04	2.72	0.808
a_{31}	901	903.198	431.620	2.20	0.00244	186300.97	1.73	0.592
a_{32}	3094.76	3092.69	588.503	-2.07	0.000669	346340.06	2.37	0.524
β_3	0.0326	0.0326	0.00397	0.0000231	0.000709	0.0000158	2.60	19.25
a_{40}	1483.95	1462.99	2293.48	-20.96	0.0141	5260489.83	2.84	0.456
a_{41}	-4633.72	-4632.87	754.99	0.85	0.000183	570017.51	1.72	0.223
a_{42}	-5522.28	-5510.9	1026.74	11.38	0.00206	1054324.53	2.37	6.47
β_4	0.902	0.902	0.00691	-0.000319	0.000354	0.0000479	2.62	2.85
	0	0						
m_{10}	1352.7	1312.59	1337.51	-40.11	0.0297	1790541.81	2.65	3.60
m_{20}	565.59	493.10	2268.26	-72.49	0.128	5150257.66	2.56	1.94
m_{30}	0	0	0	0	0	0		
m_{40}	32752	31089.85	36132.1	-1662.15	0.0507	1308291393	2.59	1.82
m_{11}	6156.92	6157.36	411.96	0.44	0.0000715	169709.90	1.83	0.147
m_{21}	0	0	0	0	0	0		
m_{31}	2105.72	2110.34	537.61	4.62	0.00219	289048.75	1.79	1.57
m_{41}	28718.62	28752.75	10230.06	34.13	0.00119	104655292.5	1.81	0.582
m_{12}	0	0	0	0	0	0		
m_{22}	7264.12	7275.88	1240.31	11.76	0.00162	1538507.19	2.28	0.644
m_{32}	5498.34	5508.51	964.80	10.17	0.00185	930942.00	2.30	1.119
m_{42}	61020	61280.98	21561.1	260.98	0.00428	464949143.8	2.28	0.799

Table 4.5: Results from bootstrapping the FGLS estimates.

Variable	True value	Mean	Std Dev	Avg. bias	Bias proportion	MSE	$\frac{MSE^{OLS}}{MSE^{GLS}}$	$\frac{Bias^{OLS}}{Bias^{GLS}}$
a_{10}	658.2	669.50	649.49	11.30	0.0172	421969.95	2.79	0.597
a_{11}	5416.14	5417.18	278.62	1.04	0.000192	77629.39	1.71	1.65
a_{12}	-1477.95	-1474.75	318.41	3.2	0.00217	101395.45	2.44	0.644
β_1	0.2	0.0200	0.00206	-0.180	0.900	0.0324	0.999	0.999
a_{20}	-1012.68	-1007.56	912.86	5.12	0.00506	833331.04	2.81	0.748
a_{21}	-1683.43	-1684.36	387.74	-0.93	0.000552	150342.07	1.69	1.42
a_{22}	3905.46	3904.46	451.83	-1	0.000256	204151.48	2.36	7.24
β_2	0.0455	0.0456	0.00292	0.0000584	0.00128	0.00000853	2.60	2.04
a_{30}	-1129.47	-1099.9	778.21	29.57	0.0262	606488.98	2.72	0.808
a_{31}	901	904.716	328.52	3.72	0.00412	107937.88	1.73	0.592
a_{32}	3094.76	3090.81	382.60	-3.95	0.00128	146395.23	2.37	0.524
β_3	0.0326	0.0326	0.00246	-0.0000012	0.0000368	0.00000606	2.60	19.25
a_{40}	1483.95	1437.97	1361.28	-45.98	0.0310	1855197.40	2.84	0.456
a_{41}	-4633.72	-4637.54	575.63	-3.82	0.000824	331362.19	1.72	0.223
a_{42}	-5522.28	-5520.52	666.40	1.76	0.000319	444088.57	2.37	6.47
β_4	0.902	0.9018881	0.00427	-0.000112	0.000124	0.0000183	2.62	2.85
m_{10}	1352.7	1341.57	821.53	-11.13	0.00823	675027.66	2.65	3.60
m_{20}	565.59	528.28	1419.15	-37.31	0.0660	2015378.46	2.56	1.94
m_{30}	0		0	0	0	0	0	
m_{40}	32752	31840.03	22443.29	-911.97	0.0278	504532955.3	2.59	1.82
m_{11}	6156.92	6153.93	304.54	-2.99	0.000486	92751.086	1.83	0.147
m_{21}	0		0	0	0	0	0	
m_{31}	2105.72	2108.66	401.53	2.94	0.00140	161234.81	1.79	1.57
m_{41}	28718.62	28660.05	7602.59	-58.57	0.00204	57802805.15	1.81	0.582
m_{12}	0		0	0	0	0	0	
m_{22}	7264.12	7282.38	820.92	18.26	0.00251	674247.21	2.28	0.644
m_{32}	5498.34	5507.43	636.47	9.09	0.00165	405175.89	2.30	1.119
m_{42}	61020	61346.68	14285.04	326.68	0.00535	204169087.6	2.28	0.799

and Zarkos (1999). Instead of drawing a vector $(\tilde{\varepsilon}_{1h},\ldots,\tilde{\varepsilon}_{Jh})$ as in step 2 above, we construct a set of vectors

$$t_h = \left(\frac{\hat{\varepsilon}_{jh}^{FGLS} - \bar{\varepsilon}_j}{\sqrt{\frac{1}{H}\sum_{l=1}^{H}\left(\hat{\varepsilon}_{jl}^{FGLS} - \bar{\varepsilon}_j\right)^2}}\right)_{1\leq j\leq J}, \text{ where } \bar{\varepsilon}_j = \frac{1}{H}\sum_h \hat{\varepsilon}_{jh}^{FGLS},$$

and then for each household draw one t_h^* with replacement from this set of vectors. In step 3, we construct new expenditure variables

$$y_{jh}^* = z_h' \hat{a}_{jh}^{FGLS} + \hat{\beta}_j^{FGLS} y_h + t_{jh}^* \hat{\varepsilon}_{jh}^{FGLS}$$

and then follow the same procedure as above. This procedure is less efficient if (4.19) is the true model, but it is robust to deviations from this specification. It is seen that although the average estimate deviates slightly from the estimated κ, we cannot reject the hypothesis of different parameters on any conventional level of significance. Out of 10000 replications, 35% resulted in a κ less than the one estimated by FGLS above. On the other hand, the bootstrapped standard error is twice the theoretical standard error estimated above in the bootstrap using parametric heteroskedasticity, and even higher in the wild bootstrap. Hence the estimator of the standard error may be inconsistent. Nonetheless, Table 4.7, which reports the results from a bootstrap similar to the one described above, but with different assumptions on the true value of κ, shows that κ is significantly different from zero at any conventional level of confidence. It is interesting to notice that it is also significantly lower than two, which corresponds to an identically distributed error term on a budget share equation.

Table 4.6: Descriptive statistics of κ.

	Mean	Std. err.	Skewness	Kurtosis	True value	Est. SE
Model (4.19)	1.31	.102	-.0477	-.119	1.32	.0524
Wild bootstrap	1.37	.137	-.221	.0200		

Table 4.7: Bootstrapped distribution of the estimated $\hat{\kappa}$ for different true values of κ

	Quantile								
True value of κ	0%	1%	5%	25%	50%	75%	95%	99%	100%
$\kappa = 0$	-.311	-.185	-.133	-.0526	.000945	.0569	.139	.190	.320
$\kappa = 1$	.594	.794	.848	.930	.990	1.05	1.13	1.19	1.34
$\kappa = 1.32$	.873	1.07	1.13	1.24	1.31	1.38	1.47	1.53	1.72
$\kappa = 2$	1.27	1.51	1.63	1.80	1.91	2.02	2.14	2.20	2.34

There are still at least two difficulties. First, the linear expenditure system may be a correct description of behaviour, but heteroskedasticity is in reality of another form than (4.19). Since the bootstrap was based on (4.19) being the true model, the results from the bootstrap do no longer necessarily contain any information. Still, if this is a good approximation to the true model, the FGLS procedure described above will probably remain superior to OLS.

Second, LES may only be an approximation to the true economic behaviour[8]. In this case, weighting the variables with $y_h^{\kappa/2}$ may change the whole approximation so that we are estimating another approximation to the true model. In this case, the OLS and FGLS estimates are not comparable. Figure 4.3 illustrates the use of OLS and FGLS in a univariate non-linear model with heteroskedasticity. Since FGLS puts more weight on the lower observations, this regression line is steeper than the OLS regression line. In this case the OLS line seems to give a better approximation to the true model, and would be preferred to FGLS. Although the example

[8]This problem is also discussed by Aasness and Nyquist (1983).

