Discussion Paper

Central Bureau of Statistics, P.B. 8131 Dep, 0033 Oslo 1, Norway

No. 77 November, 1992

INTERTEMPORAL DISCRETE CHOICE, RANDOM TASTES AND FUNCTIONAL FORM

by

John K. Dagsvik

ABSTRACT

An important problem in the analysis of intertemporal choice processes is to separate the effect of unobserved temporal persistent variables from the influence on preferences from past choice behavior (state dependence).

The present paper discusses a behavioral Axiom in the presence of random preferences relative to a discrete alternative set and demonstrates that this Axiom yields joint utility processes that belong to the class of multivariate extremal processes. Specifically, the Axiom states that if there is no effect from past choice behavior on current preferences then the distribution of the current indirect utility conditional on past choice history is independent of the past choice history. When utilities are extremal processes Dagsvik (1988) demonstrated that the corresponding choice process is Markovian with transition probabilities that have a simple structure.

Key words: Intertemporal discrete choice, habit persistence, structural state dependence, Markovian choice processes, extremal processes.

I thank Rolf Aaberge for his valuable suggestions and Anne Skoglund for excellent word processing.

1. Introduction

The purpose of the paper is to provide a theoretical justification for the structure of intertemporal utility prosesses under pure taste persistence, i.e., when there is no behavioral effect on the individual level from past choice experiences. In the present setting the agents's planning horizon is one period and the environment is assumed perfectly certain to him. On the other hand, preferences are assumed random to the agent due to his lack of ability to forecast his preferences perfectly. The case with uncertain environment will be treated elsewhere.

Heckman (1981, 1991) and others have discussed the problem of separating the effect from past behavior on current preferences from spurious effects that stem from correlation between current and past choices due to unobservables. It appears that this identification problem cannot be solved without imposing theoretical restrictions in the model. This identification problem is of considerable practical relevance. For example, in analyses of travel demand it is of interest to know when observed correlation between persons choice of a specific transport alternative at different periods of time is a result of preferences being affected by experience with this alternative or simply a result of time persistent unobservables.

In the present paper we postulate a plausible formal characterization of intertemporal random utility models with pure taste persistence and with preferences that are random relative to the agent. Specifically, this characterization is formulated as an axiom as follows: if no structural state dependence is present then the distribution of the current indirect utility function does not depend on the past choice history. In the present paper we apply the Axiom to a subclass of (intertemporal) random utility models, namely the subclass generated from max-stable processes. Recall that max-stable processes are characterized by having finite-dimensional distributions of the multivariate extreme value type. There are two reasons for studying the subclass. First, it turns out to be convenient from a mathematical point of view. Second, Dagsvik (1991) has demonstrated that the subclass of random utility models generated from max-stable processes is dense in the class of random utility models, in the sense that the corresponding choice probabilities can be approximated arbitrarily closely by the choice probabilities derived from max-stable processes. This means that there is essentially no loss of generality by restricting the utility functions to the class of max-stable processes. Furthermore, by postulating the axiom described above we get the surprising and important result that given the class of max-stable processes the utility process must be a (multivariate) extremal process. In Dagsvik (1983) and (1988) it was proved that the choice model generated from extremal utility processes has the Markov property with a particular simple structure of the transition probabilities and transition intensities. Consequently, this framework is convenient in the context of empirical applications.

The organization of the paper is as follows: In Section 2 a random utility framework is introduced. In Section 3 we discuss the class of max-stable and extremal utility processes and the implications from the axioms.

2. Preferences

The individual decision-maker (agent) is supposed to have random preferences in the sense that they depend on tasteshifters which future realizations are uncertain to the agent. This notion of random preferences is different from the traditional convention in economics where stochastic utilities usually are unobservables that are assumed perfectly foreseeable from the agent's viewpoint. In the psychological literature however, there is a long tradition dating back to Thurstone (1927) in which utilities are modeled as random. The reason for this is of course that indivduals have been found to behave inconsistently in laboratory choice experiments in the sense that a given individual makes different choices under identical experimental conditions. One explanation for this is that the agent's psychological state of mind fluctuates from one moment to the next so as to induce unpredictable shifts in his tastes. Alternatively, the agent is viewed as having difficulties with evaluating the rank order of the alternatives, cf. Simon (1988). Thus, at any given point in time neither the agent nor the observing econometrician is able to predict future tasteshifters. However, tasteshifters realized in the past are known to the agent but unobserved to the econometrician.

We shall only consider the discrete choice case. The continuous choice case is left for future work.

Let S be a finite set of n alternatives, a_1, a_2,...,a_n, and let $\mathcal{B}$ be the index set that corresponds to the collection of all non-empty subsets from S. For simplicity we assume that the economic budget constraint holds at each moment in time i.e., no savings and borrowing are allowed. To each alternative, a_j, there is associated a stochastic process $\{U_j(t)\}$, where $U_j(t)$ is the agent's (conditional indirect) utility of a_j given the information and choice history at time t. There are no transaction costs and the agent therefore chooses a_j at age t if $U_j(t)$ is the highest utility at t. Here age (time) is continuous. Let $\{J(t)\}$ denote the choice process, i.e.,

$$J(t) = j \quad \text{if} \quad U_j > \max_{k \neq j, k \in B} U_k(t)$$

where $B \in \mathcal{B}$ is the choice set. We shall assume that the choice set B is kept constant over time. We shall henceforth, for notational convenience, suppress B in the notation. Let

$$\mathbf{U}(t) = (U_1(t), U_2(t), ..., U_n(t))$$

and let $F(\mathbf{t}_r;\mathbf{u}(\mathbf{t}_r))$, $\mathbf{u}(\mathbf{t}_r) \in R^{nr}$, be the nr-dimensional distribution function of $\mathbf{U}$ at $\mathbf{t}_r = (t_1, t_2, ..., t_r)$ where $t_1 < t_2 < ... < t_r$, i.e.,

$$F(\mathbf{t}_r;\mathbf{u}(\mathbf{t}_r)) = P(\bigcap_{i=1}^{r}(\mathbf{U}(t_i) \leq \mathbf{u}(t_i))).$$

We assume that $\{\mathbf{U}(t)\}$ is separable and continuous in probability.[1)] Moreover, we assume that the mapping $\mathbf{u}(t_1) \rightarrow F(t_1;\mathbf{u}(t_1))$ is continuous. This implies that there are no ties, that is

$$P(U_i(t) = U_j(t)) = 0.$$

The class of choice models generated from $\{U(t)\}$ will be called the class of intertemporal random utility models (IRUM).

