Discussion Paper

Research Department, Central Bureau of Statistics, Norway

No. 95 July 1993

Empirical Approaches for Analysing Consumption and Labour Supply in a Life Cycle Perspective

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Abstract

During the last decade several approaches for estimation of structural life cycle models of labour supply and consumption from micro data have been proposed. Ideally, estimation requires complete, individual, life cycle data for a variety of variables such as labour supply, consumption of durables and non-durables, and their expected prices; including interest and income tax rates. No single data set includes all these variables, and the challenge has been to find specifications that can be used for estimation of the unknown parameters of interest from the data actually available. This paper surveys these approaches.

Keywords: Empirical, life cycle model, consumption, labour supply

Acknowledgement: I wish to thank my supervisors Steinar Strøm, and in particular John K. Dagsvik, for their advice all through my dr. polit. dissertation research. The research has been financially supported by the Economic Research Programme on Taxation from The Norwegian Research Council for Applied Social Science.

1 Introduction

The life cycle model introduced by Modigliani and Brumberg in 1955 has received a great deal of attention both for its micro and macro economic implications, and during the last decade many have tried to estimate structural life cycle models from micro data, cf. King [31]. A major problem in that respect is that, ideally, estimation of the life cycle model for consumption and labour supply requires complete, individual lifetime data for a great variety of variables such as household labour supply, the consumption of durables and non-durables, and their expected prices; including interest and income tax rates. Today there is no single data set containing all these variables, and the challenge has been to find specifications that can be used for estimation and identification of the parameters of interest given the data actually available. This paper surveys the *approaches* for estimating *structural* life cycle models of labour supply and consumption demand from *micro* data.

Apart from this introductory section and the concluding section, this paper is divided into three main parts. In Section 2 we discuss the theoretical framework for the econometric approaches presented in the two subsequent sections. We present (Section 2.1) the life cycle model, cf. Ghez and Becker [18] and M. King [31], including the specification of the wealth constraints and the specification of possible constraints in the labour and credit markets. Then we discuss (Section 2.2) the first order conditions in the case that the household has perfect knowledge of future prices, and point out that, in a life cycle context, the relevant demand functions are the Frisch demands, cf. MaCurdy [34], Heckman [24, 25], Browning, Deaton and Irish [11] and Blundell [7]. From an econometric point of view, the usefulness of these functions depends on the separability properties of preferences and wealth constraints, cf. Blomquist [4], and Section 2.3 focuses on this fact. This section also focuses on the fact that the practical usefulness of the intertemporal separability assumption may depend on whether there are binding constraints in the credit market in current as well as historic periods.

From Section 3 we consider in more details the various approaches for estimation. First we look at methods that assume that households have perfect knowledge of future prices. We start out (Section 3.1) with MaCurdy's fixed effect approach, cf. MaCurdy [34]. This approach uses a (first) differenced marginal utility of wealth constant function for estimation of the parameters determining the responses of labour supply to evolutionary wage changes along a given life cycle wage path. In order to explain differences in labour supply across persons, the reduced form equation for a variable related to the marginal utility of wealth, must be estimated. This estimation requires individual life cycle data, and the true relationship must typically be approximated. Section 3.1.1 presents the model specifications, and in Section 3.1.2 we view the estimation procedure. The restrictions on intra- and intertemporal preferences are discussed in Section 3.1.3, and in Section 3.1.4 we discuss this approach in the case that we allow for income taxation.

Section 3.2 reviews how Heckman and MaCurdy [27] modify MaCurdy's fixed effect approach by taking the decision of working or not into consideration in the estimation. By estimating a bivariate fixed effect Tobit model, they eliminate the possible selection bias from using a subsample of households that are unconstrained in the labour markets.

In Section 3.3 we discuss some works that utilize the fact that if preferences can be described by a Stone-Geary function, it is possible to find an explicit solution for the marginal utility of wealth and the reduced form equations for the household's decision variables, cf. Bover [8, 9] and Biørn [3]. A weakness of this approach is that estimation essentially requires complete life cycle data, and the approach typically requires rather arbitrary assumptions about lifetime prices; including interest and income tax rates.

We end (Section 3.4) the review of approaches that assume perfect certainty, by discussing how the interpretation of the life cycle theory as a two-stage budgeting process can be used for estimation of within-period preferences from cross section data only, cf. Blundell [7]. Provided that panel data are available for all goods, this approach can also be combined with a particular use of the Euler equation for the marginal utility of wealth, for estimation of the remaining parameters of lifetime utility.