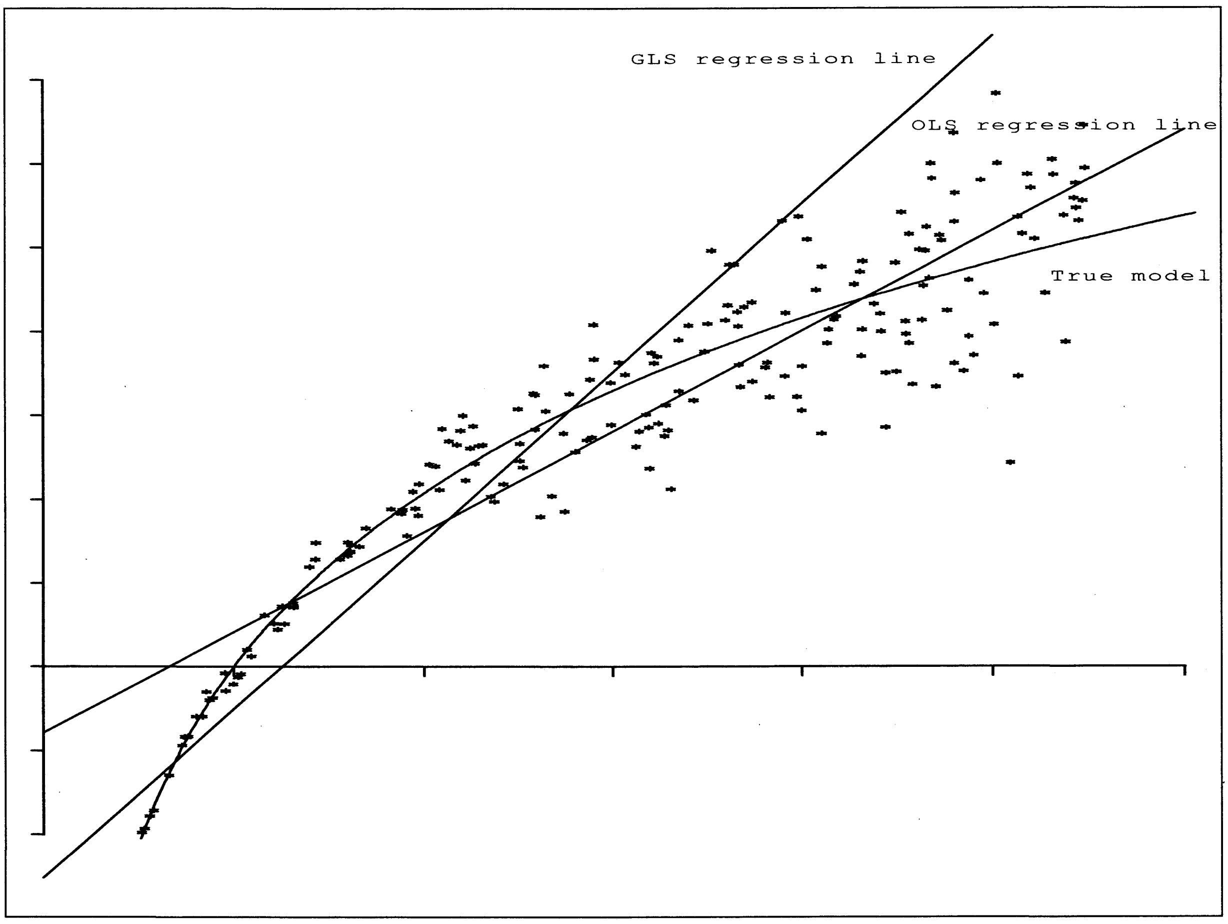

Figure 4.3: Example of OLS and FGLS in a mis-specified model.

depicted in Figure 4.3 is extremely stylized, the situation may be somewhat similar in the estimation of demand functions. If the true demand functions are non-linear functions of the regressors, every linear model is a linear approximation of the model. The weighted and the unweighted sample will lead to different approximations. Nevertheless, it is difficult to judge which of these approximations is the "best". OLS will probably give an approach that is more representative for the average of the data. WLS, on the other hand, will put relatively more weight on the poor than the rich.

To see the effect of different κ's on the estimates, we shall end this chapter by doing a sensitivity analysis. The tables in Appendix B.1 report estimation of (4.19) with a number of different κ's. It is seen that the value of κ has an important influence on the estimates. An interesting feature is that for a large number of goods, the "necessary" expenditure seems to be decreasing in κ. This could be interpreted as poor having lower levels of necessary consumption, but this is obviously far away from the LES model used in the estimation. The high sensitivity of the parameters on κ may give some support to the hypothesis that we are estimating different slopes of a non-linear curve, and may hence raise some doubts about the estimates in Table 4.3. Nevertheless, if the true demand system is non-linear, it is rather unclear what we should mean by a "good" linear approximation, so it is difficult to give any clear conclusion as to the validity of both the OLS and the FGLS estimates.

Chapter 5

Data problems

Apart from a heteroskedastic error term, it was assumed in Chapter 4 that the regressors and the error term behaved "nicely". In this chapter, we are going to take a closer look at some of the assumptions necessary for the validity of OLS and GLS. Since we are looking at a single cross-section, the list of possible difficulties is somewhat limited. Most important are probably the problems of measurement error and outliers. Furthermore, an attempt is made to include some possible omitted variables. Finally, a test on the restrictions that were made on the m's in above is presented.

5.1 Measurement error

In the analysis in Chapter 4, it was assumed that the stochastic error was uncorrelated with the regressors. This is a rather strong assumption, particularly for the relationship between the errors and total consumer expenditure. There are two ways of presenting the main problem. First, it may be seen as a simultaneous equations-problem (Deaton 1986). Consider the following system of equations:

$$\begin{aligned} y_{1h} &= m_1' z_h + \beta_1 \left(y_h - \textstyle\sum_l m_l' z_h \right) + \varepsilon_{1h} \\ &\vdots \\ y_{Jh} &= m_J' z_h + \beta_J \left(y_h - \textstyle\sum_l m_l' z_h \right) + \varepsilon_{Jh} \\ y_h &= y_{1h} + \ldots + y_{Jh} \end{aligned} \tag{5.1}$$

The consumer expenditure on every good is a function of total consumer expenditure, but by definition total consumer expenditure is also the sum of the expenditure on every good. Consequently, y_h is correlated with the ε_{jh}'s. A remedy for this is to use instrumental variables to estimate the system.

The problem may also be seen as a measurement error problem. Among others Aasness (1990, Essay 5) and Aasness et al. (1993) finds strong evidence supporting errors in measurement of total consumer expenditure[1]. The main problem is that a fraction of consumption, such as most durables, is purchased in "lumps". If a household purchases an expensive piece of clothing set during the bookkeeping period, this will lead to an artificially high figure for consumer expenditure on clothing, and hence for total consumer expenditure for the household in this period. Consequently, total consumer expenditure should be treated as a latent variable, and observed total consumer expenditure over a limited period of time does not necessarily equal the true expenditure. It is well-known that in the presence of measurement error, OLS, and hence

[1] There are studies that reject the presence of measurement error though. Murthi (1994) argues (without the use of any formal tests) that in his sample of Sri Lankan data, the use of instrumental variables do not change the data sufficiently to support the hypothesis of measurement error.

Table 5.1: Estimation of demand system by 2SLS. Standard errors in parenthesis.

	Estimates					Implied values		
Group	a_{0j}	a_{1j}	a_{2j}	β_j	R^2	m_{0j}	m_{1j}	m_{2j}
Child goods	612.97	6273.33	-4039.90	.0351	.31	5238.22	7428.36	0
	(1257.74)	(394.86)	(577.18)	(.00482)		(2786.05)	(515.21)	(0)
Adult goods	-1659.16	-1252.38	5041.65	.0381	.15	3355.96	0	9422.07
	(1606.59)	(504.38)	(737.27)	(.00615)		(2997.53)	(0)	(781.06)
Neutral goods	-2855.05	1298.43	5231.90	.0217	.11	0	2011.40	7725.63
	(1685.77)	(529.24)	(773.61)	(.00646)		(0)	(510.05)	(718.70)
Other goods	3901.25	-6319.38	-6233.65	.905	.91	123169.68	23464.50	97940.62
	(2677.39)	(840.55)	(1228.66)	(.0103)		(65394.28)	(10837.25)	(12784.43)
Sum	0	0	0	1		131763.86	32904.26	115088.31
						(70530.62)	(11437.64)	(13574.54)

SUR, is inconsistent[2]. It is evident that this is another dimension of the simultaneity-problem presented above, and instrumental variables is still a possible remedy.

To get consistent estimates, we need a vector of instruments that are correlated with the latent total consumer expenditure, but uncorrelated with the measurement error of the observed consumer expenditure. Two such instruments are net and gross income since households with higher income are likely to have a higher total consumer expenditure (see Statistics Norway (1996) for a definition of these variables). The demographic variables are treated as exogenous, and hence are included as instruments. Denote by w_h the vector of instruments, y_h^* the latent total consumer expenditure and by X and W the matrices of regressors and instruments.. We then have the model

$$\begin{aligned} y_{jh} &= a_j' z_h + \beta_j y_h^* + \varepsilon_{jh},\ 1 \le j \le J \\ y_h &= y_h^* + \nu_h \qquad (5.2) \\ E\varepsilon_h \varepsilon_h' &= \Sigma \text{ where } \varepsilon_h = (\varepsilon_{1h}, \dots \varepsilon_{Jh})' \\ E\nu_h w_h &= 0 \\ E\varepsilon_h w_h &= 0 \\ Ey_h^* w_h &\neq 0 \qquad (5.3) \end{aligned}$$

Estimating (5.2) using two step least squares (2SLS) on each equation separately, we get the results reported in Table 5.1.

To test whether there really are problems of measurement error, we can use a test developed by Durbin, Hausman and Wu (see eg. Godfrey (1988) or Hausman (1978)). This is actually not a test for measurement error, but will test whether there are errors in measurement that influences the parameter estimates. Under the null hypothesis of no measurement error, we have that for a single equation,

$$\sqrt{H}\left(\hat{\theta}^{OLS} - \hat{\theta}^{2SLS}\right) \xrightarrow{L} N(0, V). \qquad (5.4)$$

where

$$V = V^{2SLS} - V^{OLS}, \qquad (5.5)$$

is asymptotically (almost surely) positive definite. An estimator of V which is positive definite in finite samples as well is

$$\hat{V} = s^2 \left[\left(\frac{1}{H} X'W (W'W)^{-1} W'X \right)^{-1} - \left(\frac{1}{H} X'X \right)^{-1} \right], \qquad (5.6)$$

[2]It should be remarked that we do not have "classical" measurement error, i.e. we do not generally have $E\varepsilon_{jh}\nu_h = 0$. Nonetheless, unless $\operatorname{Var}\varepsilon_{jh} = \beta_j \operatorname{Var}\nu_h$, the estimates are still inconsistent (Biørn and Aasness 1989).

where s^2 is the usual estimator of the variance of the error using the 2SLS-estimates (Greene 1997, 443). (5.4) is then used to construct a test that under the null hypothesis has a χ^2-distribution with degrees of freedom equal to the number of variables possibly measured with error.