When the finite dimensional distributions have been specified it is in principle possible to derive joint choice probabilities for a sequence of choices. In practice however, it turns out to be rather difficult to find stochastic processes that are good candidates for utility processes in the sense that they imply tractable expressions for the choice probabilities in the intertemporal context. More importantly, the class of intertemporal random utility models is quite large and it is thus of substantial interest to restrict this class on the basis of theoretical grounds.

One important theoretical problem in this context is to characterize the preferences under different assumptions about the effect from past choice experience.

The main purpose of the present work is to characterize preferences and the choice probabilities when there are no effects from past experiences on future preferences nor on future choice opportunities. To this end we start with the following definition:

<u>Definition</u>:

By pure-taste-persistent preferences (PTPP) we mean that there are no effects on the agent's preferences from previous choices.

Thus PTPP means that preferences are exogenous relative to the choice process. Heckman (1981) calls PTPP "habit persistence". We prefer however the notion PTPP since habit persistence may yield association to dependence on past choice experience.

We now introduce a fundamental assumption stated as Axiom 1 below.

Axiom 1

The indirect utility, $max_k U_k(t)$, is stochastically independent of $\{J(s), \forall s<t\}$.

Axiom 1 states that the distribution of the indirect utility at time t across series of observationally identical choice experiments, does not depend on the choice history prior to t. Clearly, this Axiom follows from PTPP. However, PTPP is not necessarily implied by the Axiom. In fact, PTPP would imply that $U_j(t)$, for each j, is independent of $\{J(s), \forall s<t\}$.

In order to clarify the interpretation consider the case in which the preferences also depend on a process, $\eta = \{\eta_t, t>0\}$, that is random to the observer but perfectly foreseeable to the agent. Since the conventional assumption is that preferences are deterministic to the agent this case thus represents a generalization of the traditional setting. Then the obvious modification of Axiom 1 to be consistent with PTPP is

$$P(\max_k U_k(t) \leq y \mid J(s), s<t, \eta) = P(\max_k U_k(t) \leq y \mid \eta). \tag{2.1}$$

If we also require that

$$P(\max_k U_k(t) \leq y \mid J(s), s<t) = P(\max_k U_k(t) \leq y) \tag{2.2}$$

then the process η must be stochastically independent of $\{J(s), s<t\}$. The difference in interpretation between (2.2) and (2.1) is the following: While (2.1) follows from PTPP (on the individual level) (2.2) corresponds to an aggregate analogue - namely that PTPP holds on average.

In Sections 3 and 4 we shall investigate the implications from Axiom 1.

3. The class of intertemporal generalized extreme value models (IGEV)

The IGEV is generated from utility processes that are max-stable. A max-stable process has finite dimensional distributions of the multivariate extreme value type. This means that maximum of independent copies of a max-stable process is max-stable (cf. de Haan, 1984). As is wellknown, there are three types within this class. We shall here consider type III which has finite-dimensional distributions (normalized) that are characterized by

$$\log F(t_r;u(t_r)) = e^{-z}\log F(t_r;u(t_r) - z\mathbf{1}),\ z\in R \tag{3.1}$$

where $\mathbf{1} = (1,1,\ldots,1)$. In particular, the one-dimensional marginal distribution has the form

$$F_1(t;u_1) = \exp(-e^{v_1-u_1}).$$

In Dagsvik (1991) it is proved that under suitable conditions the class of IGEV models is dense in the class of IRUM. By this it is meant that IRUM choice probabilities for a specific sequence of alternatives can be approximated arbitrarily closely to the corresponding choice probabilities of an IGEV model. The implication of this result is that there is no loss of generality by restricting the class of IRUM to the class of IGEV. Thus in the following we shall assume that the utility function is a multivariate max-stable process which is continuous in probability.

A very important subclass of the class of max-stable processes is called extremal processes. Multivariate extremal processes have been characterized by de Haan and Resnick (1977) and Dagsvik (1988). Following Dagsvik (1988) let $\{H_t(\cdot),\ t\geq 0\}$ be a family of multivariate extreme value distribution functions that satisfies $H_0=1$. Suppose also that $H_t(w)/H_s(w)$, for $t>s$, is a distribution function in $w\in R^n$. The multivariate extremal process has the same finite-dimensional distribution as $\{Y(t),\ t\geq 0\}$ defined by

$$Y(t) = \max(Y(s),W(s,t)),\ s<t,\ Y(0) = -\infty \tag{3.2}$$

where maximum is taken componentwise, $W(s,t)$ and $W(s',t')$ are independent when $(s,t)\cap(s',t')=\emptyset$ and

$$P(W(s,t) \le w) = H_t(w)/H_s(w). \tag{3.3}$$

Also Y(s) and W(s,t) are independent. It can be demonstrated that a multivariate extremal process is a pure jump Markov process (cf. de Haan and Resnick, 1977).

In Appendix 2 we summarize some of the properties of multivariate extremal processes. The class of multivariate extremal processes turns out to be of particular importance in choice theory as the next result indicates.

Theorem 1

Assume that the choice model belongs to the IGEV class with utility process that is continuous in probability. Then Axiom 1 implies that the utility function is a multivariate extremal process.

The proof of Theorem 1 is given in the appendix.

Theorem 1 provides the necessary theoretical foundation for postulating utility processes that are of the extremal type, at least as a point of departure. Theorem 1 has some very important implications which are summarized in Theorem 2 and Corollary 1.

Theorem 2

Suppose that the utility function is a multivariate extremal process (type III) with c.d.f. $H_t(y)$ at t. Then the choice process $\{J(t)\}$ is a Markov chain with marginal - and transition probabilities given by

$$P_j(t) \equiv P(J(t) = j) = -\frac{\partial_j G_t(0)}{G_t(0)}, \tag{3.4}$$

$$Q_{ij}(s,t) \equiv P(J(t)=j \mid J(s)=i) = \frac{-\partial_j G_t(0) + \partial_j G_s(0)}{G_t(0)}, \tag{3.5}$$

for i≠j, and

$$Q_{ii}(s,t) = \frac{-\partial_i G_t(0) + \partial_i G_s(0) + G_s(0)}{G_t(0)} \tag{3.6}$$

where $G_t(y)=-\log H_t(y)$ and ∂_j denotes the partial derivative with respect to the j-th component.

Theorem 2 has been proved in Dagsvik (1983) in the case where utilities are independent across alternatives. The more general case with interdependent utilities is proved in Dagsvik (1988). Resnick and Roy (1990) give a proof that does not require that the partial derivatives of $H_t(y)$ exist.