In section 4 we turn to methods that assume that households do not have perfect knowledge of future prices and variables influencing future preferences. These methods assume that households maximize expected utility. First we comment on the changes in the optimization problem and the first order conditions compared with the perfect certainty case (Section 4.1). Section 4.2 discusses the problems related to MaCurdy's fixed effect approach in the uncertainty case, cf. [36]. This approach turns out to be unsuccessful, and in [35], MaCurdy suggests to use the marginal rate of substitution functions for estimation (Section 4.3). This method allows for more flexible functional forms than the fixed effect approach, but it requires that we observe prices and consumption for at least two goods for the current period.

The marginal rate of substitution functions cannot be used for estimation of the parameters of the transformation of within-period utilities, but MaCurdy shows that this estimation is possible through a particular use of the Euler equation for marginal utility of wealth. This method can be viewed as a special case of the generalized method of moments, cf. Hansen and Singleton [22]. The generalized method of moments exploits the fact that the Euler equation for the marginal utility of wealth, and the other first order conditions, imply a set of population orthogonality conditions that can be used for estimation of the parameters of the utility function. In section 4.4 we discuss this approach, and comment on some of its limitations. Section 5 summarizes the paper.

2 Theoretical Framework

This section presents and discusses the life cycle model of labour supply and consumption including durable goods. The aim of the discussion is to clarify some aspects that are relevant for estimation.

2.1 Consumption and Labour Supply Behaviour

Assume the household consists of one adult whose lifetime preferences can be described by the utility function

$$V_0 = \sum_{t=0}^{T} (1+\rho)^{-t} U_t(C_t,\, K_t,\, H_t), \tag{1}$$

where T is the planning horizon, ρ is the time preference rate, subscript t denotes period, $U_t(C_t,\, K_t,\, H_t)$ is within-period utility, C_t and K_t are Hicks composite goods grouped as consumption of non-durables and durables, and H_t is labour supply. Within-period utility is assumed to be concave in consumption of goods and leisure.

While standard economic theory allows for non-separable preferences both within and between periods, empirical specifications usually assume at least intertemporal separability, cf. page 12, and we will assume this throughout the paper. This assumption requires cardinal utility and that the demand for a particular good is not influenced by habits. Recently several authors have emphasized the importance of allowing for non-separable intertemporal utility, cf. Bover [9], Hotz, Kydland and Sedlacek [29], Johnson and Pencavel [30] and Muellbauer [38], but with one exception we shall not discuss non-separable specifications.

The household faces the within-period wealth constraints,

$$A_t - A_{t-1} = w_t H_t + r_t A_{t-1} - S_t(r_t A_{t-1},\, w_t H_t) - p_t C_t - q_t\left[K_t - (1-\delta_f)K_{t-1}\right], \tag{2}$$

where $t = 0, 1, 2, \cdots, T$, and δ_f is the depreciation rate for durables. The prices p, q and w are nominal prices of non-durables, durables and leisure, and r is the nominal pre-tax interest rate. Since households are assumed to have perfect knowledge of future prices and variables influencing future preferences, expected prices coincide with realized prices. The variable A_t is nominal net value of interest-bearing assets at the end of period t, and if debt exceeds interest-bearing claims, it is negative. The possibility of capital gains on durables is taken account of through changes in q_t from one period to another, and the assumption that durables do not depreciate totally during one period, that is, $0 \leq \delta_f < 1$.

The specification of the tax function captures that income taxes $S_t(r_t A_{t-1}, w_t H_t)$ are assessed on wage and interest incomes. If the household's net interest income is negative, it allows for deduction of interests on debt by the tax assessment. We also assume that the tax function is continuous and convex, and that it has continuous first- and second order partial derivatives. Wealth taxes are ignored.

The specification of the wealth constraint implies that we disregard that households can place their wealth into various kinds of securities, such as bonds, stocks and shares, pension funds, arts and antiques. With the exception of the fact that inheritance at the beginning of the planning period can be (exogenously) included into the value of A_0, we do not either consider the possibilities of inheritance and bequest. In particular, this means that we treat A_T as exogenous.