Using the Hausman-test on the estimates above, we get the results reported in Table 5.2. When considering each equation separately, it is seen that the parameter on child goods is significantly affected by measurement error. The effect on other goods is also significant at the 5% level, whereas the effect on adult and neutral goods is not significant at any conventional level of significance. Testing the effect on the parameters on child, adult, and neutral goods together (other goods is excluded since the adding up constraint would induce a singular covariance matrix), the estimates of the whole system is significantly different when taking measurement error into account. Consequently, when ignoring measurement errors, as was done in Chapter 4, we will probably get inconsistent estimates. Nevertheless, the actual change in the parameter estimates in not dramatic compared to the OLS estimates.

Table 5.2: Results from Hausman test of measurement error

	Statistic	Degrees of freedom	p-value
Child goods	18.34	1	.0000185
Adult goods	.869	1	.351
Neutral goods	.133	1	.716
Other goods	5.48	1	.0192
Complete system	18.92	3	.000285

It should be remarked that there are still strong signs of heteroskedasticity, particularly with regard to total consumer expenditure. Consequently, 2SLS is inefficient although it is consistent. It might be possible to construct asymptotically efficient estimators in this case, but this is outside the scope of the present work.

5.2 Outliers and attempts at robust estimation

Consumer expenditure surveys are quite likely to contain a few extreme observations that have a large influence on empirical moments, and hence on OLS estimates. As long as expected value of all the errors is zero, this does not lead to inconsistent estimates, but it will normally reduce the efficiency of OLS. Due to this, Aasness et al. (1993, Appendix A) argues in favour of *winsorizing* the sample. A $\alpha\%$ winsorization of a variable x consists of replacing x by x^* where

$$x^* = \begin{cases} x_{\alpha/2} & \text{if } x \leq x_{\alpha/2} \\ x & \text{if } x_{\alpha/2} < x < x_{1-\alpha/2} \\ x_{100-\alpha/2} & \text{if } x \geq x_{100-\alpha/2} \end{cases} \tag{5.7}$$

where $x_{\alpha/2}$ is the $\alpha/2\%$-percentile of x. In the present case, we shall use 2% winsorization, i.e. 1% of the sample is truncated in each tail. The percentiles are shown in Table 5.3. Since measurement error seems to have a significant impact on the estimates of the parameters, 2SLS was also used on the winsorized sample. The measures of income that are used as instruments were also winsorized, whereas the new total consumer expenditure was defined as the sum of the winsorized expenditures on the four groups of goods. The estimates from this transformed sample are presented in table 5.4.

There are difference between the estimates in Table 5.1 and Table 5.4, although most of the new estimates lie within a 95% confidence interval of the standard 2SLS estimates. Nonetheless, since this difference arises from changing the values on less than 30 observations, it should be relatively clear that there are some extreme observations that have a strong influence on the

Table 5.3: Percentiles used in winsorization

Variable	Percentile 1%	Percentile 99%
Expenditure on child goods	0	72047
Expenditure on adult goods	0	93946
Expenditure on neutral goods	1352	88075
Expenditure on other goods	37455	974663
Net income	19621	667488
Gross income	21525	956337

Table 5.4: Estimation of demand system by 2SLS using winsorized data. Standard errors in parenthesis.

Group	Estimates a_{0j}	a_{1j}	a_{2j}	β_j	R^2	Implied values m_{0j}	m_{1j}	m_{2j}
Child goods	698.43	6188.33	-3823.84	.0335	.33	4468.97	7370.86	0
	(1168.36)	(357.53)	(517.69)	(.00420)		(1896.00)	(417.88)	(0)
Adult goods	-2362.10	-1699.65	4062.41	.0481	.18	3057.30	0	9558.04
	(1513.82)	(463.24)	(670.76)	(.00545)		(2532.76)	(0)	(842.95)
Neutral goods	-2646.68	1398.66	4549.95	.0235	.18	0	2228.72	7234.04
	(1189.34)	(363.95)	(526.99)	(.00428)		(0)	(368.31)	(556.16)
Other goods	4310.35	-5887.35	-4788.52	.895	.94	105024	25698.71	97348.65
	(2307.05)	(705.98)	(1022.23)	(.0083)		(42259.62)	(8057.76)	(12313.67)
Sum	0	0	0	1		112550.28	35298.29	114141.11
						(45923.47)	(8500.02)	(13233.34)

estimates. Whether the results in Table 5.4 are better than those presented above is on the other hand an open question. It might be that the data from the extreme observations are extremely valuable since they give information about behaviour for values far from the average. On the other hand, these extreme observations may also arise from pure measurement error or particular circumstances that do not conform to the theory, so that it would be better to ignore the observations or reducing their influence.

The existence of a relatively large number of influential observations is also found by other techniques. Belsley et al. (1980) call observations with a hat value[3] above $2p/H$, where p is the number of regressors, a *leverage point*. In the present sample, we get 93 such leverage point, which according to Belsley et al. should be studied carefully each one. Another measure of the influence of an observation is the standardized difference of fitted values DFITS (see Belsley et al. (1980) or Staudte and Sheather (1990) for details). Staudte and Sheather (1990) argues that observations with a DFITS above $1.5 \cdot \sqrt{\frac{p+1}{H-p-1}}$ should be studied. In the present sample, there are 77, 78, 50 and 64 such observations in the four OLS regressions respectively. With this number of influential data points, it is impossible to go into detail on everyone. Instead of using least squares techniques as above, techniques that are more robust to outliers, such as least absolute deviation (LAD), may be appropriate. The use of OLS on the winsorized sample presented above may be seen as an attempt at robust estimation, although to the best of my knowledge, the statistical and robustness properties of this estimator are not established.

5.3 Further explanatory variables

Until this point, the demand for different goods has been assumed to be a function of a household's income and demographic composition only. This is unproblematic if other explanatory variables are orthogonal to these variables, but this is unlikely. Education and social class was

[3]The hat values are the diagonal elements of the *hat matrix* $X\left(X'X\right)^{-1}X'$ where X is the matrix of regressors, and gives information on how much that particular observation influeneces the value of the predicted values.

discussed in Chapter 4, and will not be considered here, mostly because of lack of data. Instead, it will be focused on region and area of residence, socio-economic status, book-keeping period and some other characteristics of the household and the main income earner. The estimate of the m_{0j}'s is based on the estimate of the intercept, so the m's are estimated for average values of the control variables. That is, for a set of control variables X, we define the average values $\bar{x} = \frac{1}{H}\iota' X$, and substitute a_{0j} by $a_{0j} + \bar{x}\theta_j$ where θ_j is the OLS estimates for the parameters of the variables X. By this procedure, we get estimates for the m's that are reported in Table 5.5. Full details of all the variables and the regression parameters are given in Appendix B.2.

Table 5.5: Estimates of the m's calculated at the average of the sample. Standard errors in parenthesis.

	Estimates					Implied values		
Group	$a_{0j} - \bar{x}\theta_j$	a_{1j}	a_{2j}	β_j	R^2	m_{0j}	m_{1j}	m_{2j}
Child goods	3327.67	6748.85	-2280.56	.0133	.39	6980.12	7524.03	0
	(1408.92)	(374.35)	(550.51)	(.00198)		(1820.72)	(443.31)	(0)
Adult goods	2002.54	-1646.07	5012.69	.0283	.27	9758.39	0	9855.38
	(1896.84)	(503.99)	(741.15)	(.00266)		(2976.03)	(0)	(1500.00)
Neutral goods	-6084.06	1416.36	6534.59	.0222	.21	0	2707.62	10333.42
	(1990.72)	(528.94)	(777.83)	(.0028)		(0)	(645.83)	(1336.93)
Other goods	753.85	-6519.14	-9266.72	.936	.98	257404.44	47951.34	150983.53
	(3119.58)	(828.88)	(1218.91)	(.00438)		(82943.43)	(17288.88)	(42656.30)
Sum	0	0	0	1		274842.95	58182.99	171172.32
						(86475.89)	(17881.23)	(44940.40)

The estimated cost of living is rather high, and the standard errors are also quite high, particularly on the "cost of running a household" m_0. Some of this may be due to the heteroskedasticity problem discussed in Section 4.5. Using the same technique as above, we get an estimate of the degree of heteroskedasticity $\hat{\kappa} = 1.05$ (.054). We could construct a FGLS estimator, which probably would give better estimates, but this is outside the scope of the present work. Remark also that for some policy questions, the gross coefficients estimated in Chapter 4 may be more interesting than the net coefficients presented here.

Chapter 6

Estimation of equivalence scales

In Chapter 2 and 3 we discussed to what extent it was possible to estimate equivalence scales, mainly using expenditure data. Although it was seen that this required rather strong assumptions, we shall now proceed by trying to estimate equivalence scales using the empirical results from Chapters 4 and 5.

6.1 Deriving equivalence scales from the LES

If we know a CCC utility function for every individual in a household and knows the exact structure of the BSWF, we can derive a household cost function and hence equivalence scales from equation (3.4). Unfortunately, it is generally not possible to derive these functions from knowledge of demand behaviour. We can then either make general assumptions on the utility function that makes it possible to identify it, such as the assumption of a money metric utility function, or we can assume a specific functional form. Neither approach is satisfactory, but we are going to assume that every agent has a utility function of the form (4.2), for which we derived estimates of each parameter in the preceding chapters. We can then derive an estimate of household equivalence scales. Although this assumption is virtually impossible to justify, the structure is convenient since the equivalence scales will be the same for every concave AG AN welfare function. This is because when every individual has got her "necessary consumption", the marginal utility of money is identical for every individual. This also makes it possible to partially escape from the Pangloss problem of Chapter 3.