From Theorem 2 it is easy to obtain the corresponding transition intensities. Recall that the transition intensities for $\{J(t)\}$ are defined by

$$\gamma_{ij}(t) = \lim_{\Delta t \to 0} \frac{Q_{ij}(t,t+\Delta t)}{\Delta t}, \quad \text{for } i \neq j$$

and

$$\gamma_{ii}(t) = \lim_{\Delta t \to 0} \left(\frac{Q_{ii}(t,t+\Delta t) - 1}{\Delta t} \right).$$

According to Dagsvik (1988) we have the following:

<u>Corollary 1</u>

Suppose $G_t(y)$ is differentiable with respect to t. Let $g_t(y) \equiv \partial G_t(y)/\partial t$. Then

$$\gamma_{ij}(t) = \frac{-\partial_j g_t(0)}{G_t(0)} \tag{3.7}$$

for i≠j, and

$$\gamma_{ii}(t) = -\sum_{k\neq i}\gamma_{ik}(t) = \frac{\partial_i g_i(0) - g_i(0)}{G_t(0)}. \tag{3.8}$$

4. Interpretation of results

From Theorem 1 and Corollary 1 we can draw a number of important conclusions. First, note that (3.4) is the wellknown formula for the choice probability in the GEV class (Generalized extreme value models), cf. McFadden (1981). The interpretation of $G_t(0)$ is as

$$E\left(\max_k U_k(t)\right) = \log G_t(0) + 0.5772.... \tag{4.1}$$

Similar to Dagsvik (1983) it can be proved that

$$\operatorname{corr}\left\{\max_k U_k(s), \max_k U_k(t)\right\} = \rho\left(\frac{G_s(0)}{G_t(0)}\right), \quad s \leq t \tag{4.2}$$

where $\rho:[0,1]\rightarrow[0,1]$ is an increasing function with $\rho(0)=0$ and $\rho(1)=1$.[2)] By combining (3.4) and (3.5) we get

$$Q_{ij}(s,t) = P_j(t) - \frac{G_s(0)P_j(s)}{G_t(0)}, \quad s \leq t, \quad i \neq j \tag{4.3}$$

and

$$Q_{ii}(s,t) = P_i(t) + \frac{G_s(0)}{G_t(0)}(1 - P_i(s)) \tag{4.4}$$

where we emphasis that, apart from a monoton mapping $G_s(0)/G_t(0)$ is the autocorrelation function of the indirect utility function. In other words, in the absence of structural state dependence there is a simple relationship between the transition probabilities, the marginal probabilities (3.4) and the autocorrelation function of the indirect utility function.

The transition intensities (3.7) and (3.8) can be given a particular interpretation. Let

$$p_j(t) = -\frac{\partial_j g_t(0)}{g_t(0)}. \tag{4.5}$$

Recall that $\{\max_k U_k(t)\}$ is a pure jump Markov process. The interpretation of (4.5) is that given that a jump of the process $\{\max_k U_k(t)\}$ occurs at time t, $p_j(t)$ is the probability that the highest utility is attained in state j. To see this note that by Theorem A1 (iv) in Appendix 2

$$1 - \frac{g_t(y)}{g_t(x\mathbf{1})} \quad \text{for} \;\; y > x\mathbf{1}, \;\; x \in R, \;\; y \in R^n$$

is the c.d.f. of the utility process given that a jump occurs at t and given that $\max_k U_k(t)=x$. Note also that $g_t(y)$ has the property

$$g_t(y) = e^{-x} g_t(y - x\mathbf{1}) \tag{4.6}$$

which follows immediately from (3.1). The probability that $J(t)=j$, given that a jump occurs at t and given $\max_k U_k(t-)=x$, is therefore easily demonstrated to be

$$\frac{-\int_x^\infty \partial_j g_t(y\mathbf{1})dy}{g_t(x\mathbf{1})}, \quad y \in R$$

which by (4.6) reduces to

$$-\frac{\partial_j g_t(0)}{g_t(0)} \int_x^\infty e^{-(y-x)} dy = p_j(t).$$

From (3.7) we thus obtain that

$$\gamma_{ij}(t) = p_j(t)\frac{g_t(\mathbf{0})}{G_t(\mathbf{0})} = p_j(t)\partial E(\max_k U_k(t))/\partial t, \text{ for } i \neq j. \tag{4.7}$$

The last equality follows from the wellknown property that

$$E(\max_k U_k(t)) = \log G_t(\mathbf{0}) + 0.5772....$$

The second equality of (4.7) tells us that the transition intensity is the product of the rate of change in the mean indirect utility and the conditional probability that the highest utility is attained at alternative j given that a jump of the indirect utility occurs at t.

The conditional probability, $\pi_{ij}(t)$, of moving to state j given that state i is left follows from

$$\gamma_{ij}(t) = \pi_{ij}(t)\sum_{r \neq i}\gamma_{ir}(t) = -\pi_{ij}(t)\gamma_{ii}(t).$$

Hence (4.7) implies

$$\pi_{ij}(t) = \frac{p_j(t)}{1-p_i(t)}. \tag{4.8}$$

The results discussed above are related to the particular utility function representation. However, we can use Theorem 1 and Theorem 2 to obtain a characterization result of the choice model that is independent of the particular utility structure provided the utility process $\{U(t), t>0\}$ satisfies the condition

$$E\max(1, \exp(a(\sup_{s \in K}(\max_k U_k(s)))) < \infty \tag{4.9}$$

for some $a>0$ and any Borel set $K \subset (0,\infty)$ that is compact.

Theorem 3

Assume that the choice model is a random utility model where the utility process satisfies (4.9) and the first order partial derivatives of the corresponding finite-dimensional distributions exist. Assume furthermore that Axiom 1 holds. Then the choice process $\{J(t),t>0\}$ is a Markov chain and the transition probabilities, $\{\tilde{Q}_{ij}(s,t)\}$, from state (alternative) i to state j have the form

$$Q_{ij}(s,t) = P(J(t)=j) - K(s,t)P(J(s)=j) \quad for \quad j\neq i, \tag{4.10}$$

and

$$Q_{ii}(s,t) = P(J(t)=i) + (1-P(J(s)=i))K(s,t) \tag{4.11}$$

for some suitable positive function, $K(s,t)$.