The period-specific wealth constraints can often be more conveniently represented by amalgamating them into a lifetime wealth constraint

$$\begin{aligned} d_T A_T - A_0 &= \sum_{t=0}^{T} d_t \{w_t H_t - S_t(r_t A_{t-1},\, w_t H_t)\} \\ &- \sum_{t=0}^{T} d_t \Big\{ p_t C_t + q_t [K_t - (1-\delta_f) K_{t-1}] \Big\}, \end{aligned} \tag{3}$$

where $d_t \equiv 1/[(1+r_0)(1+r_1)\cdots(1+r_t)]$, for $t = 0, 1, \cdots, T$, is the discount rate that converts income in period t into its period 0 equivalent, and where $r_0 \equiv 0$.

In order to get an idea of the problems involved when we consider credit market constraints, we follow Mariger [37] and assume that households must borrow against mortgage in property. That is, net debt, $-A_t$, cannot exceed a fraction κ of the market value of the property, $q_t K_t$,

$$-A_t \leq \kappa q_t K_t, \quad t = 0,\, 1, \cdots, T. \tag{4}$$

Here κ is the fraction between maximum debt and the market value of durables, and it is assumed that it is equal to all persons. In particular, this means that it is independent of income and changes in institutionally determined constraints.

The household may also be constrained in the labour market, for example due to institutional constraints, but this will not be discussed here. Only the non-negative restriction will be accounted for, i.e.,

$$H_t \geq 0, \quad t = 0, 1, \cdots, T. \tag{5}$$

Maximization of lifetime utility (1) subject to the within-period wealth constraint (2), the borrowing constraint (4), the non-negative constraint (5), and given values of initial stock of assets and durables, and terminal stock of assets, yields the following first order conditions,

$$\frac{\partial U_t}{\partial C_t} = \lambda_t p_t, \quad t = 0, 1, \cdots, T, \tag{6}$$

$$\frac{\partial U_t}{\partial K_t} = \lambda_t q_t - \frac{1 - \delta_f}{1 + \rho} \lambda_{t+1} q_{t+1} - \gamma_t \kappa q_t, \quad t = 0, 1, \cdots, T, \tag{7}$$

$$-\frac{\partial U_t}{\partial H_t} = \lambda_t m_t + \alpha_t, \quad t = 0, 1, \cdots, T, \tag{8}$$

and the Euler equation

$$\lambda_t = \frac{1}{1 + \rho}(1 + R_{t+1})\lambda_{t+1} + \gamma_t, \quad t = 0, 1, \cdots, T, \tag{9}$$

where

$$m_t = w_t \left[1 - \frac{\partial S_t(r_t A_{t-1}, w_t H_t)}{\partial (w_t H_t)}\right] \tag{10}$$

and

$$R_{t+1} \equiv r_{t+1} \left[1 - \frac{\partial S_{t+1}(r_{t+1} A_t, w_{t+1} H_{t+1})}{\partial (r_{t+1} A_t)}\right] \tag{11}$$

are the after-tax marginal wage and interest rate, that are both endogenous for the household. The Lagrange multipliers $\lambda_t \equiv (1 + \rho)^t \lambda_t^*$, $\gamma_t \equiv (1 + \rho)^t \gamma_t^*$ and $\alpha_t \equiv (1 + \rho)^t \alpha_t^*$ can be associated with the wealth constraint, the borrowing constraint and the labour supply constraint respectively. Superscript $*$ means that the values of the multipliers are discounted to period 0, and multipliers without superscript are current values.

According to the Envelope Theorem, the Lagrange multipliers signify the marginal rate of change of the maximum value of the utility with respect to a change in the constraint, cf. Takayama [42]. Thus they express the shadow price of the actual resource. In particular, λ_t^* can be interpreted as the marginal utility of wealth in period t discounted into the first planning period 0, while λ_t is measured in current values.

All the Lagrange multipliers are non-negative, and if a constraint is not binding, the multiplier is zero. Hence, $\gamma > 0$ if and only if the borrowing constraint (4) is binding. Similarly, $\alpha > 0$ if and only if the labour supply constraint (5) is binding. Since the wealth constraint is always binding, $\lambda_t > 0$ for all t.

2.2 Interpretation of the First Order Conditions

The first order conditions and all the constraints constitute a simultaneous equation system. This system implicitly defines the lifetime demand path for all goods, and the Lagrange parameters, as functions of initial and terminal value of assets, initial stock of durables, lifetime prices; including the interest rate and all the formal income tax rates in the tax tables, and the preferences. In most cases it is, however, impossible to obtain a closed form solution for these functions.

