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How large is the class of generalized extreme value random utility models?

by

John K. Dagsvik

Research Department
Microeconometric Research Division

Abstract

The Generalized Extreme Value Model (GEV) was developed by McFadden (cf. McFadden, 1981) with the purpose of extending the Luce Model to account for interdependent utilities. While the Luce model satisfies the IIA property it has not been clear whether or not the GEV class implies theoretical restrictions on the choice probabilities other than those that follow from the random utility framework.

The present paper extends the GEV class to the intertemporal situation and proves that the choice probabilities generated from random utility processes can be approximated arbitrarily closely by choice probabilities from an intertemporal GEV model.

Key words: Max-stable processes, generalized extreme value models, intertemporal discrete choice, random utility models.

1. Introduction

McFadden (1977) introduced the class of Generalized Extreme Value (GEV) random utility models for discrete choice. This class contains the special cases known as the Luce and the nested logit model. The GEV class is generated from utility functions that have distributions of the multivariate extreme value type. This class of distribution functions yields strong restrictions on the interdependence between the utilities of different alternatives. For example, the correlation between joint extreme valued distributed utilities is always non-negative. The GEV class is tractable since it enables us to express choice probabilities on closed form. The GEV class is also appealing from a theoretical point of view since it is consistent with certain invariance properties, cf. Strauss (1979) and Robertson and Strauss (1981). In particular this class of utility functions is closed under aggregation of alternatives, i.e., the utility function relative to any aggregate version of the alternative space is multivariate extreme value distributed provided the original alternative set has multivariate extreme value distributed utilities. Until recently is has not been known if the GEV class yields apriori theoretical restrictions on the choice probabilities. However, Dagsvik (1990) demonstrated that the GEV class is dense in the class of random utility models. This means that any random utility choice model can be approximated arbitrarily closely by choice models belonging to the GEV class. Accordingly, the GEV class yields no theoretical restrictions on the choice probabilities other than those that follow from the random utility hypothesis. Necessary and sufficient conditions for a discrete choice model to be consistent with a random utility model were given by Falmagne (1978).

In the present paper we extend the result of Dagsvik (1990) to the intertemporal case in which the agent makes discrete choices at different points of time without transition costs. In the present paper the choice environment is assumed to be perfectly certain apart from future preferences which are allowed to be random to the agent. The interpretation of intrapersonal randomness due to psychological factors dates back to the work by Thurstone (1927) and is supported by many laboratory experiments since then. If the random components at different points in time are independent the conventional static model framework applies. However, in many experiments it is natural to assume that psychological factors that affect tastes show some stability over time. In addition, there may be interpersonel randomness due to variables that are perfectly certain to the agent but unobserved by the analyst.

The extension of GEV to the intertemporal setting (IGEV) is obtained by introducing max-stable utility processes. A max-stable utility process is characterized by the property that its finite-dimensional distributions are of the multivariate extreme value type.

The paper is organized as follows. In Section 2 the general intertemporal random utility framework is defined. In Section 3 the subclass of random utility processes generated by the class of max-stable random utility models is characterized and discussed. This section also contains the proof of the property that the class of max-stable random utility models is dense in the class of random utility models.

2. The intertemporal random utility model

The choice setting is defined as follows. Let S be a set of finite alternatives, a_1, a_2, ..., a_n, and let $\mathcal{S}$ be the collection of all non-empty subsets from S. To each alternative, a_j, there is associated a utility process, $U_j = \{U_j(t), t>0\}$, where U_j is a separable stochastic process that is assumed to be continuous in probability. Let $\mathbf{U} = (U_1,U_2,...,U_n)$, denote the multivariate process. The choice environment is assumed to be perfectly certain. Thus the choice setting here is analogous to Heckman (1981), McFadden (1984) and Dagsvik (1983).

The agents choice process, $J = \{J(t), t>0\}$, is defined by

$$J(t) = j \quad \text{if} \quad U_j(t) = \max_k U_k(t).$$

This means that there are no transaction costs, i.e., the agent can move "frictionless" from one alternative (state) to another in continuous time. The motivation for a random utility framework is that the agent may have tastes that fluctuate over time according to his psychological state of mind which to him is not perfectly foreseeable. An additional justification for randomness in the agents utility function is that there may be variables that are perfectly foreseeable to the agent but unobservable to the analyst.