Proof:

Assume first that the random utility model belongs to the IGEV class. Then by Theorem 1 and 2 $\{J(t), t>0\}$ is a Markov chain with transition probabilities that have the structure given in (4.3) and (4.4). Dagsvik (1992) has proved that under condition (4.9) the class of IGEV models is dense in the class of random utility models. This means that for any $\delta>0$ it is possible to find an IGEV model with transition probabilities $\{Q_{ij}(s,t)\}$ such that

$$|P(J(t)=j\,|\,J(t_n)=i,\ J(t_r), \forall r<n) - Q_{ij}(t_n,t)| < \delta \tag{4.12}$$

holds for any $t_1 < t_2 < ... < t_n < t$.

Suppose now that the Theorem is not true, i.e., for some time epochs $t_1, t_2,..., t_n$, t and some choice sequence

$$|P(J(t)=j\,|\,J(t_n)=i,\ J(t_r),\ \forall r<n) - \tilde{Q}_{ij}(t_n,t)| > \delta$$

for any choice of K(s,t). But since $\tilde{Q}_{ij}(s,t)$ has the same structure as $Q_{ij}(t_n,t)$ we may choose K(s,t) such that $\tilde{Q}_{ij}(s,t) = Q_{ij}(s,t)$ which yields a contradiction of (4.12). Thus $\{J(t), t>0\}$ is a Markov chain and the transition probabilities have the structure stated in the Theorem.

Q.E.D.

The result in Theorem 3 is of particular interest - since it provides a nonparametric test for the hypothesis that no structural state dependence effects are present.

Recall that in the present paper we have assumed that the choice set is constant over time[3)] and that there are no transaction costs.

From Corollary 1 it follows that the hazard function, $h_i(t)$, of state i is given by

$$h_i(t) \equiv -\gamma_{ii}(t) = \frac{g_t(0) - \partial_i g_t(0)}{G_t(0)}. \tag{4.13}$$

Let $T_i(s)$ be the length of time in state i given that state i was entered at time s. Then it follows immediately from (4.13) that the survivor function of the choice process $\{J(t)\}$ is given by

$$P(T_i(s)>t-s) = \exp\left(-\int_s^t h_i(u)du\right) = \frac{G_s(0)}{G_t(0)}\exp\left(-\int_s^t \frac{\partial_i g_u(0)du}{G_u(0)}\right). \tag{4.14}$$

5. Special cases

In the case where the utilities are independently distributed across alternatives we obtain

$$G_t(y) = \exp(\beta(t)) \sum_k \exp(v_k(t) - y_k) \tag{5.1}$$

and

$$P_j(t) = \frac{\exp(v_j(t))}{\sum_k \exp(v_k(t))} \tag{5.2}$$

where $\beta(t)$ is some function of t and where $v_j(t)$ has the interpretation

$$EU_j(t) = v_j(t) + \beta(t). \tag{5.3}$$

From Corollary 1 we get that the transition intensities in this case reduce to

$$\gamma_{ij}(t) = P_j(t)(v_j'(t) + \beta'(t)) = \frac{(v_j'(t) + \beta'(t))\exp(v_j(t))}{\sum_k \exp(v_k(t))} \tag{5.4}$$

for $i \neq j$, provided $v_j(t)$ and $\beta(t)$ are differentiable. We see from (5.4) that we must have $v_j'(t) + \beta'(t) > 0$ for all j. By (4.13) the corresponding hazard function reduces to

$$h_i(t) = \sum_k P_k(t) v_k'(t) + \beta'(t) - P_i(t)(v_i'(t) + \beta'(t)). \tag{5.5}$$

Consider next the particular case with independent utilities and $v_j(t) = v_j$. Then (5.5) reduces to

$$h_i(t) = \beta'(t)(1 - P_i) \tag{5.6}$$

and the survivor function (4.14) becomes

$$P(T_i(s)>t-s) = \exp(-(1-P_i)(\beta(t)-\beta(s))) \tag{5.7}$$

where P_i is given by (5.2) when the mean utilities $v_k(t)$ are substituted by v_k, $k=1,2,\ldots,n$.

If in addition $\beta(t)$ is linear in t; $\beta(t)=t\theta$, then $\{J(t)\}$ becomes a time homogenous Markov chain with exponentially distributed holding times i.e.,

$$P(T_i(s)>t-s) = \exp(-(t-s)(1-P_i)\theta) \tag{5.8}$$

and transition intensities

$$\gamma_{ij} = \theta P_j \quad \text{for} \quad i \neq j \tag{5.9}$$

and

$$h_i \equiv -\gamma_{ii} = \theta(1-P_i). \tag{5.10}$$

There is an important observation to be made here. Recall that the well-known specification of the transition intensities due to Cox (see Andersen et al., 1991) in the multistate case can be written

$$\gamma_{ij}(t) = \lambda_{ij}(t)\exp(f(Z_i, Z_j; \alpha)) \tag{5.11}$$

where $f(\cdot)$ is some specified function and Z_i is an individual specific time invariant vector of state specific covariates that characterize state i and α is a vector of parameters. $\lambda_{ij}(t)$ is called the baseline hazard. Let us now compare the structure (5.11) with the result of Corollary 1 in the time homogenous case when the utilities are independent across alternatives and

$$v_j = f(Z_j, \alpha). \tag{5.12}$$

Then we get

$$\gamma_{ij}(t) = \beta'(t) P_j = \frac{\beta'(t) \exp(f(Z_j,\alpha))}{\sum_k \exp(f(Z_k,\alpha))} \quad i \neq j. \tag{5.13}$$

We realize that (5.13) is essentially different from (5.11) in that it depends on all the covariates in a particular way while (5.11) only depends on the covariates related to state i and j. Therefore, the standard proportional hazard specification (5.11), which is often applied in duration analysis, is inconsistent with a random utility formulation.

Footnotes

1) Recall that a process $\{Y(t), t>0\}$ being continuous in probability means that $\exists\delta>0$ such that for any $\eta_1, \eta_2>0$

$$P(\|Y(t)-Y(s)\|>\eta_1) < \eta_2$$

whenever $|t-s|<\delta$, where $\|\cdot\|$ is the standard Eudician metric.

2) The function $\rho(\cdot)$ has the form

$$\rho(y) = -\frac{6}{\pi^2}\int_0^y \frac{\log x\,dx}{1-x}.$$

(cf. Tiago de Oliveira, 1973).

3) It is possible to extend the results of this paper to the case with non-decreasing choice sets. (See Dagsvik, 1983).

APPENDIX 1

Lemma 1

Let F(x,y) be a bivariate (type III) extreme value distribution. Then -logF(-x,-y) is convex. If F(x,y) is continuous the left and right derivatives, $\partial F_{\pm}(x,y)/\partial x$ and $\partial F_{\pm}(x,y)/\partial y$, exist and are non-decreasing.