To find the utility of a household with composition z and income y, we substitute the demand functions 4.4 into the utility function 4.2. This yields

$$u^i = \prod_{j=1}^{J} \left(\frac{\gamma_j^p}{\gamma + \gamma^p} \frac{y^\dagger}{p_j} \right)^{\gamma_j^p} \times \prod_{j=1}^{J} \left(\frac{\gamma_j}{\gamma + \gamma^p} \frac{y^\dagger}{N p_j} \right)^{\gamma_j} \tag{6.1}$$

where $y^\dagger = y - p' \left(\mu^p + \sum_k z_k \mu_k \right)$ is the income net of expenditures on necessary consumption. Rearranging, we get

$$\begin{aligned} u^i &= \Gamma N^{-\gamma} \left(\frac{y^\dagger}{\gamma + \gamma^p} \right)^{\gamma + \gamma^p} && (6.2) \\ \text{where } \Gamma &= \prod_{j=1}^{J} \left(\frac{\gamma_j^p}{p_j} \right)^{\gamma_j^p} \left(\frac{\gamma_j}{p_j} \right)^{\gamma_j} . \end{aligned}$$

Assume now that the marginal household welfare of increased utility to every household member is decreasing. A sufficient (but by no means necessary) condition is that $\gamma + \gamma^p \leq 1$ and the BSWF is concave. Using the indirect utility function (6.2) in an AG welfare function, we get the indirect welfare function

$$V^*(p,y,z) = \Gamma N^{-\gamma} \left(\frac{y^\dagger}{\gamma + \gamma^p} \right)^{\gamma + \gamma^p}, \tag{6.3}$$

since every household member will get exactly the same utility level.

Solving (6.3) for y, we get an expression for the household cost function:

$$C^*(p,\mathcal{W},z) = p' \left(\mu^p + \sum_k z_k \mu_k \right) + \left(\frac{\mathcal{W}}{\Gamma} \right)^{\frac{1}{\gamma+\gamma^p}} N^{\frac{\gamma}{\gamma+\gamma^p}}. \tag{6.4}$$

Measuring the base level of welfare in money for a reference household, we get the equivalence scales

$$L(p,y,z,z_0) = \frac{\zeta_z + \frac{y-\zeta_{z_0}}{\gamma+\gamma^p} \left(\frac{N}{N_0} \right)^{\frac{\gamma}{\gamma+\gamma^p}}}{\zeta_{z_0} + \frac{y-\zeta_{z_0}}{\gamma+\gamma^p}} \tag{6.5}$$

where $\zeta_z = p' \left(\mu^p + \sum_k z_k \mu_k \right)$ is the cost of giving a household with composition z their necessary consumption. There are two big difficulties with this expression. Ideally, we would like to estimate a set of regression equations of the form

$$y_{jh} = m_j' z_h + \left(\frac{\gamma_j}{\gamma} + \frac{\gamma_j^p}{\gamma^p} \right) \left(y_h - \sum_{l=1}^{J} m_l' z_h \right). \tag{6.6}$$

It is clear that it is not possible to identify γ_j and γ_j^p separately, nor to estimate the sums γ or γ^p. Equivalence scales will depend on $\gamma + \gamma^p$, which is a parameter determining the curvature of the utility of money-function. According to the "classical economists", the marginal utility of money was falling (Cooter and Rappoport 1984), which corresponds to $\gamma + \gamma^p < 1$. From demand data, it is not possible to verify this, though. Furthermore, $\frac{\gamma}{\gamma+\gamma^p} \equiv \alpha$, which gives an expression for the proportion of utility that is derived from the consumption of private goods, is necessary for an estimate of equivalence scales. One interesting case is $\alpha = 1$, which corresponds to a situation where public goods are only consumed as necessary goods, and all consumption above the minimum level is private.

It is seen that

$$L(p,\zeta_{z_0},z,z_0) = \frac{\zeta_z}{\zeta_{z_0}}, \tag{6.7}$$

so for a household welfare equal nought, the equivalence scales correspond to a class of equivalence scales similar in structure to the OECD scale , where there is a constant cost of every household member and also a fixed cost of "running a household". On the other hand,

$$\lim_{y \to \infty} L(p,y,z,z_0) = \left(\frac{N}{N_0} \right)^{\alpha}, \tag{6.8}$$

so when the income, and hence welfare, approaches infinity, the equivalence scales converge to the *constant elasticity of family size* scale favoured by Buhmann et al. (1988) and NOU (1996:13). This means that we have a foundation for two widely used equivalence scales based on consumer theory. Unfortunately, we are not able to estimate the family size elasticity of need from demand data, so this value has to be chosen discretionarily.

The utility of money-parameter $\gamma + \gamma^p$ describes the shape of the equivalence scales between the two limits described above. The higher is $\gamma + \gamma^p$, the slower is the convergence towards $(N/N_0)^{\alpha}$.

6.2 Estimates and discussion

Using the estimates from Chapter 4 and 5 and expression (6.5), we can derive equivalence scales for different values of α, $\gamma + \gamma^p$ and different household compositions. In this analysis, we shall restrict attention to a reference household consisting of a single adult.

Consider first the case $\gamma + \gamma^p = 1$. Equation (6.5) now simplifies to

$$L(p, y, z, z_0) = N^\alpha + \frac{m'(z - N^\alpha z_0)}{y}. \tag{6.9}$$

Using the FGLS estimates from Table 4.3, Figure 6.1 shows the equivalence scales for some household compositions with no returns to scale ($\alpha = 0$). Similar estimates are presented in Figure 6.2 for households with some returns to scale ($\alpha = 0.75$). To illustrate the effect of decreasing marginal utility of money, Figure 6.3 depicts equivalence scales for different households where there are no returns to scale ($\alpha = 1$), but where $\gamma + \gamma^p = 0.5$. It is seen that the curves approaches the asymptotic level faster, but the shapes are relatively similar.

It is clear that for these estimates to be useful for practical policy making, it is necessary to choose an income level for the reference household, such as a poverty line. One approach is to assume that the poverty line is welfare equaling nought. One difficulty with this approach is that the utility function is not defined for households below the poverty line. Still, this may be a useful approach. Using this base welfare level, we can construct equivalence scales of the form $1 + s_1 \times$ (number of children) $+ s_2 \times$ (number of adults-1). Using different estimates from Chapter 4 and 5, we get estimates that are reproduced in Table 6.1. The estimates from FGLS are relatively similar to those in the widely used OECD and "modified OECD" scales[1]. The difference between OLS and 2SLS is not very important, and both seems to give very low estimates of the cost of children. Still, they are both relatively close to the modified OECD scale.

It should be emphasized that the results in Table 6.1 and Figures 6.1 to 6.3 are expressed in 1994 prices. Equivalence scales with different prices will generally be different. This is because the m's used in the calculation are defined as $m_{ij} = p_j \mu_{ij}$. With the knowledge of the 1994 prices and the new prices, a calculation of the new equivalence scales is trivial.

Hence it is seen that if we impose enough restrictions on preferences, we are able to estimate seemingly reasonable equivalence scales. However, the validity of these scales will obviously depend upon the validity of the assumptions.

[1] The modified OECD scale appears in e.g. Eurostat (1997).

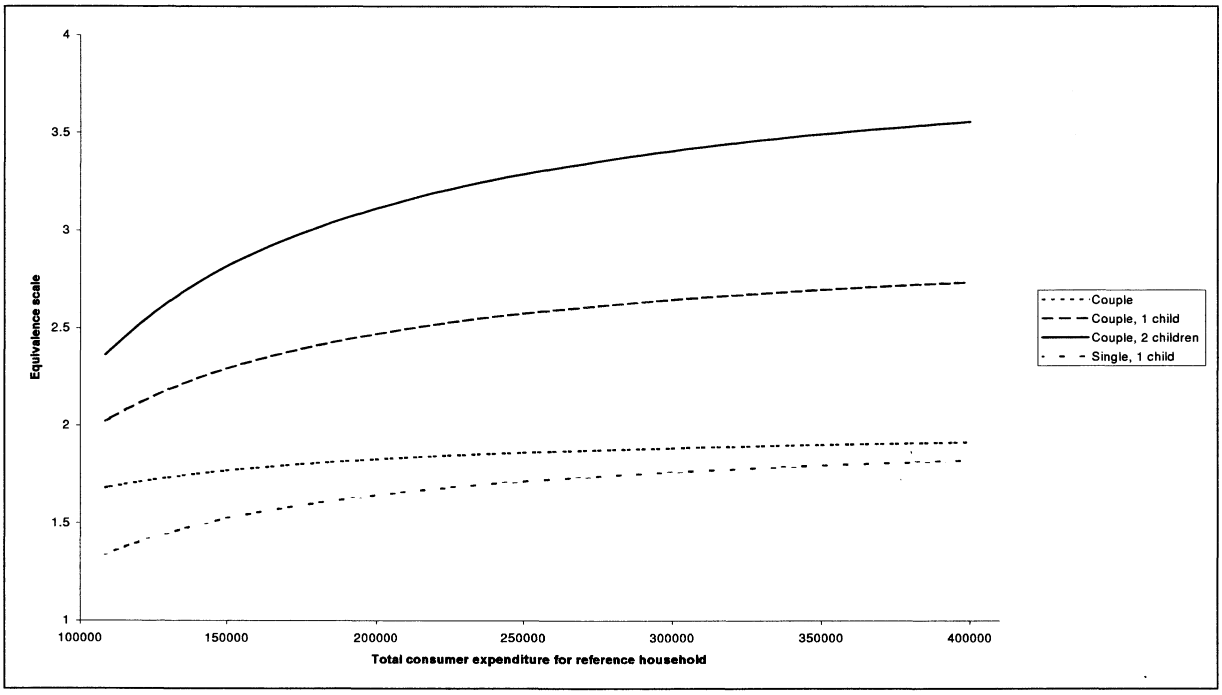

Figure 6.1: Equivalence scales for some household compositions with $\alpha = 1$ and $\gamma + \gamma^p = 1$.