Let $F(\mathbf{t}_m;\mathbf{u}(\mathbf{t}_m))$ be the n×m-dimensional distribution of U where $\mathbf{t}_m = (t_1,t_2,...,t_m)$ are m points in time, $\mathbf{u}(t_j) = (u_1(t_j), u_2(t_j),...,u_m(t_j))$ and $\mathbf{u}(\mathbf{t}_m) = (\mathbf{u}(t_1), \mathbf{u}(t_2),...,\mathbf{u}(t_m))$. Thus

$$F(\mathbf{t}_m;\mathbf{u}(\mathbf{t}_m)) = P\left\{\bigcap_{i=1}^{m}\bigcap_{j=1}^{n}(U_j(t_i)\leq u_j(t_i))\right\}.$$

In the following we shall assume that $F(\mathbf{t}_n;\mathbf{u}(\mathbf{t}_m))$ is continuous in $\mathbf{u}(\mathbf{t}_m)$ which implies that $P(U_i(t)=U_j(t)) = 0$. Moreover, we shall assume that $\sup_{s\in K}U_j(s)$ is a random variable when $K\subset R_+$ is a Borel set and finally we require that when $a\in(0,\bar{a})$ for some $\bar{a}>0$

$$E\max[1,\exp(a(\sup_{s\in K}(\max_k U_k(s))))] < \infty. \tag{2.1}$$

The probability of a particular choice career is given by

$$p(\mathbf{t}_m,\mathbf{j}_m) = P(\bigcap_{i=1}^{m}(J(t_i)=j_i)) = P(\bigcap_{i=1}^{m}(U_{j_i}(t_i)=\max_k U_k(t_i))). \tag{2.2}$$

where $\mathbf{j}_m = (j_1,j_2,...,j_m)$.

When the finite-dimensional distributions are specified the choice probabilities (2.2) can in principle be calculated. Let $d(\mathbf{j}_m,\mathbf{t}_m)F$ denote the differential operator of F with respect to the components $(j_1,j_2,...,j_m)$, at time epochs $(t_1,t_2,...,t_m)$, By straight forward calculus it follows that

$$p(\mathbf{t}_m,\mathbf{j}_m) = \int_{R^m} d(\mathbf{j}_m,\mathbf{t}_m)F(\mathbf{t}_m;u_1\mathbf{1}_n,u_2\mathbf{1}_n,...,u_m\mathbf{1}_n) \tag{2.3}$$

where $\mathbf{1}_n = (1,1,1,...,1)$. For example with n=m=2 we get

$$p(t_1,t_2;2,1) = \int_{R^2} F(t_1,t_2;x,dx,dy,y). \tag{2.4}$$

Except for special cases it is very difficult to obtain tractable expressions for the choice probabilities. This is wellknown for the one period case and it is even more difficult in the multiperiod case. For example, while multivariate extreme value c.d.f. imply closed form expressions in the one-period case this is no longer true in the multiperiod case except in special cases.

3. The class of max-stable intertemporal random utility models

The class of intertemporal generalized extreme value random utility models (IGEV) is generated from utility processes that are max-stable. The class of max-stable processes is precisely the class of stochastic processes that have finite-dimensional distributions of the multivariate extreme value type, cf. de Haan (1984). As is wellknown there are three types of extreme value distributions and we shall restrict our analysis to max-stable processes with type III marginals (cf. Resnick, 1987). In the intertemporal context this class has been investigated by Dagsvik (1983). A special subclass of max-stable processes is the class of extremal processes. This class turns out to yield tractable expressions for the choice probabilities, as was shown by Dagsvik (1983). Subsequently, Dagsvik (1988) and Resnick and Roy (1990) have investigated this class in detail and extended the results of Dagsvik (1983).

The IGEV class is of particular interest for a number of theoretical reasons. First it can be viewed as an extension of the GEV class to the intertemporal context. The GEV class contains the Luce model and the extension of the Luce model to the intertemporal setting is obtained by letting $U_1,U_2,...,$ be independent max-stable processes. Second, it has the property that it is closed under aggregation of alternatives in S in the sense that the (multivariate) utility process relative to a univers of aggregate alternatives is also a max-stable process. Strauss and Robertson (1981) have introduced other invariance assumptions that characterize the GEV class and accordingly the IGEV class.