Proof:

Let L(x,y) = -logF(-x,-y). Since F is a c.d.f. it follows that L is non-decreasing. Moreover, since F is a bivariate extreme value distribution it follows by Proposition 5.11, p. 272 in Resnick (1987) that there exists a finite measure μ on

$$\Delta = \left\{ z \in R_+^2 : z_1^2 + z_2^2 = 1 \right\}$$

such that

$$L(x,y) = \int \max(z_1 e^{x}, z_2 e^{y}) \mu(dx,dy).$$

Since $z_1 e^x$ and $z_2 e^y$ are convex functions it follows that L(x,y) is convex. Since L(x,y) is convex the left and right derivatives of F(x,y) exist. (See for example Kawata, Theorem 1.11.1 p. 27.)

Q.E.D.

Proof of Theorem 1

Resnick and Roy (1990) have demonstrated that there is no loss of generality by assuming the components of the utility process are independent. Although their discussion only regards multivariate extremal processes it is clear that their argument goes through also in the general max-stable case. Thus we assume that the components $U_j(t)$, j=1,2,...,n, of U(t) are independent. Also we assume n=2 since the general case is completely analogous.

Consider the case of choices at two moments in time, s and t. Let $F(s,t;\, x_1,x_2,y_1,y_2)$ be the corresponding c.d.f. of $(U(s), U(t))$. By the independence assumption

$$F(s,t;\, x_1,x_2,y_1,y_2) = F_1(s,t;\, x_1,y_1)F_2(s,t;\, x_2,y_2) \tag{A.1}$$

where F_j is the c.d.f. of $(U_j(s), U_j(t))$. Note that since $\{U_j(t), t\geq 0\}$ is assumed continuous in probability it follows that $F_j(s,t;\, x,y)$ is continuous in (x,y).

Let

$$G_i(x,y) = -\log F_i(s,t;\, x,y).$$

The probability distribution, $M(s,t;\, j_1, j_2, z)$, of $\{J(s), J(t), \max_k U_k(t)\}$ is given by

$$M(s,t;\, 2,1,z) \equiv P(J(s)=2, J(t)=1, \max_k U_k(t) \leq z) = \int_{y\leq z} F(s,t;\, x,dx,dy,y). \tag{A.2}$$

and similarly for other values of j_1 and j_2.

By Lemma 1 the first order left and right derivatives of $G_i(x,y)$ exsist. From now on we shall use the notion derivative of $G_i(x,y)$ meaning the respective (first order) right derivatives.

Since F by assumption is a multivariate extreme value distribution it follows from (3.1) that for $z \in R$

$$G_i(x,y) = e^{-z} G_i(x-z, y-z).$$

Hence

$$\begin{aligned} F(s,t;\, x,dx,dy,y) = {} & [\exp(-e^{-y}(G_1(x-y,0)+G_2(x-y,0)))] \\ & \cdot e^{-2y} \partial_2 G_1(x-y,0) \partial_1 G_2(x-y,0) dx dy \end{aligned} \tag{A.3}$$

where ∂_j denotes the partial derivative with respect to component j. Let

$$h_i(x) = G_i(-x,0).$$

From the relationship

$$G_i(x,y) = e^{-y}G_i(x-y,0) = e^{-y}h_i(y-x)$$

it follows that

$$\partial_2 G_i(-x,0) = -h_i(x) + h_i'(x). \tag{A.4}$$

By Lemma 1 $h_i(x)$ is convex and has derivatives that are non-decreasing. From (A.2), (A.3) and (A.4) it follow after the change of variable $x=-u+y$ that

$$M(s,t;\, 2,1,dy) = e^{-2y}dy\int_R [\exp(-e^{-y}(h_1(u)+h_2(u)))]\, h_1'(u)(h_2(u)-h_2'(u))du. \tag{A.5}$$

Suppose now that $x = r \geq -\infty$ is the largest point at which $h_1'(x) + h_2'(x) = 0$. Then since $h_i'(x)$ is nondecreasing it must be true that $h_1'(x)=h_2'(x)=0$ for $x \leq r$. As a consequence the mapping $\psi : R_+ \to [r,\infty)$ defined by

$$z = h_1(\psi(z)) + h_2(\psi(z)) - h_1(0) - h_2(0) \tag{A.6}$$

exists, is invertible and has (right) derivative everywhere on R_+. By change of variable

$$u \to \psi^{-1}(u) = z$$

(A.5) takes the form

$$M(s,t;2,1,dy) = e^{-2y}dy\exp(-be^{-y})\int_q^\infty (\exp(-e^{-y}z))\gamma(z)dz \tag{A.7}$$

where

$$q = h_1(-\infty) + h_2(-\infty) - h_1(0) - h_2(0) = h_1(r) + h_2(r) - h_1(0) - h_2(0),$$

$$\gamma(z) = h_1'(\psi(z))\psi'(z)(h_2(\psi(z)) - h_2'(\psi(z))) \tag{A.8}$$

and

$$b = h_1(0) + h_2(0).$$

According to Axiom 1 we must have

$$M(s,t;\ i,j,dy) = M(s,t;\ i,j,\infty)P(\max_k U_k(t) \in dy). \tag{A.9}$$

The (marginal) distribution of $\max_k U_k(t)$ for i, j = 1,2, is easily demonstrated to be extreme value as below

$$P(\max_k U_k(t) \leq y) = \exp(-e^{-y}b). \tag{A.10}$$

With $\theta = \exp(-y)$ we thus obtain from (A.5), (A.6) and (A.7)

$$\theta e^{-b\theta}\int_q^\infty (\exp(-\theta z))\gamma(z)dz = M(s,t;\ 2,1,\infty)be^{-b\theta}. \tag{A.11}$$

Note that (A.11) implies that the Laplace transform of $\gamma(z)$ has the form c/θ where c is a constant. But this implies that $\gamma(z) = 0$ for $z<q$ and

$$\gamma(z) = M(s,t;2,1,\infty)b, \quad z \geq q. \tag{A.12}$$

From the definition of $\psi(z)$ we get

$$1 = (h_1'(\psi(z)) + h_2'(\psi(z))\psi'(z). \tag{A.13}$$

Hence (A.8), (A.12) and (A.13) with $u=\psi(z)$ yield

$$h_1'(u)(h_2(u)-h_2'(u)) = h'(u)C_{21}, \quad \text{for} \quad u>r, \tag{A.14}$$

where $h(u) = h_1(u)+h_2(u)$ and $C_{ij} = M(s, t; i, j, \infty)b$. Similarly we get

$$h_2'(u)(h_1(u)-h_1'(u)) = h'(u)C_{12}, \quad \text{for} \quad u>r. \tag{A.15}$$