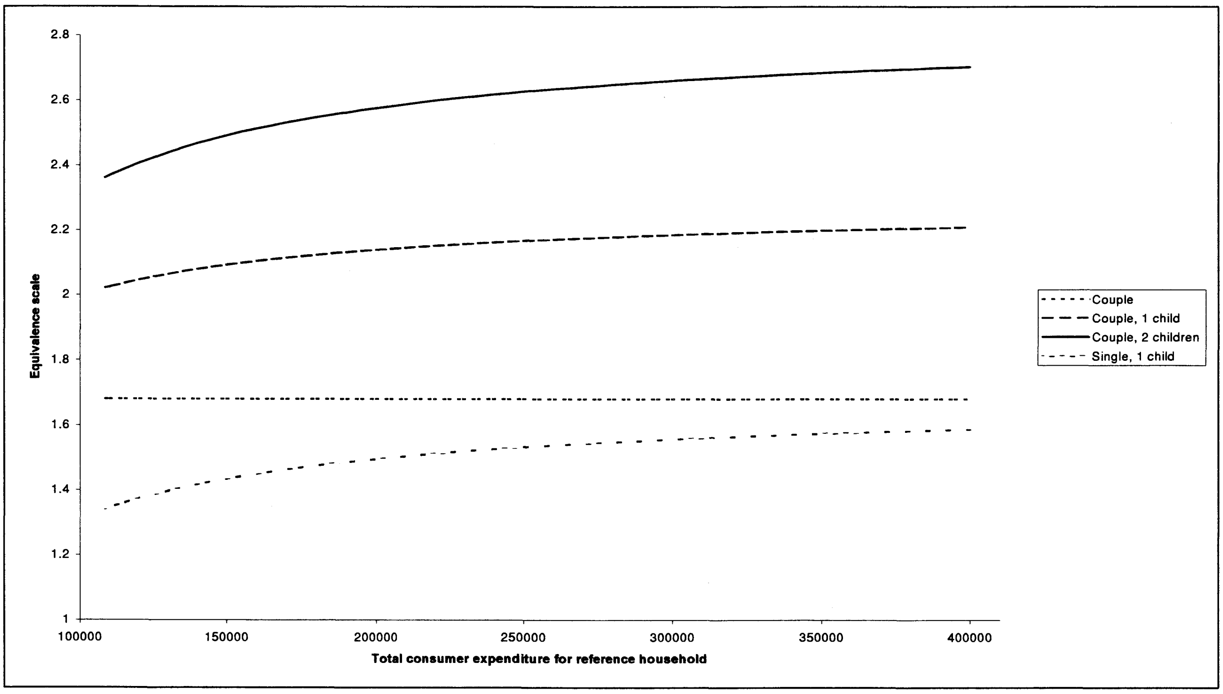

Figure 6.2: Equivalence scales for some household compositions with $\alpha = 0.75$ and $\gamma + \gamma^p = 1$.

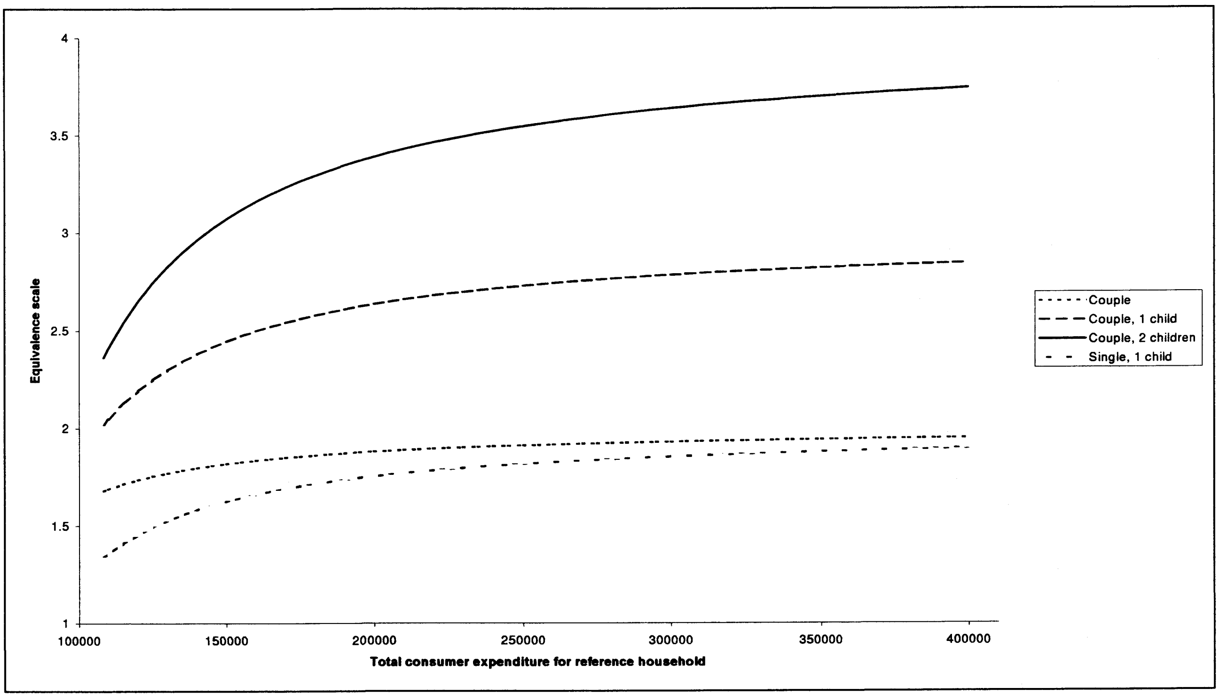

Figure 6.3: Equivalence scales for some household compositions with no returns to scale and decreasing marginal utility of money ($\gamma + \gamma^p = 0.5$).

Table 6.1: Some estimates of equivalence scales based on estimated necessary consumption (Asymptotic standard errors in parenthesis, asymptotic p-values in brackets)

	Estimation technique				
	OLS	FGLS	2SLS	OECD	Modified OECD
m_0	129963.48	34671.22	131763.28		
	(63905.27)	(23596.66)	(70530.62)		
m_1	31360.48	36981.27	32904.26		
	(13159.12)	(7977.37)	(11437.64)		
m_2	159525.29	73782.47	115088.31		
	(27849.01)	(15097.69)	(13574.54)		
s_1	0.11	0.34	0.13	0.50	0.30
	(.0525)	(.113)	(.0607)		
s_2	0.55	0.68	0.47	0.70	0.50
	(.127)	(.152)	(.135)		
Couple	1.55	1.68	1.47	1.70	1.50
	(.127)	(.152)	(.135)		
Couple, 1 child	1.66	2.02	1.60	2.20	1.80
	(.160)	(.247)	(.180)		
Couple, 2 children	1.77	2.36	1.73	2.70	2.10
	(.202)	(.353)	(.233)		
Single, 1 child	1.11	1.34	1.13	1.50	1.30
	(.0525)	(.113)	(.0607)		
Wald test against OECD scale	73.53	4.21	62.62		
	[$1.08 \cdot 10^{-16}$]	[.122]	[$2.53 \cdot 10^{-14}$]		
Wald test against modified OECD	14.45	.280	12.94		
	[.000162]	[.869]	[.00155]		

Chapter 7

Recapitulation

To make inter-household comparisons of welfare based on household income or expenditure, we can use equivalence scales to transform incomes to comparable magnitudes. Although scales constructed from calculations of costs of covering needs may serve, it is tempting to use demand data to try to make estimates based on actual behaviour. Nevertheless, it was shown that without further restrictions on preferences, every set of demand behaviour is compatible with almost any equivalence scale. This is mainly because demand data makes it possible to identify a set of indifference curves, but not to compare the welfare level between the indifference curves of different agents. Consequently, if we want to use demand data to estimate equivalence scales, we have to add further assumptions that makes it possible to identify the welfare level associated with a given consumption vector. Some assumptions that have been used to identify equivalence scales in the literature were presented, but none of them seems to be satisfactory.

Conventional approaches to estimating equivalence scales may also be criticized for ignoring how consumption is distributed within the household. The household's consumption decision depends on the tastes of the household members as well as the distribution of power within the household. Estimates based on a household utility function then implicitly take the intra-household distribution as the optimal distribution. This was called the Pangloss problem of equivalence scales. It was seen that there are mainly two solutions to the problem. One solution is to postulate a welfare function based on some judgements of equity and distribution which corresponds to the social planners value judgements. This approach was not pursued further, but it is an interesting topic for future studies. It was also shown that for some specifications of individual preferences, the welfare function doesn't influence the equivalence scales. The other approach is to assume that individual preferences belong to this class of utility functions. Then it is possible to estimate equivalence scales from the knowledge of the individual utility functions alone.

A considerable amount of space was devoted to the estimation of a linear expenditure system. First it was shown how this demand system might be seen as the result of households maximizing a BSWF. A well-known problem with the LES is that in cross-sections, the parameters determining necessary consumption are unidentifiable from demand data alone. Constructing groups of goods for which it was assumed that there would not be any necessary consumption solved this. We could then proceed by estimating the parameters of the model by ordinary least squares. Nevertheless, there were signs of strong heteroskedasticity, so a feasible generalized least squares estimator was proposed. Some bootstrap analyses indicate that this estimator probably performs well.

Previous studies have found signs of measurement error in total consumer expenditure that had a significant influence on the estimated parameters. A new estimate based on instrumental variables was suggested. Most of the new estimates were quite similar to the OLS estimates, but a Hausman test revealed that there were significant changes. A large number of outliers were also found. No real attempts of robust estimation were made, but this is an important area for future research. Finally, a regression which included a number of possible omitted variables was presented, but these estimates did not differ much from the original OLS estimates.