In the present section we shall prove that the IGEV class is dense in the class of intertemporal random utility models. To this end we need to introduce some definitions.

Let N denote the class of n-dimensional random utility processes and for any subclass $D \subset N$ let $\mathscr{F}_D$ denote the class of intertemporal random utility models generated from D.

Definition

Let M be a subclass of N. We say that $\mathscr{F}_M$ is dense in $\mathscr{F}_N$ if for any $U \in N$ for any t_m and $\varepsilon > 0$ there exist a $U^* \in M$ such that

$$|p^*(t_m;j_m) - p(t_m;j_m)| < \varepsilon$$

for all j_m where p^* is generated by U^* and p is generated by U.

Before we state the main result of the paper we shall discuss a particular representation result of max-stable processes given by de Haan (1984).

Theorem 1

Suppose Y is a n-dimensional type III max-stable process which is continuous in probability. Then there exists a finite measure λ on R such that if (X_k, ε_k) is an enumeration of the points in the Poisson process on R^2 with intensity measure $\lambda(dx)e^{-\varepsilon}d\varepsilon$, then the process, V, defined by

$$V_j(t) = max_k(v_t(j,X_k) + \varepsilon_k)$$

with suitable L_1-functions, $exp(v_t(j,\cdot))$, $j=1,2,...,n$, $t \in R_+$, has the same finite-dimensional distributions as Y.

Recall that the condition that $\exp(v_t(j,\cdot))$ is L_1 means that

$$\int \exp(v_t(j,x))\lambda(dx) < \infty.$$

The result above is called the max-spectral representation result by de Haan, and $v_t(j,\cdot)$ is denoted the spectral function.

In the context of choice theory the result above allows interesting interpretations.

Interpretation I

Consider the following general example. Each agent in a population faces a set of choice alternatives, $a(i,j,k)$, $j=1,2,\ldots,m$, $k=1,2,\ldots$, where i indexes the agent and (j,k) indexes the alternatives. This means that the choice sets are agent-specific. Alternative $a(i,j,k)$ is characterized by attributes $(Z_j(t),X_{ik})$ where $Z_j(t)\in R^2$, $X_{ik}\in R$. Thus for fixed j every agent faces the same Z-attribute. However, the set of feasible X-attributes vary from one agent to another. Suppose now that the set of feasible attributes are not observed by the analyst nor is the X-attribute of the chosen alternative observed. Let the utility function be defined by

$$U_{jk}(i,t) = f(Z_j(t),X_{ik}) + \varepsilon_{ik}$$

where f is a deterministic function that is the same for every agent and depends on the alternatives solely through their attributes. The term ε_{ik} is a taste-shifter that is supposed to account for differences in tastes across agents and across alternatives due to unobservables. Here it is assumed that the differences in utility between $a(i,j,k)$ and $a(i,j^*,k)$ is perfectly accounted for by $(Z_j(t),X_{ik})$ and $(Z_{j^*}(t),X_{ik})$. In addition we also have unobserved heterogeneity in opportunities since the set of feasible X-attributes differ from one agent to another. The set of feasible X- and ε-attributes

$$\wp_i = \{(X_{ik},\varepsilon_{ik}),\ k=1,2,\ldots\},\quad i=1,2,\ldots$$

are realizations from independent copies of a Poisson process with intensity measure

$$\lambda(dx)\,e^{-\varepsilon}\,d\varepsilon\,.$$

Thus two different agents face two different Poisson realizations. The coordinates of these points represent the unobserved attributes and unobserved taste variables, respectively. The utility that corresponds to the observable Z-attributes is

$$U_j(i,t) = \max_k U_{jk}(i,t) = \max_k (f(Z_j(t), X_{ik}) + \varepsilon_{ik}).$$

Now if the function $\exp(f(Z_j(t),\cdot))$ is L_1-integrable it follows that the utility $U_j(i,t)$ can be viewed as a realization of a max-stable stochastic process.