By substracting (A.14) from (A.15) we get

$$h'(u)h_1(u) - h_1'(u)h(u) = h'(u)(C_{12}-C_{21})$$

which, when dividing by $h(u)^2$ becomes equal to

$$\frac{h_1'(u)h(u)-h_1(u)h'(u)}{h(u)^2} = \frac{h'(u)(C_{21}-C_{12})}{h(u)^2}. \tag{A.16}$$

Next, integrating both sides of (A.16) yields

$$\frac{h_1(u)}{h(u)} = \frac{C_{12}-C_{21}}{h(u)} + d, \quad \text{for} \quad u>r,$$

where d is a constant. Hence we obtain

$$h_1(u) = C_{12} - C_{21} + h(u)d, \quad \text{for} \quad u>r. \tag{A.17}$$

By inserting (A.17) into (A.14) we get

$$h'(u)(h_2(u)-h_2'(u))d = h'(u)C_{21}, \quad \text{for} \quad u>r,$$

which is equivalent to

$$h_2(u) - h_2'(u) = C_{21}/d. \tag{A.18a}$$

Similarly

$$h_1(u) - h_1'(u) = C_{12}/d, \text{ for } u>r. \tag{A.18b}$$

Eq. (A.18) is a first order differential equation which has a solution of the form

$$h_j(u) = \alpha_j + \beta_j e^u, \text{ for } u>r, \ j=1,2. \tag{A.19a}$$

Since $h_j'(u)=0$ for $u\leq r$ and $h_j(u)$ is continuous we get from (A.19a) that

$$h_j(u) = \alpha_j + \beta_j e^r, \text{ for } u\leq r. \tag{A.19b}$$

As a consequence

$$G_j(x,y) = e^{-y} h_j(y-x) = \alpha_j e^{-y} + \beta_j \exp(-\min(x,y-r)). \tag{A.20}$$

From (A.20) we obtain that for $s<t$

$$P(U_j(t)\leq y \,|\, U_j(s)=x) = 0 \text{ when } y<x+r \tag{A.21}$$

and

$$P(U_j(t)\leq y \,|\, U_j(s)=x) = P(U_j(t)\leq y) \text{ when } y>x+r. \tag{A.22}$$

Eq. (A.21) means that $\{U_i(t)\}$ is non-decreasing with probability one. Eq. (A.22) means that conditional on $U_i(t)>U_i(s)$ then $U_i(t)$ is stochastically independent of $U_i(s)$. But then we must have that $\{U_i(t)\}$ is equivalent to the utility process defined by

$$U_i(t) = \max(U_i(s), W_i(s,t)) + r \tag{A.23}$$

where $W_i(s,t)$ is extreme value distributed and independent of $U_i(s)$. Since $U_1(t)$-$U_2(t)$ is independent of r for any t we may without loss of generality choose r=0. But then (A.23) defines the extremal process as defined by Tiago de Oliveira and others, (cf. Dagsvik, 1983, 1988) which was to be proved.

So far we have proved that conditional on a particular choice history, at two points in time, Axiom 1 implies utilities that are extremal processes. We have not yet demonstrated that the class of choice models with extremal utility processes fullfills the requirement of Axiom 1 when we condition on $\{J(\tau), \forall\tau\leq t\}$. Fortunately, however, this has been proved by Resnick and Roy (1990), p.p. 321.

Q.E.D.

APPENDIX 2

Theorem A1

Suppose $H_t(w)$ is differentiable in t. Then the multivariate extremal process $\{Y(t)\}$ has the following properties

(i) It is continuous in probability.

(ii) With probability one it has only a finite number og jumps in a finite time interval.

(iii) The transition probability function is given by

$$P(Y(t) \leq y | Y(s) = w) = \begin{cases} H_t(y)/H_s(y), \; y\geq w, \; s<t \\ 0, \; otherwise \end{cases} \tag{A.29}$$

(iv) Given that a jump occurs the process jumps from x into $[y,\infty)$ with probability

$$\Pi_t(w,[y,\infty)) = \begin{cases} \dfrac{g_t(y)}{g_t(w)} & if\ \ y>w \\ 0 & if\ \ y\leq w \end{cases} \tag{A.30}$$

where

$$g_t(y) = \frac{-\partial log H_t(y)}{\partial t}. \tag{A.31}$$

Proof:

(i) This result is demonstrated by extending the proof by Resnick (1987) p. 182 to the multivariate non-homogeneous case.

(ii) This is Theorem 1 in Dagsvik (1988).

(iii) Dagsvik (1988), p. 33 gives this result.

It thus only remains to prove (iv). From (iii) it follows that the intensity of a jump out of state w is given by

$$\lim_{t\to s}(P(Y(t) > w|Y(s) = w)/(t-s)) = g_t(w).$$

Recall that $\Pi_t(w, [y,\infty))$ is the probability that $\{Y(t)\}$ jumps from w into $[y,\infty)$ given that a jump occurs. Since $\{Y(t)\}$ is a Markov process we have

$$\lim_{t\to s}(P(Y(t) > y|Y(s) = w)/(t-s)) = g_t(w)\Pi_t(w,[y,\infty)).$$

But by (iii) we get

$$\lim_{t\to s}(P(Y(t) > y|Y(s) = w)/(t-s)) = g_t(y)$$

for $y\geq w$ and zero otherwise. Thus by combining the last two equations yields

$$\Pi_t(\mathbf{w},[\mathbf{y},\infty)) = \frac{g_t(\mathbf{y})}{g_t(\mathbf{w})}.$$

Q.E.D.

REFERENCES

Andersen, P.K., Hansen, L.S. and Keiding, N. (1991): "Non- and Semi-parametric Estimation of Transition Probabilities from Censored Observation of a Non-homogeneous Markov Process". Scand. J. Statist., **18**, 153-167.

Chamberlain, G. (1985): "Heterogeneity, Omitted Variable Bias and Duration Dependence". In J.J. Heckman and B. Singer (eds.), Longitudinal analysis of labor market data, Cambridge University Press, London.

Dagsvik, J.K. (1983): "Discrete Dynamic Choice: An Extension of the Choice Models of Luce and Thurstone". J. Math. Psychology, **27**, 1-43.

Dagsvik, J.K. (1988): "Markov Chains Generated by Maximizing Components of Multidimensional Extremal Processes". Stochastic Proc. Appl., **28**, 31-45.