The estimates were subsequently used to calculate equivalence scales. To identify the scales, it was assumed that every household member had utilities that could be described by a Stone-Geary utility function. Since this functional form yields parallel linear Engel curves, the particular functional form for the welfare function does not matter as long as it satisfies some relatively general conditions, so the Pangloss criticism does not apply. It was seen that for low levels of utility, most of the estimated scales were relatively close to the "modified OECD scale", although the estimated scales are not constant for different utility levels. Particularly, the equivalence scales tend toward a constant elasticity of household size scale when utility tends to infinity. This elasticity depends upon the share of private goods in the production of utility. Unfortunately, I was not able to estimate it from demand data. Nevertheless, the finding is useful since it may give an economic interpretation to this parameter.

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Appendix A

Proofs of propositions

A.1 Proof of Proposition 1

To prove the existence of such a cost function, let $\tilde{C}$ be any cost function that rationalizes D, and denote by $\tilde{V}$ the associated indirect utility function. Then $\tilde{C}\left[p, \tilde{V}(p,y,z), z\right] = y$ for all $(p,y,z) \in \mathbb{R}_+^J \times \mathbb{R} \times \mathcal{Z}$. Define $C(p,\mathcal{U},z) = \tilde{C}\left[p, \tilde{V}\left(p^0, g(z,z_0,\mathcal{U})\,\varrho(\mathcal{U}), z\right), z\right]$. Then C satisfies $C\left(p^0,\mathcal{U},z\right) = g(z,z_0,\mathcal{U})\,\varrho(\mathcal{U})$, so $\frac{C(p^0,\mathcal{U},x)}{C(p^0,\mathcal{U},z^0)} = \frac{g(z,z_0,\mathcal{U})\varrho(\mathcal{U})}{g(z_0,z_0,\mathcal{U})\varrho(\mathcal{U})} = g(z,z_0,\mathcal{U})$. Furthermore, since $g(z,z_0,\mathcal{U})\,\varrho(\mathcal{U})$ is strictly increasing in $\mathcal{U}$, C and $\tilde{C}$ represent the same preferences, so C rationalizes D as well.

To see the uniqueness of C, take another $\hat{C} \neq \tilde{C}$ such that also $\hat{C}$ rationalizes D. Then we can write $\tilde{C}(p, f(\mathcal{U},z), z) = \hat{C}(p,\mathcal{U},z)$ for some monotonic function f. Moreover, the indirect utility function associated with $\hat{C}$, $\hat{V}$, satisfies $f\left[\hat{V}(p,y,z), z\right] = \tilde{V}(p,y,z)$. Since f is monotonic in its first argument, it follows that $f^{-1}\left[\tilde{V}(p,y,z), z\right] = \hat{V}(p,y,z)$ where f^{-1} is the inverse of f with regard to the first argument. Constructing C from $\hat{C}$ instead of $\tilde{C}$, we get
$C(p,\mathcal{U},z) = \hat{C}\left[p, \hat{V}\left(p^0, g(z,z_0,\mathcal{U})\,\varrho(\mathcal{U}), z\right), z\right]$
$= \tilde{C}\left[p, f \circ f^{-1}\left(\tilde{V}\left(p^0, g(z,z_0,\mathcal{U})\,\varrho(\mathcal{U}), z\right), z\right), z\right]$, which is the same expression as above since $f \circ f^{-1}$ is the identity. ■

A.2 Proof of Lemma 7

Continuity follows from Mas-Colell et al. (1995, Theorem M.K.6) or Sydsæter et al. (1990, Setning 4.30). To show quasi-concavity, define for all $\bar{q} \in \mathcal{Q}$ the set $P(\bar{q}) = \left\{q \in \mathcal{Q} \,\middle|\, U^h(q) \geq U^h(\bar{q})\right\}$ where the z has been ignored for notational simplicity. We need to show that $P(\bar{q})$ is convex for all $\bar{q} \in \mathcal{Q}$. Let $q, q' \in P(\bar{q})$, and denote by $\{q_i\}$, $\{q_i'\}$ and $\{\bar{q}_i\}$ the optimal intra-household distributions of consumption associated with q, q' and $\bar{q}$. Then by definition, $U^h(q) \geq U^h(\bar{q})$ and $U^h(q') \geq U^h(\bar{q})$. Furthermore, quasi-concavity gives that for any $\lambda \in (0,1)$, $u^i(\lambda q_i + (1-\lambda)\,q_i') \geq \lambda u^i(q_i) + (1-\lambda)\,u^i(q_i')$. Then it follows that $U^h(\lambda q + (1-\lambda)\,q') \geq W\left(\left\{u^i(\lambda q_i + (1-\lambda)\,q_i')\right\}\right) \geq W\left(\left\{u^i(\lambda q_i) + (1-\lambda)\,u^i(q_i')\right\}\right)$, and by the quasi-concavity of W, $W\left(\left\{\lambda u^i(q_i) + (1-\lambda)\,u^i(q_i')\right\}\right) \geq \min\left[U^h(q), U^h(q')\right] \geq U^h(\bar{q})$, so $\lambda q + (1-\lambda)\,q' \in P(\bar{q})$. Then $P(\bar{q})$ is convex, so U is quasi-concave in q. ■

A.3 Proof of Lemma 8

Denote by $\{q^{1i}\}$ and $\{q^{2i}\}$ the intra-household allocations of goods generated by W^1 and W^2. Consider first the case $\{q^{1i}\} = \{q^{2i}\}$. From the FOCs from welfare maximization, we have for two individuals i_1 and i_2 an some good j_1 and j_2

$$\begin{aligned} \frac{W^1_{i_1}}{W^1_{i_2}} \frac{u^{i_1}_{j_1}}{u^{i_2}_{j_2}} &= \frac{p_{j_1}}{p_{j_2}} = \frac{W^2_{i_1}}{W^2_{i_2}} \frac{u^{i_1}_{j_1}}{u^{i_2}_{j_2}} \\ &\Rightarrow \frac{W^1_{i_1}}{W^1_{i_2}} = \frac{W^2_{i_1}}{W^2_{i_2}}. \end{aligned} \tag{A.1}$$

This condition can only be true for all i_1 and i_2 if there is a function f such that $W^1 = f \circ W^2$.

Consider now the case $\{q^{1i}\} \neq \{q^{2i}\}$. Denote by D^i the individual demand function for individual i and by λ^i agent i's share of household income as defined above. Then household demand D satisfies

$$D(p, y) = \sum_i D^i \left(p, \lambda^i (p, y) y\right). \tag{A.2}$$

In this case, we will generally have different functions λ for W^1 and W^2. Since the functions D^i can take almost any shape, this equality will not hold unless the λs are equal. ∎

A.4 Proof of Lemma 9

Assume that a household with composition $z \in \mathcal{Z}$ has a welfare function W that satisfies AG and generates D given $\{u^i\}$. Since there is a unique solution to (3.7), Lemma 8 gives that any other welfare function generating D is a monotonic transformation of W. Assume further that there is a transformation $f : \mathbb{R} \times \mathcal{Z} \to \mathbb{R}$ such that $f(z) \circ W$ is a new welfare function for the household that is also AG and satisfies D. Then for any $u \in \mathbb{R}$, we have $W(u\iota) = u$ and $f(W(u\iota), z) = u$, so $f(u, z) = u$ for any $u, z \in \mathbb{R} \times \mathcal{Z}$. Consequently f has to be the identity with regard to u. ∎

A.5 Proof of Proposition 11

Since welfare functions are AG, they are the identity for a household with composition ι^k. The indifference map of an agent of type k is then obtainable from observation of a household with composition ι^k, and since utilities are money metric wrt. p_0, a unique CCC individual utility function is obtainable for every agent since $\iota^k \in \mathcal{Z}$ for every k. With the knowledge of every individual utility function, the uniqueness of equivalence scales follows from Corollary 10 ∎.

Appendix B

Additional estimation results

B.1 Sensitivity to the degree of heteroskedasticity

Sensitivity of the FGLS estimates to the value of κ (Standard errors in parenthesis)

	$\kappa = 0$	$\kappa = 0.5$	$\kappa = 1$	$\kappa = 1.32$	$\kappa = 1.5$	$\kappa = 2$
a_{10}	2739.48	1799.43	1046.22	658.20	479.35	92.41
	(1110.44)	(929.41)	(755.61)	(651.62)	(598.68)	(465.16)
a_{11}	7080.33	6473.68	5833.34	5416.14	5192.88	4596.84
	(333.44)	(310.76)	(288.66)	(274.78)	(267.38)	(247.05)
a_{12}	-2569.74	-2181.14	-1758.10	-1477.95	-1327.03	-921.06
	(445.86)	(403.82)	(354.38)	(319.26)	(299.51)	(243.24)
β_1	0.0161	0.0180	0.0194	0.0200	0.0202	.0202
	(0.00181)	(0.00193)	(0.00202)	(0.00206)	(0.00208)	(.00209)
a_{20}	-1067.93	-1159.16	-1050.00	-1012.68	-1028.46	-1252.24
	(1473.39)	(1229.46)	(1023.22)	(919.75)	(874.97)	(792.91)
a_{21}	-1028.01	-1332.75	-1577.9	-1683.43	-1717.07	-1683.27
	(442.42)	(411.09)	(390.90)	(387.85)	(390.77)	(421.13)
a_{22}	5450.40	4597.13	4009.62	3905.46	3971.31	4700.41
	(591.59)	(534.19)	(479.89)	(450.63)	(437.73)	(414.62)
β_2	0.0328	0.0394	0.044	0.0455	0.0452	.0389
	(0.00240)	(0.00255)	(0.00274)	(0.00291)	(0.00303)	(.00355)
a_{30}	-3097.61	-2438.20	-1645.83	-1129.47	-860.37	-178.69
	(1548.38)	(1211.67)	(927.37)	(775.09)	(702.69)	(535.87)
a_{31}	1206.38	1096.76	982.40	901.00	853.49	712.00
	(464.94)	(405.14)	(354.28)	(326.85)	(313.83)	(284.61)
a_{32}	5064.20	4298.01	3545.61	3094.76	2870.84	2363.37
	(621.70)	(526.46)	(434.93)	(379.76)	(351.54)	(280.21)
β_3	0.0238	0.0272	0.0305	0.0326	0.0336	.0359
	(0.00252)	(0.00251)	(0.00248)	(0.00245)	(0.00244)	(.00240)
a_{40}	1426.06	1797.93	1649.60	1483.95	1409.47	1338.52
	(2431.55)	(1961.64)	(1564.65)	(1357.24)	(1262.46)	(1065.76)
a_{41}	-7258.70	-6237.69	-5237.82	-4633.72	-4329.30	-3625.57
	(730.14)	(655.90)	(597.74)	(572.34)	(563.83)	(566.04)
a_{42}	-7944.86	-6714.01	-5797.13	-5522.28	-5515.11	-6142.72
	(976.31)	(852.32)	(733.82)	(664.98)	(631.58)	(557.29)
β_4	0.927	0.915	0.906	0.902	0.900	.905
	(0.00396)	(0.00407)	(0.00419)	(0.00429)	(0.00438)	(.00478)