A concrete example is choice of occupation and job. Suppose there are n occupations with wages Z_j, j=1,2,...,n. Within each occupation there are different jobs with non-pecuniary attributes. To agent i only a particular subset of jobs within occupation j is feasible, say $\{a(i,j,k)\}$, with non-pecuniary attributes X_{ik}, k=1,2,... The measure $\lambda(\cdot)$ represents an "aggregate" or mean measure of the availability of attributes. For example

$$\frac{\lambda(x)}{\lambda(1)}$$

can be interpreted as the distribution of feasible X-attributes relative to an (arbitrary selected) agent. For a more detailed discussion on this interpretation, see Dagsvik (1990, a,b). In empirical applications it may be cases where one has auxiliary aggregate data on total number of alternatives with specific attribute values. From these auxiliary data it would therefore be possible to obtain estimates of $\lambda(x)/\lambda(1)$.

Interpretation II

Now suppose that the agent does not perceive or alternatively, simply does not take into account the whole set of attributes available to him. Specifically, the set of attributes (stimuli) taken into account varies randomly from one experiment to another due to unpredictable fluctuations in the agent's ability to perceive stimuli. The fraction $\lambda(x)/\lambda(1)$ can now be interpreted as the mean fraction (across experiments) of stimuli taken account of by the agent. Thus in this case $\lambda(x)/\lambda(1)$ is a measure of the agent's perceived choice set or, alternatively, the set of attributes - or signals - the agent is informed about.

Next let us turn to the main result of the paper.

Theorem 2

Assume that (2.1) holds and that the first order partial derivatives, $\partial F(t_m;u(t_m))/\partial u_k(t_i)$ exist for all t_i, $i\leq m$, and all m and $k\leq n$. Then the class of IGEV is dense in the class of intertemporal random utility models.

Proof:

For notational simplicity we shall present the proof for the special case with m=n=2. The proof in the general case is completely analogous. Let

$$G_a(t_1,t_2;x_1,x_2,y_1,y_2) = \exp(-\int_{z\in R^4}(\exp(a\max(z_1-x_1,z_2-x_2,z_3-y_1,z_4-y_2)))F(t_1,t_2;dz)). \tag{3.1}$$

Let $\{p_a(t_1,t_2;i,j)\}$ be the choice probabilities generated by $G_a(t_1,t_2;x_1,x_2,y_1,y_2)$. According to (2.4) we have for i=2 and j=1

$$p_a(t_1,t_2,2,1) = \iint G_a(t_1,t_2;x,dx,dy,y). \tag{3.2}$$

The c.d.f. (3.1) is a type III multivariate extreme value distribution. Let $(U(t_1), U(t_2))$ be the utility vectors with joint c.d.f. $F(t_1,t_2;x_1,x_2,y_1,y_2)$ and let $\{p(t_1,t_2;i,j)\}$ denote the corresponding choice probabilities. For given x_1,x_2,y_1y_2 define

$$Z = \max(U_1(t_1)-x_1, U_2(t_1)-x_2, U_1(t_2)-y_1, U_2(t_2)-y_2). \tag{3.3}$$

The c.d.f. of Z is given by

$$P(Z\leq z) = F(t_1,t_2;x_1+z,x_2+z,y_1+z,y_2+z), \quad z\in R. \tag{3.4}$$

For notational convenience let

$$H(x_1,x_2,y_1,y_2;a) = -\log G_a(t_1,t_2;x_1,x_2,y_1,y_2).$$

From (3.1) and (3.4) it follows that

$$\begin{aligned} H(x_1,x_2,y_1,y_2;a) &= E\left(e^{aZ}\right) \\ &= \int e^{az}\sum_k F_k(t_1,t_2;x_1+z,x_2+z,y_1+z,y_2+z)\,dz \\ &= a\int e^{az}(1-F(t_1,t_2;x_1+z,x_2+z,y_1+z,y_2+z))\,dz \end{aligned} \tag{3.5}$$

where $F_k(t_1,t_2;\cdot)$ denotes the partial derivative with respect to the k-th argument and the last equality follows from integration by parts.