It turns out that in a life cycle perspective, the relevant demand functions are the so called Frisch demands, cf. Frisch [17]. These functions are characterized by demand being conditioned on the marginal utility of wealth, and if preferences are intraperiod separable (in addition to interperiod separable), each of the first order conditions (6) to (8) implicitly define the Frisch demands. In what follows we make that assumption. This means that when we now turn to the discussion of some properties of the first order conditions that are relevant for estimation, we also discuss the properties of the Frisch demand functions.

The first order condition for non-durables, cf. equation (6), and the Euler equation (9) imply

$$\frac{\partial U_{t+1}}{\partial C_{t+1}} = \frac{1+\rho}{1+R_{t+1}} \left[\frac{\partial U_t}{\partial C_t} - \gamma_t \right], \tag{12}$$

when $p_t = 1$ for all $t = 0, 1, \cdots, T$. That is, if preferences are both inter- and intraperiod separable, if there are no binding borrowing constraints, and if R and p as well as preferences are constant over time, then consumption (or its marginal utility) at age t is the only relevant variable to predict consumption (or its marginal utility) at age $t+1$. Hall [20] also assumes that $(1+\rho)/(1+R_{t+1})$ is constant[1] (at least over a decade or two), and tests this implication on macro time series data.

If the tax function is convex, the first order condition for labour supply and leisure demand, cf. equation (8), can be rewritten as

$$\tilde{H}_t = \begin{cases} 0 & \text{if } -\frac{1}{\lambda_t} \frac{\partial U_t}{\partial H_t}\Big|_{H=0} \geq m_t|_{H=0} \\ -\frac{1}{\lambda_t}\frac{\partial \tilde{U}_t}{\partial \tilde{H}_t} = \tilde{m}_t & \text{otherwise,} \end{cases} \tag{13}$$

where "tilde" denotes that the variable is evaluated at optimum. That is, the decision of working or not is determined by comparing the marginal utility of leisure at zero hours work (measured in money), $-\frac{1}{\lambda_t} \frac{\partial U_t}{\partial H_t}\Big|_{H=0}$, with the price of leisure, that is, the after-tax marginal wage rate at zero hours labour supply, $m_t|_{H=0}$. If the marginal utility of leisure at zero hours work exceeds the after-tax marginal wage rate at zero hours, the person chooses not to work. Contrary, if the marginal utility of leisure at zero hours work is smaller than the wage rate, he chooses to work, and decreases leisure until the marginal utility of leisure equals the marginal wage rate. Labour supply then has two different dimensions, annual participation in the work force, and annual hours of work. Heckman and MaCurdy [27] utilize this property and estimate a bivariate Tobit life cycle model of female labour supply, cf. section 3.2.

These conditions are necessary, but not sufficient conditions for optimum[2], since we do not require that the tax function is strictly convex. But as long as the household maximizes utility, the data observed by the econometrician yield maximum utility, and by plugging these values into the first order conditions, we obtain the marginal utilities that are relevant for estimation.

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[1] The constancy of $(1+\rho)/(1+R_{t+1})$ is of cause a strong simplification even if we disregard interest income taxation. While ρ is a constant, even the pre-tax interest rate varies across time and households.

[2] These criteria are often referred to as local criteria as opposed to global criteria, where we need to compare the utility levels of various combinations of labour supply and consumption.

Notice also that the marginal tax rate on labour income at zero hours is endogenous, and can well be positive, since taxation of labour income depends on the level of interest income. It also depends on the income of the other spouse if the household consists of two adults that are treated as one taxpayer by the tax assessment. In Norway this was the case for married couples with at least one low wage income until 1992.

The specification of the credit market constraint (4) implies that whether the credit market constraint is binding or not depends on the demand for durables. However, in order to simplify the discussion of the first order condition for durables, we now argue as if this on/off decision is independent of consumer behaviour. The demand for durables, cf. equation (7), can then be written as

$$\tilde{K}_t = \begin{cases} -\frac{A_t}{\kappa q_t} \quad \text{if} \quad \left.\frac{\partial U_t}{\partial K_t}\right|_{k=-\frac{A}{\kappa q}} < \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1} q_{t+1} \\ \frac{\partial \tilde{U}_t}{\partial K_t} = \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1} q_{t+1} \quad \text{otherwise,} \end{cases} \tag{14}$$

where we recall that "tilde" denotes that the variable is evaluated at the optimum. The marginal cost, $\lambda_t q_t - \frac{1-\delta_f}{1+\rho}\lambda_{t+1} q_{t+1}$, is now measured in terms of utility.