Sensitivity of the estimates of the m's to the value of κ (Standard errors in parenthesis)

	$\kappa = 0$	$\kappa = 0.5$	$\kappa = 1$	$\kappa = 1.32$	$\kappa = 1.5$	$\kappa = 2$
m_{10}	4833.01	3409.80	2093.42	1352.70	996.96	192.65
	(1541.27)	(1248.47)	(979.57)	(825.30)	(749.24)	(567.12)
m_{20}	3192.35	2368.80	1337.41	565.59	128.47	-1058.84
	(2492.75)	(2081.97)	(1653.90)	(1394.16)	(1264.95)	(966.99)
m_{30}	0	0	0	0	0	0
	(0)	(0)	(0)	(0)	(0)	(0)
m_{40}	121938.12	83784.86	50492.75	32752.92	24459.95	5838.28
	(60814.97)	(41504)	(28205.30)	(22055.55)	(19383.54)	(14007.89)
m_{11}	7585.50	7082.03	6525.47	6156.92	5961.09	5469.30
	(383.10)	(347.38)	(317.34)	(302.13)	(296.03)	(296.82)
m_{21}	0	0	0	0	0	0
	(0)	(0)	(0)	(0)	(0)	(0)
m_{31}	1953.84	2017.83	2070.18	2105.72	2130.42	2267.31
	(539.43)	(468.75)	(416.62)	(397.20)	(393.49)	(432.05)
m_{41}	21821.13	24734.47	27044.27	28718.62	29881.39	35540.00
	(12674.76)	(9630.55)	(7974.37)	(7575.52)	(7614.68)	(9291.07)
m_{12}	0	0	0	0	0	0
	(0)	(0)	(0)	(0)	(0)	(0)
m_{22}	10679.72	9375.54	8017.79	7264.12	6937.41	6477.46
	(1075.77)	(995.00)	(890.33)	(807.10)	(756.33)	(596.18)
m_{32}	8866.41	7600.41	6308.75	5498.34	5076.63	4005.33
	(936.55)	(805.12)	(692.30)	(628.10)	(595.47)	(508.95)
m_{42}	139979.16	104332.3	76204.54	61020.01	53581.33	35204.76
	(26269.74)	(20311.76)	(16033.94)	(13908.71)	(12924.05)	(10712.80)
m_0	129963.48	89563.46	53923.58	34671.22	25585.38	4972.08
	(63905.27)	(44013.08)	(30116.84)	(23596.66)	(20733.28)	(14862.74)
m_1	31360.48	33834.33	35639.92	36981.27	37972.9	43276.61
	(13159.12)	(10054.85)	(8370.65)	(7977.37)	(8030.82)	(9826.10)
m_2	159525.29	121308.25	90531.09	73782.47	65595.37	45687.56
	(27849.01)	(21771.45)	(17342.48)	(15097.69)	(14039.02)	(11578.22)

B.2 Estimation of the demand system with additional explanatory variables

Variable	Child		Adult		Neutral		Other	
	Parameter	SE	Parameter	SE	Parameter	SE	Parameter	SE
Intercept	27703.00	5468.98	4207.14	7362.89	-7872.53	7727.32	-24038.00	12109.00
Number of children	6748.85	374.35	-1646.07	503.99	1416.37	528.94	-6519.14	828.88
Number of adults	-2280.56	550.51	5012.69	741.15	6534.59	777.83	-9266.72	1218.91
Total consumer expenditure	0.0133	0.00198	0.0283	0.00266	0.0222	0.00279	0.936	0.00438
Region								
Oslo and Akershus*	507.69	1543.55	2458.88	2078.08	-4665.77	2180.94	1699.20	3417.66
Rest of eastern Norway*	-25.56	1315.82	-656.97	1771.48	-3457.46	1859.16	4139.99	2913.41
Agder and Rogaland*	-88.63	1467.26	324.62	1975.37	-3714.47	2073.14	3478.48	3248.73
Western Norway*	-1391.39	1419.83	-346.29	1911.51	-2864.75	2006.12	4602.43	3143.71
Trøndelag*	2088.72	1651.63	-903.62	2223.58	-4550.01	2333.64	3364.91	3656.95
Type of residenc area								
Sparsely populated area*	-2279.27	1276.95	-2744.03	1719.16	-3841.13	1804.25	8864.42	2827.36
Densely populated area (200-1 999)*	-962.81	1363.78	-1147.50	1836.05	-5123.29	1926.93	7233.60	3019.61
Densely populated area (2 000-19 999)*	-1354.61	1227.29	-1445.31	1652.30	-1802.88	1734.08	4602.79	2717.41
Densely populated area (20 000-99 999)*	-1019.14	1411.67	-873.29	1900.53	-1419.98	1994.60	3312.41	3125.65
Unskilled worker*	1461.29	2817.74	-4962.29	3793.53	2959.34	3981.29	541.67	6238.91
Socio-economic status								
Skilled worker*	2163.42	2832.92	-1665.07	3813.96	853.41	4002.73	-1351.76	6272.51
Salaried, low level*	803.63	3024.14	-2507.11	4071.39	1225.98	4272.91	477.49	6695.89
Salaried, mean level*	3680.98	2731.84	-1790.67	3677.88	4938.91	3859.91	-6829.22	6048.70
Salaried, high level*	3537.47	2790.76	-1988.05	3757.20	7763.49	3943.16	-9312.91	6179.16
Farmer or fishermen*	-962.25	3262.33	-2145.85	4392.07	6352.98	4609.46	-3244.88	7223.29
Other self-employed*	-390.05	2972.33	-717.31	4001.65	5112.74	4199.71	-4005.38	6581.18
Student or pupil*	5411.06	3925.11	-328.84	5284.37	3989.40	5545.92	-9071.63	8690.78
Pensioner*	3099.36	2840.38	4675.56	3824.00	-3315.04	4013.27	-4459.88	6289.02
Homeworker*	3510.58	3457.96	-976.63	4655.45	18.27	4885.87	-2552.22	7656.44
Other household characteristics								
Age of main inc. earner	-1108.22	192.21	544.60	258.78	314.21	271.58	249.40	425.59
Age squared	10.25	2.01	-7.34	2.70	-2.32	2.83	-0.59	4.44
Female main inc. earner*	218.19	1007.48	-1273.53	1356.36	-1108.32	1423.50	2163.66	2230.70
No. economically active members	2362.82	690.94	2221.10	930.22	-3712.94	976.26	-870.97	1529.86
Book-keeping period								
Book-keeping period 1*	-831.80	2720.97	-10938.00	3663.24	-672.64	3844.56	12442.00	6024.64
Book-keeping period 2*	-3805.71	2692.45	-10152.00	3624.84	-1288.94	3804.25	15247.00	5961.48
Book-keeping period 3*	-2288.99	2712.85	-13496.00	3652.31	2949.92	3833.09	12835.00	6006.67
Book-keeping period 4*	-4470.59	2731.19	-11187.00	3677.00	543.01	3859.00	15115.00	6047.27
Book-keeping period 5*	-3280.39	2720.43	-14061.00	3662.51	1812.93	3843.79	15529.00	6023.43
Book-keeping period 6*	-1596.57	2718.24	-12469.00	3659.57	-1536.32	3840.70	15602.00	6018.59
Book-keeping period 7*	-3850.68	2800.92	-11489.00	3770.88	-264.46	3957.52	15604.00	6201.66
Book-keeping period 8*	23.75	2778.21	-14272.00	3740.31	2312.97	3925.43	11936.00	6151.38
Book-keeping period 9*	-4729.62	2790.54	-8573.69	3756.90	2922.49	3942.84	10381.00	6178.66
Book-keeping period 10*	-2209.13	2799.34	-10312.00	3768.75	2400.44	3955.28	10121.00	6198.16
Book-keeping period 11*	-1702.01	2736.09	-12677.00	3683.60	7571.18	3865.92	6808.20	6058.12
Book-keeping period 12*	496.44	2719.97	-6870.16	3661.90	1241.95	3843.15	5131.77	6022.43
Book-keeping period 13*	-2740.06	2761.04	-9895.45	3717.19	7823.52	3901.17	4811.99	6113.36
Book-keeping period 14*	-511.55	2854.42	-10350.00	3842.90	8216.67	4033.11	2644.82	6320.11
Book-keeping period 15*	-5314.67	2927.52	-9344.59	3941.32	7117.54	4136.40	7541.72	6481.97
Book-keeping period 16*	-1955.30	2876.20	-7701.92	3872.23	1505.97	4063.89	8151.25	6368.34
Book-keeping period 17*	-3635.38	2682.21	-12993.00	3611.05	-661.59	3789.78	17290.00	5938.81
Book-keeping period 18*	-3333.79	2603.89	-10215.00	3505.62	1950.87	3679.13	11598.00	5765.40
Book-keeping period 19*	-296.74	2716.08	-8843.37	3656.66	1969.07	3837.64	7171.04	6013.81
Book-keeping period 20*	311.10	2724.56	-8570.88	3668.07	-223.45	3849.62	8483.24	6032.57
Book-keeping period 21*	-2362.39	2704.61	-12086.00	3641.21	237.57	3821.44	14211.00	5988.41
Book-keeping period 22*	-568.00	2810.72	-13831.00	3784.07	6706.57	3971.36	7692.80	6223.35
Book-keeping period 23*	-134.07	2675.07	-10260.00	3601.44	278.32	3779.69	10116.00	5923.00
Book-keeping period 24*	-5312.29	2616.40	-12874.00	3522.46	1350.66	3696.80	16835.00	5793.10
Book-keeping period 25*	-4213.75	2770.39	-10202.00	3729.77	4381.74	3914.37	10034.00	6134.05

* denotes a dummy variable

The excluded dummy variables are Region Northern Norway, Residence in area with 100 000 residents or more, Other socio-economic status and Book-keeping period 26.