Notice that from (3.1) we get

$$H(x_1,x_2,y_1,y_2;a) = e^{-az}H(x_1-z,x_2-z,y_1-z,y_2-z) \tag{3.6}$$

for any $z\in R$. By combining (3.6) and (3.2) we get

$$p_a(t_1,t_2;2,1) \tag{3.7}$$
$$= \iint (\exp(-e^{-ax}H(0,0,y-x,y-x;a)))H_2(0,0,y-x,y-x;a)H_3(0,0,y-x,y-x;a)e^{-2ax}dxdy$$
$$- \iint (\exp(-e^{-ax}H(0,0,y-x,y-x;a)))d_3H_2(0,0,y-x,y-x;a)e^{-ax}dx$$

where d_k denotes the differential with respect to the k-th component. By applying (3.6) it is easily veryfied that

$$\frac{H_3(0,0,u,u;a)}{H(0,0,u,u;a)} = \frac{H_3(-u,-u,0,0;a)}{H(-u,-u,0,0;a)}. \tag{3.8}$$

Now make the following change of variable; u=y-x in (3.7). Then integration with respect to x gives

$$p_a(t_1,t_2;2,1)$$
$$= \iint (\exp(-e^{-ax}H(0,0,u,u;a)))H_2(0,0,u,u;a)H_3(0,0,u,u;a)e^{-2ax}dxdu$$
$$- \iint (\exp(-e^{-ax}H(0,0,u,u;a)))d_3H_2(0,0,u,u;a)e^{-ax}dxdu \tag{3.9}$$
$$= a\int \frac{H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)du}{a^2H(0,0,u,u;a)H(-u,-u,0,0;a)} - \int \frac{d_3H_2(0,0,u,u;a)}{aH(0,0,u,u;a)}.$$

Since

$$e^{az}F_2(t_1,t_2;z,z,z+u,z+u) \le \max(1,e^{\bar{a}z})F_2(t_1;z,z) \tag{3.10}$$

for $a \in (0,\bar{a})$, we get from (3.5), (2.1) and the Lebesgue Dominated Convergence Theorem that

$$\lim_{a\to 0} H_2(0,0,u,u;a)/a = -\lim_{a\to 0} \int e^{az}F_2(t_1,t_2;z,z,z+u,z+u)dz \tag{3.11a}$$
$$\overset{c}{=} -\int F_2(t_1,t_2;z,z,z+u,z+u)dz.$$

Similarly it follows that

$$\lim_{a\to 0} H_3(-u,-u,0,0;a)/a = -\int F_3(t_1,t_2;z-u,z-u,z,z)\,dz \tag{3.11b}$$

and

$$\begin{aligned}\lim_{a\to 0} H(0,0,u,u;a) &= \sum_k \int F_k(t_1,t_2;z,z,z+u,z+u)\,dz \\ &= \Big|_{z=-\infty}^{z=\infty} F(t_1,t_2;z,z,z+u,z+u) = 1 = \lim_{a\to 0} H(-u,-u,0,0;a).\end{aligned} \tag{3.12}$$

Moreover

$$H(0,0,u,u;a) \geq H(0,0,\infty,\infty;a)$$

and

$$H(-u,-u,0,0;a) \geq H(\infty,\infty,0,0;a)$$

which imply that

$$0 < \int \frac{H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)\,du}{a^2 H(0,0,u,u;a)H(-u,-u,0,0;a)} \leq \frac{\int H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)\,du}{a^2 H(0,0,\infty,\infty;a)H(\infty,\infty,0,0;a)}. \tag{3.13}$$

Also from (3.5) we get

$$\begin{aligned}&\frac{\int H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)\,du}{a^2} \\ &\leq \int\Big[\int \max(1,e^{az})F_2(t_1,t_2;z,z,z+u,z+u)\,dz \int \max(1,e^{av})F_3(t_1,t_2;v-u,v-u,v,v)\,dv\Big]du.\end{aligned} \tag{3.14}$$