If the household is constrained in the credit market, the demand for durables is determined by the constraint, and the marginal utility of durables, evaluated at the constraint, is less than the marginal cost. Hence, credit market constraints increases the demand for durables relative to the demand in the case of no binding constraints. Contrary, if the household is not constrained in the credit market, the household adjusts the demand for durables until the marginal utility equals the marginal cost.

Economic theory gives few guidelines with respect to how to measure demand for durables. It should be measured in physical units or in real values, but apart from that the theory is of little help. An example of the problems involved, is the measuring of housing consumption with all its dimensions such as location, number of rooms and quality. Problems with this definition also lead to problems with the definition of purchase and user price of durables, cf. Kornstad [32].

Notice that our treatment of durables assumes that there are no fixed costs in the demand for durables, and that durables can be treated as continuous variables. These approximations may be reasonable for white goods, but not for other kinds of durables such as housing.

While the first order conditions (6) to (8) determine allocation of resources within a particular period, the Euler equation (9) determines allocation of resources over time. It can be rewritten as

$$\lambda_t = \frac{1}{1+\rho}(1+R_{t+1})\lambda_{t+1} \quad \text{if} - A_t < \kappa q_t K_t \tag{15}$$

and

$$\lambda_t \geq \frac{1}{1+\rho}(1+R_{t+1})\lambda_{t+1} \quad \text{if} - A_t = \kappa q_t K_t. \tag{16}$$

If the household is unconstrained in the credit market, saving should be adjusted until the marginal utility of wealth in period t equals the (discounted) marginal utility of using the same resources next year added after-tax interest incomes. If the household is constrained, the marginal utility in period t exceeds the marginal utility of postponing consumption one period.

In the present case of perfect certainty, all the λ's can be calculated once and for ever, and the updating mechanism is given by the Euler equation (9). This means that if the household is unconstrained in all markets, and is able to calculate the marginal utility of wealth for a particular period, cf. equation (29), it can use the Euler equation to calculate λ for all t, and plug these values into the first order conditions to find consumption and labour supply.

As will be evident from the following analysis, equation (15) and (16) have been utilized in various ways to obtain equations that are suitable for estimation. Since some households can be constrained, the possibility of borrowing constraints complicates estimation. One solution is to estimate the model from a subgroup of unconstrained households, but then we should correct for the possible selection problem, cf. Heckman [26]. Another possibility is to use the complete sample and estimate a switching regimes model, cf. Hajivassiliou and Ioannides [19], but most works disregard the possibility of this constraint since one usually cannot identify what households are constrained, cf. Hall [20], MaCurdy [34, 36] and Blundell [7].

2.3 Separability, Constraints and Frisch Demands

The functional form of the Frisch demands depends only on the specification of within-period utility and, as we have seen, on whether corner solutions are present. Their usefulness, however, depends on the separability properties of preferences, the possibility of binding constraints in the labour and credit markets, and the specification of lifetime wage rates and the income tax system, and this section focuses on these topics.

Apart from the difficulties related to α_t and γ_t being latent and the endogeneity of R_{t+1} and m_t, the fact that λ_t is latent and varying across age complicates the estimation of the Frisch demand functions. The problem related to λ_t being age-specific can be circumvented by expressing these equations as functions of λ_0 (or λ_0^*). Substituting the Euler equations successively into each other, leads to

$$\lambda_t = (1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}, \tag{17}$$

where

$$D_t = \frac{1}{(1+R_0)(1+R_1)\cdots(1+R_t)} \tag{18}$$

is the after-tax marginal discount rate that transforms income in period t into its period 0 equivalent, and

$$\overline{\gamma}_{t-1} = \frac{1+\rho}{1+R_t}\gamma_{t-1} + \frac{(1+\rho)^2}{(1+R_t)(1+R_{t-1})}\gamma_{t-2} + \cdots D_t(1+\rho)^t\gamma_0.$$

Hence, the marginal utility of wealth at age t can be expressed as a difference consisting of a function of the marginal utility of wealth at age 0, and a weighted sum of the multipliers of the borrowing constraints.