Dagsvik, J.K. (1991): "How Large is the Class of Generalized Extreme Value Random Utility Models?" Mimeo, Central Bureau of Statistics, Oslo.

de Haan, L. (1984): "A Spectral Representation for Max-stable Processes". Ann. Probab., **12**, 1194-1204.

de Haan, L. and Resnick, S. (1977): "Limit Theory for Multivariate Sample Extremes". Z. Wahrscheinlichkeitsth., **40**, 317-337.

Heckman, J.J. (1981): "Statistical Models for the Analysis of Discrete Panel Data". In C.F. Manski and D. McFadden (eds.), Structural analysis of discrete data. MIT Press, Cambridge.

Heckman, J.J. (1991): "Identifying the Hand of the Past: Distinguishing State Dependence from Heterogeneity". Am. Econ. Rev., **81**, 75-79.

Kawata, T. (1972): "Fourier analysis in probability theory". Academic Press, New. York.

McFadden, D. (1981): "Probabilistic Theories of Choice". In C.F. Manski and D. McFadden (eds.), Structural analysis of discrete data. MIT Press, Cambridge.

Resnick, S. (1987): "Extreme value, regular variation and point processes". Springer Verlag, New York.

Resnick, S. and Roy, R. (1990): "Multivariate Extremal Processes, Leader Processes and Dynamic Choice Models". Adv. Appl. Prob., **22**, 309-331.

Simon, H.A. (1988): "Rationality as Process and as Product of Thought". In D.E. Bell, H. Raiffa and A. Tversky (eds.), Decision making; descriptive, normative, and prescriptive interactions. Cambridge University Press, Cambridge.

Thurstone, L. L. (1927): "A Law of Comparative Judgment". Psychological Rev., **34**, 272-286.

Tiago de Oliveira, J. (1973): "An Extreme Markovian Stationary Process". Proceedings of the fourth conference in probability theory, Acad. Romania, Brasov, pp. 217-225.

ISSUED IN THE SERIES DISCUSSION PAPER

No. 1 *I. Aslaksen and O. Bjerkholt (1985)*: Certainty Equivalence Procedures in the Macroeconomic Planning of an Oil Economy.

No. 3 *E. Biørn (1985)*: On the Prediction of Population Totals from Sample surveys Based on Rotating Panels.

No. 4 *P. Frenger (1985)*: A Short Run Dynamic Equilibrium Model of the Norwegian Production Sectors.

No. 5 *I. Aslaksen and O. Bjerkholt (1985)*: Certainty Equivalence Procedures in Decision-Making under Uncertainty: An Empirical Application.

No. 6 *E. Biørn (1985)*: Depreciation Profiles and the User Cost of Capital.

No. 7 *P. Frenger (1985)*: A Directional Shadow Elasticity of Substitution.

No. 8 *S. Longva, L. Lorentsen and Ø. Olsen (1985)*: The Multi-Sectoral Model MSG-4, Formal Structure and Empirical Characteristics.

No. 9 *J. Fagerberg and G. Sollie (1985)*: The Method of Constant Market Shares Revisited.

No. 10 *E. Biørn (1985)*: Specification of Consumer Demand Models with Stochastic Elements in the Utility Function and the first Order Conditions.

No. 11 *E. Biørn, E. Holmøy and Ø. Olsen (1985)*: Gross and Net Capital, Productivity and the form of the Survival Function. Some Norwegian Evidence.

No. 12 *J.K. Dagsvik (1985)*: Markov Chains Generated by Maximizing Components of Multidimensional Extremal Processes.

No. 13 *E. Biørn, M. Jensen and M. Reymert (1985)*: KVARTS - A Quarterly Model of the Norwegian Economy.

No. 14 *R. Aaberge (1986)*: On the Problem of Measuring Inequality.

No. 15 *A.-M. Jensen and T. Schweder (1986)*: The Engine of Fertility - Influenced by Interbirth Employment.

No. 16 *E. Biørn (1986)*: Energy Price Changes, and Induced Scrapping and Revaluation of Capital - A Putty-Clay Model.

No. 17 *E. Biørn and P. Frenger (1986)*: Expectations, Substitution, and Scrapping in a Putty-Clay Model.

No. 18 *R. Bergan, Å. Cappelen, S. Longva and N.M. Stølen (1986)*: MODAG A - A Medium Term Annual Macroeconomic Model of the Norwegian Economy.

No. 19 *E. Biørn and H. Olsen (1986)*: A Generalized Single Equation Error Correction Model and its Application to Quarterly Data.

No. 20 *K.H. Alfsen, D.A. Hanson and S. Glomsrød (1986)*: Direct and Indirect Effects of reducing SO_2 Emissions: Experimental Calculations of the MSG-4E Model.

No. 21 *J.K. Dagsvik (1987)*: Econometric Analysis of Labor Supply in a Life Cycle Context with Uncertainty.

No. 22 *K.A. Brekke, E. Gjelsvik and B.H. Vatne (1987)*: A Dynamic Supply Side Game Applied to the European Gas Market.

No. 23 *S. Bartlett, J.K. Dagsvik, Ø. Olsen and S. Strøm (1987)*: Fuel Choice and the Demand for Natural Gas in Western European Households.

No. 24 *J.K. Dagsvik and R. Aaberge (1987)*: Stochastic Properties and Functional Forms of Life Cycle Models for Transitions into and out of Employment.

No. 25 *T.J. Klette (1987)*: Taxing or Subsidising an Exporting Industry.

No. 26 *K.J. Berger, O. Bjerkholt and Ø. Olsen (1987)*: What are the Options for non-OPEC Countries.

No. 27 *A. Aaheim (1987)*: Depletion of Large Gas Fields with Thin Oil Layers and Uncertain Stocks.

No. 28 *J.K. Dagsvik (1987)*: A Modification of Heckman's Two Stage Estimation Procedure that is Applicable when the Budget Set is Convex.

No. 29 *K. Berger, Å. Cappelen and I. Svendsen (1988)*: Investment Booms in an Oil Economy -The Norwegian Case.

No. 30 *A. Rygh Swensen (1988)*: Estimating Change in a Proportion by Combining Measurements from a True and a Fallible Classifier.

No. 31 *J.K. Dagsvik (1988)*: The Continuous Generalized Extreme Value Model with Special Reference to Static Models of Labor Supply.

No. 32 *K. Berger, M. Hoel, S. Holden and Ø. Olsen (1988)*: The Oil Market as an Oligopoly.