Appendix C

Classification of consumer goods

The classification is mainly based on discretion. The commodity numbers (vXXX) are documented in e.g. Wold (1996).

Child goods

	Child clothing and footwear
v294	Shirts, children
v300	Nightwear, children
v304	Dresses, blouses and tunics, girls
v312	Suits, boys
v314	Jackets and waistcoats, boys
v316	Slacks, boys
v320	Suits, pan suits, skirts and jackets, girls
v326	Slacks, girls
v328	Dungaree clothing, ski clothing etc., children
v337	Coats etc., children
v341	Outer wear of plastic, children
v347	Stockings and socks, children
v355	Underwear, cotton, children
v358	Other underwear, children
v359	Infants' garment
v363	Cardigans and sweaters of wool, children
v365	Other cardigans and sweaters, children
v412	Skiing boots and sporting shoes, children
v415	Other leather footwear, children
v423	Rubber footwear, children
v427	Other footwear, children
	Other child goods
v256	Prepared food for infants
v720	Play equipment
v806	Baby carriages
v901	Child-care, friends and relatives
v902	Child-care, maids and nannies
v903	Childminders
v904	Public childminders
v905	Kindergarten (6h or more a day)
v906	Kindergarten (less than 6h a day)
v907	Outdoor kindergartens
v908	After-school care out of scool
v909	After-school care at school

Adult goods

	Adult clothing and footwear
v293	Sports and work shirts, adult

v295	Other shirts of cotton, adult
v297	Shirts of other materials than cotton, adult
v299	Nightwear, adult
v303	Dresses, women
v305	Blouses and tunics, women
v311	Suits, men
v313	Jackets, men
v315	Slacks, men
v319	Suits and pant suits, women
v321	Shirts, women
v323	Jackets, women
v325	Slacks, women
v327	Ski clothing and parkas, adult
v329	Dungaree clothing, adult
v331	Smocks etc.
v336	Coats etc., men
v338	Coats etc., women
v340	Outer wear of plastic, adult
v345	Stockings and socks, women
v346	Stockings and socks, men
v353	Underwear, cotton, women
v354	Underwear,cotton, men
v356	Other underwear, women
v357	Other underwear, men
v362	Cardigans and sweater of wool, adult
v364	Other cardigans and sweaters, adult
v411	Skiing boots and sporting shoes, adult
v413	Other leather footwear, adult
v422	Rubber footwear, adult
v426	Other footwear, adult
	Other adult goods
v268	Light beer
v269	Lager, dark and light
v270	Strong beer
v272	Non-alcoholic wines
v273	Red wines
v274	White wines
v275	Port and sherry
v276	Other wines
v277	Aqua vitae
v278	Cognac and whisky
v279	Liquor
v280	Liqueur and punch
v283	Cigars and cheroots
v284	Cigarettes
v286	Smoking tobacco
v288	Chewing tobacco and snuff
v290	Cigarette paper
v707	Weapons and ammunition
v745	Lotteries and pools
v746	Bingo
v848	Expenses for burial places
v887	Union subscription

Neutral goods

	Neutral foodstuff
v001	Wheat flour
v003	Rye flour
v004	Other kinds of flour

v005	Oat meal
v006	Rice
v007	Other kinds of meal
v008	Health food, flour and meal
v010	Crispbread
v012	Unsweetened biscuits
v015	Dark rye bread
v016	Rye bread
v017	Brown bread
v019	White bread
v020	Other kinds of bread
v022	Health food, bread
v025	Pastry
v027	Other cakes
v028	Cream biscuits
v032	Cake biscuits
v034	Other kinds of bakery products
v038	Macaroni and spaghetti
v039	Puffed rice and cornflakes
v146	Full cream milk
v149	Skimmed milk
v152	Liquid milk
v191	Apples and pears
v192	Plums and cherries
v193	Oranges
v194	Grapes and peaches
v195	Bananas
	Other neutral goods
v666	Driving lessons
v667	Railway
v668	Tram and suburban railway
v669	Ship
v670	Airline
v671	Bus, monthly tickets
v672	Bus, cliptickets
v673	Bus, single tickets
v737	Cinemas
v739	Theatres
v741	Concerts, museums and exhibitions
v742	Athletic sports, sports meetings, festivals, etc.
v750	Expenses for hobby courses

Appendix D

Symbols, abbreviations, and notation

The following notational conventions are employed in the present work:

- Vectors and matrices are written as ordinary variables, but generally small letters denote vectors and capital letters matrices.
- All vectors are column vectors.
- The transpose of a matrix M is denoted by M'
- The identity matrix is denoted by I and is assumed to be of the dimension to make matrix operations defined. The vector ι is a vector of ones, and is also assumed to be of the appropriate dimension.
- For two vectors x and y, $x \geq y$ means that for all i, $x_i \geq y_i$, and $x \gg y$ means that for all i, $x_i > y_i$
- A sequence of elements $(a_1, \ldots a_N)$ is sometimes written as $\{a_i\}_{i=1}^N$ to simplify notation. When the range of the index should be clear from the context, it is omitted from the expression.
- For a sequence of stochastic vectors $\{x_i\}_{i=1}^\infty$, a vector x and a stochastic vector Y, $x_i \xrightarrow{p} x$ means that the sequence $\{x_i\}_{i=1}^\infty$ converges to x in probability and $x_i \xrightarrow{L} x$ means that $\{x_i\}_{i=1}^\infty$ converges to Y in law (or distribution). See e.g. Lehmann (1999) for definitions and properties of these concepts. We have chosen to work with convergence in probablility in the present work, but on most occations, convergence in probability may be replaced by almost sure convergence.
- Composite functions are denoted by the operator $\circ$, that is, if we have two functions $f : \mathcal{A} \to \mathcal{B}$ and $g : \mathcal{B} \to \mathcal{C}$, then $h = g \circ f$ is the function such that $h(x) = g[f(x)]$ for all $x \in \mathcal{A}$. Furthermore, if $\tilde{g} : (\mathcal{B}, \mathcal{D}) \to \mathcal{C}$, then $h = \tilde{g}(d) \circ f$ is the function such that $h(x) = \tilde{g}[f(x), d]$ for all $x \in \mathcal{A}$ for some $d \in \mathcal{D}$.
- A gradient is denoted by $\bigtriangledown$, and the subscript denotes which variables we take the derivative with regard to. That is, for some function f, $\bigtriangledown_p f(p) = \left[\frac{\partial f(p)}{\partial p_1}, \ldots, \frac{\partial f(p)}{\partial p_J}\right]'$.
- 0 is assumed to be a naturan number, i.e. $0 \in \mathbb{N}$.

The following abreviations occur:

BSWF	Bergson-Samuelson welfare function
ONC	Ordinal non-comparability
CNC	Cardinal non-comparability
OLC	Ordinal level comparability
CFC	Cardinal full comparability
CRS	Cardinal ratio-scales
CCC	Complete cardinal comparability
IB	Independent of base
AN	Anonymity
AG	Agreeing
PP	Paretian property
SPP	Strict Paretian property
LES	Linear expenditure system
FOC	First order condition
OLS	Ordinary least squares
SUR	Seemingly unrelated regression
BLUE	Best linear unbiased estimator
GLS	Generalized least squares
FGLS	Feasible GLS
MSE	Mean square error
2SLS	Two-step least squares
ML	Maximum likelihood
LR	Likelihood ratio
CLT	Central limit theorem
iid	Independently and identically distributed

The following symbols are widely used:

J	Number of consumption goods
j	Index on a typical consumer good
N	Number of household members
i	Index on a typical household member
K	Number of demographic groups
k	Index on a typical demographic group or a member of this group
H	Number of households in sample
h	Index on a typical household
$\mathcal{Q}$	Consumption set
$\mathcal{Z}$	Set of possible demographic compositions
m_{ij}	Agent i's necessary consumption of good j
m_{ij}^{p}	Agent i's necessary consumption of public good j. The i is sometimes omitted
γ_{ij}	Agent i's coef. on good j (i may be omitted)
γ_{ij}^{p}	Agent i's coef on public good j (i may be omitted)
β_j	Household coefficient on good j, $\beta_j = \sum_i \gamma_{ij} + \gamma_{ij}^{p}$
α	Fraction of private goods, $\alpha = \frac{\gamma}{\gamma+\gamma^{p}}$
u^i	Utility function for individual i
W^z	Bergson-Samuelson welfare function for household with composition z
U	Household utility function
U^z	Household utility function for household with composition z
$\mathcal{U}, \mathcal{V}$	Utility levels
V	Indirect utility function. A $*$ denotes indirect welfare function.
C	Expenditure function. A $*$ denotes a household expenditure function.
p	Vector of prices assumed to be constant and identical for all agents
L	Equivalence scale
q	Consumption vector. A supscript i denotes for agent i
D	Marshallian demand function $\left(D(p, y, z) \in \mathbb{R}^{J}\right)$
Θ	Space of vectors of utility functions
Φ	Space of BSWFs
λ^i	Agent i's share of household income (λ is the vector og λ^is)
$\mathcal{D}$	Space of demand functions that may be generated by a BSWF

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