Since $\max(1,e^{az})$ is nondecreasing in a it follows by the Lebesgue's Monotone Convergence Theorem that the right hand side of (3.14) converges towards

$$\int\Big[\int F_2(t_1,t_2;z,z,z+u,z+u)\,dz \int F_3(t_1,t_2;v-u,v-u,v,v)\,dv\Big]du$$

as $a \to 0$. Now observe that

$$\begin{aligned}&\int F_2(t_1,t_2;z,z,z+u,z+u)\,dz\\&= P(U_2(t_1) > \max(U_1(t_1),U_1(t_2)-u,U_2(t_2)-u)) \le P(u>U_2(t_2)-U_2(t_1))\end{aligned} \tag{3.15}$$

and

$$\begin{aligned}&\int F_3(t_1,t_2;z-u,z-u,z,z)\,dz\\&= P(U_1(t_2) > \max(U_1(t_1)+u,U_2(t_1)+u,U_2(t_2)) \le P(u<U_1(t_2)-U_1(t_1)).\end{aligned} \tag{3.16}$$

When we combine (3.14), (3.15) and (3.16) we obtain

$$\begin{aligned}&\frac{\int H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)\,du}{a^2}\\&\le \int P(u>U_2(t_2)-U_2(t_1))P(u<U_1(t_2)-U_1(t_1))\,du\\&\le \int_{-\infty}^{0} P(u>U_2(t_2)-U_2(t_1))\,du + \int_{0}^{\infty} P(u<U_1(t_2)-U_1(t_1))\,du\\&= -\int_{-\infty}^{0} u\,dP(u\ge U_2(t_2)-U_2(t_1)) + \int_{0}^{\infty} u\,dP(u\ge U_1(t_2)-U_1(t_1))\,du\\&\le E\,|U_2(t_1)-U_2(t_2)| + E\,|U_1(t_2)-U_1(t_1)| < \infty.\end{aligned} \tag{3.17}$$

The last inequality follows because (2.1) implies that the expectation $EU_k(t)$ exists. Consequently (3.12), (3.13) and (3.17) yield

$$\begin{aligned}&\lim_{a\to 0}\int \frac{H_2(0,0,u,u;a)H_3(-u,-u,0,0;a)\,du}{a^2H(0,0,u,u;a)H(-u,-u,0,0;a)} \le \frac{\sum_{k=1}^{2} E|U_k(t_2)-U_k(t_1)|}{\lim_{a\to 0} H(0,0,\infty,\infty;a)H(\infty,\infty,0,0,;a)}\\&= \sum_{k=1}^{2} E|U_k(t_2)-U_k(t_1)|\end{aligned} \tag{3.18}$$

and by (3.9) we therefore obtain

$$\lim_{a\to 0} p_a(t_1,t_2;2,1) = -\lim_{a\to 0}\int \frac{d_3H_2(0,0,u,u;a)}{aH(0,0,u,u;a)}. \tag{3.19}$$

Furthermore, due to (3.12), H(0,0,u,u;a)>H(0,0,∞,∞;a)>K when a is sufficiently close to zero and K is a constant.

Hence by (3.5)

$$0 < \frac{-d_3H_2(0,0,u,u;a)}{aH(0,0,u,u;a)} < \frac{\int \max(1,e^{\bar{a}z})d_3F_2(t_1,t_2;z,z,z+u,z+u)\,dz}{K}. \tag{3.20}$$

when a is sufficiently small. The right hand side of (3.20) is integrable with respect to u because

$$\begin{aligned}&\iint \max(1,e^{\bar{a}z})d_3F_2(t_1,t_2;z,z,z+u,z+u)\,dz \le \iint \max(1,e^{\bar{a}z})d_3F_2(t_1,t_2;\infty,z,z+u,\infty)\,dz\\ &= \int \max(1,e^{\bar{a}z})F_2(t_1,t_2;\infty,z,\infty,\infty)\,dz\\ &= E\max[1,\exp(\bar{a}U_2(t))] < \infty.\end{aligned} \tag{3.21}$$

We are now ready to apply the Lebesgue Dominating Convergence Theorem, which gives

$$\begin{aligned}\lim_{a\to 0} p_a(t_1,t_2;2,1) &= -\lim_{a\to 0}\int \frac{d_3H_2(0,0,u,u;a)}{aH(0,0,u,u;a)} = -\int \frac{\lim_{a\to 0} d_3H_2(0,0,u,u;a)/a}{\lim_{a\to 0} H(0,0,u,u;a)}\\ &= \iint d_3F_2(t_1,t_2;z,z,z+u,z+u)\,dz = p(t_1,t_2;2,1)\end{aligned} \tag{3.22}$$

which concludes the proof.

Q.E.D.

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