No. 33 *I.A.K. Anderson, J.K. Dagsvik, S. Strøm and T. Wennemo (1988)*: Non-Convex Budget Set, Hours Restrictions and Labor Supply in Sweden.

No. 34 *E. Holmøy and Ø. Olsen (1988)*: A Note on Myopic Decision Rules in the Neoclassical Theory of Producer Behaviour, 1988.

No. 35 *E. Biørn and H. Olsen (1988)*: Production - Demand Adjustment in Norwegian Manufacturing: A Quarterly Error Correction Model, 1988.

No. 36 *J.K. Dagsvik and S. Strøm (1988)*: A Labor Supply Model for Married Couples with Non-Convex Budget Sets and Latent Rationing, 1988.

No. 37 *T. Skoglund and A. Stokka (1988)*: Problems of Linking Single-Region and Multiregional Economic Models, 1988.

No. 38 *T.J. Klette (1988)*: The Norwegian Aluminium Industry, Electricity prices and Welfare, 1988.

No. 39 *I. Aslaksen, O. Bjerkholt and K.A. Brekke (1988)*: Optimal Sequencing of Hydroelectric and Thermal Power Generation under Energy Price Uncertainty and Demand Fluctuations, 1988.

No. 40 *O. Bjerkholt and K.A. Brekke (1988)*: Optimal Starting and Stopping Rules for Resource Depletion when Price is Exogenous and Stochastic, 1988.

No. 41 *J. Aasness, E. Biørn and T. Skjerpen (1988)*: Engel Functions, Panel Data and Latent Variables, 1988.

No. 42 *R. Aaberge, Ø. Kravdal and T. Wennemo (1989)*: Unobserved Heterogeneity in Models of Marriage Dissolution, 1989.

No. 43 *K.A. Mork, H.T. Mysen and Ø. Olsen (1989)*: Business Cycles and Oil Price Fluctuations: Some evidence for six OECD countries. 1989.

No. 44 *B. Bye, T. Bye and L. Lorentsen (1989)*: SIMEN. Studies of Industry, Environment and Energy towards 2000, 1989.

No. 45 *O. Bjerkholt, E. Gjelsvik and Ø. Olsen (1989)*: Gas Trade and Demand in Northwest Europe: Regulation, Bargaining and Competition.

No. 46 *L.S. Stambøl and K.Ø. Sørensen (1989)*: Migration Analysis and Regional Population Projections, 1989.

No. 47 *V. Christiansen (1990)*: A Note on the Short Run Versus Long Run Welfare Gain from a Tax Reform, 1990.

No. 48 *S. Glomsrød, H. Vennemo and T. Johnsen (1990)*: Stabilization of emissions of CO_2: A computable general equilibrium assessment, 1990.

No. 49 *J. Aasness (1990)*: Properties of demand functions for linear consumption aggregates, 1990.

No. 50 *J.G. de Leon (1990)*: Empirical EDA Models to Fit and Project Time Series of Age-Specific Mortality Rates, 1990.

No. 51 *J.G. de Leon (1990)*: Recent Developments in Parity Progression Intensities in Norway. An Analysis Based on Population Register Data.

No. 52 *R. Aaberge and T. Wennemo (1990)*: Non-Stationary Inflow and Duration of Unemployment.

No. 53 *R. Aaberge, J.K. Dagsvik and S. Strøm (1990)*: Labor Supply, Income Distribution and Excess Burden of Personal Income Taxation in Sweden.

No. 54 *R. Aaberge, J.K. Dagsvik and S. Strøm (1990)*: Labor Supply, Income Distribution and Excess Burden of Personal Income Taxation in Norway.

No. 55 *H. Vennemo (1990)*: Optimal Taxation in Applied General Equilibrium Models Adopting the Armington Assumption.

No. 56 *N.M. Stølen (1990)*: Is there a NAIRU in Norway?

No. 57 *Å. Cappelen (1991)*: Macroeconomic Modelling: The Norwegian Experience.

No. 58 *J. Dagsvik and R. Aaberge (1991)*: Household Production, Consumption and Time Allocation in Peru.

No. 59 *R. Aaberge and J. Dagsvik (1991)*: Inequality in Distribution of Hours of Work and Consumption in Peru.

No. 60 *T.J. Klette (1991)*: On the Importance of R&D and Ownership for Productivity Growth. Evidence from Norwegian Micro-Data 1976-85.

No. 61 *K.H. Alfsen (1991)*: Use of macroeconomic models in analysis of environmental problems in Norway and consequences for environmental statistics.

No. 62 *H. Vennemo (1991)*: An Applied General Equilibrium Assessment of the Marginal Cost of Public Funds in Norway.

No. 63 *H. Vennemo (1991)*: The marginal cost of public funds: A comment on the literature.

No. 64 *A. Brendemoen and H. Vennemo (1991)*: Aclimate convention and the Norwegian economy: A CGE assessment.

No. 65 *K. A. Brekke (1991)*: Net National Product as a Welfare Indicator.

No. 66 *E. Bowitz and E. Storm (1991)*: Will restrictive demand policy improve public sector balance?

No. 67 *Å. Cappelen (1991)*: MODAG. A Medium Term Macroeconomic Model of the Norwegian Economy.

No. 68 *B. Bye (1992)*: Modelling Consumers' Energy Demand.

No. 69 *K. H. Alfsen, A. Brendemoen and S. Glomsrød (1992)*: Benefits of Climate Policies: Some Tentative Calculations.

No. 70 *R. Aaberge, Xiaojie Chen, Jing Li and Xuezeng Li (1992)*: The structure of economic inequality among households living in urban Sichuan and Liaoning, 1990.

No. 71 *K.H. Alfsen, K.A. Brekke, F. Brunvoll, H. Lurås, K. Nyborg and H.W. Sæbø (1992)*: Environmental Indicators.

No. 72 *B. Bye and E. Holmøy (1992)*: Dynamic equilibrium adjustments to a terms of trade disturbance

No. 73 *O. Aukrust (1992):* The Scandinavian contribution to national accounting

No. 74 *J. Aasness, E, Eide and T. Skjerpen (1992):* A criminometric study using panel data and latent variables (will be issued later)

No. 75 *R. Aaberge and Xuezeng Li (1992):* The trend in income inequality in urban Sichuan and Liaoning, 1986-1990

No. 76 *J.K. Dagsvik and Steinar Strøm (1992):* Labor sypply with non-convex budget sets, hours restriction and non-pecuniary job-attributes

No. 77 *J.K. Dagsvik (1992)*: Intertemporal discrete choice, random tastes and functional form