Substituting equation (17) into the first order conditions (6) to (8), implies

$$\frac{\partial U_t}{\partial C_t} = \left[(1+\rho)^t \lambda_0 D_t - \overline{\gamma}_{t-1}\right] p_t, \tag{19}$$

$$\frac{\partial U_t}{\partial K_t} = \left[(1+\rho)^t\lambda_0 D_t - \overline{\gamma}_{t-1}\right] q_{ut} - \left[\kappa q_t - \frac{1-\delta_f}{1+R_{t+1}} q_{t+1}\right] \gamma_t \tag{20}$$

and

$$-\frac{\partial U_t}{\partial H_t} = \left[(1+\rho)^t\lambda_0 D_t - \overline{\gamma}_{t-1}\right] m_t + \alpha_t, \tag{21}$$

where

$$q_{ut} \equiv q_t - \frac{1-\delta_f}{1+R_{t+1}} q_{t+1}$$

is the user price of durables. Since R depends on the marginal taxation of interest incomes, the user price is endogenous.

At this moment we notice that if preferences are non-separable both between and within periods, all marginal utilities depend on the consumption of all goods in all periods, and estimation of a particular first order condition is very data demanding even if we could observe λ. If preferences are separable between, but not within periods, the marginal utilities for a particular period all depend on the consumption of all goods in that period, and estimation of a particular first order condition requires less data. If, however, preferences are both intra- and intertemporal separable, the marginal utilities only depend on the consumption of the actual good, and estimation is considerably simplified.

In the case of no binding credit market constraints in any historic period as well as no binding constraints in the labour and credit market in the current period, γ_t, $\overline{\gamma}_{t-1}$ and α_t equal zero. If we also assume additive within-period utility, the Frisch demand functions become

$$C_t = C_t\left[(1+\rho)^t\lambda_0 D_t p_t\right], \quad t = 0, 1, \cdots, T, \tag{22}$$

$$K_t = K_t\left[(1+\rho)^t\lambda_0 D_t q_{ut}\right], \quad t = 0, 1, \cdots, T, \tag{23}$$

and

$$H_t = H_t\left[(1+\rho)^t\lambda_0 D_t m_t\right], \quad t = 0, 1, \cdots, T, \tag{24}$$

where the C_t-, K_t- and H_t-functions are the inverse of the functions for the subutilities of C, K and H respectively.

This specification illustrates the advantage of what MaCurdy [34] labels the λ-constant demand functions. In the case of no binding constraints in any historic period in the credit market, no current binding constraints in the credit and labour markets, and inter- and intraperiod separable preferences, the arguments of the demand functions, apart from the discounting rate, are reduced to prices observable within the *current* period and the (latent) *life cycle component* λ_0. This means that with the exception of the information that is included in D_t, m_t and q_{ut}, λ_0 summarizes all historic and future information relevant to the household's current decisions.

If there is no income taxation, D_t, m_t and q_{ut} are exogenous, and λ_0 summarizes all historic and future information relevant to the household's current decisions. The marginal utility of wealth at age zero can then be thought of as a statistic[3] representing the household's (perfect) expectations about future pre-tax wage and interest rates, the purchase prices of durables and non-durables and all the (formal) income tax rates in the tax tables, besides realized values of these variables earlier in life. Initial and terminal (net) wealth also influence consumption and labour supply through the marginal utility of wealth.

Since λ_0 is independent of time, it can be treated as a fixed effect during estimation, and if the Frisch demand functions are additive in λ_0 or its logarithm, the problem related to λ_0 being latent can be overcome by differencing the Frisch demands, cf. MaCurdy [34] and section 3.1. This approach also reduces the problem related to the fact that estimation of the λ-constant demand functions is quite data demanding even if we could observe λ_0, since the after-tax discount rate D_t is endogenous and depends on $R_0, R_1, \cdots, R_t$, cf. equation (40).

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[3] Friedman (1957) argues that the consumer's aggregate consumption is related to "permanent" and "transitory" income. Mincer (1962) transfers this theory to the labour supply market, and assumes that labour supply is related to transitory and permanent wage rate and incomes. Comparing these theories with our marginal utility of wealth constant functions, we find that permanent income and wage rate play the same role as the marginal utility of wealth, while transitory incomes and wage rate play the role of the current prices.

While empirical analyses typically assume that life cycle wage rates are independent of labour supply, they can also be thought of as being affected by work experience, cf. Heckman [28] and Nakamura and Nakamura [39][4]. This assumption introduces a kind of non-separability through the wealth constraint, since current labour supply decisions can be viewed as an investment to increase future wages. The marginal wage rate then depends on future hours, and both future wage rates and hours enter into the first order conditions. The data requirement thus increases.

The introduction of income taxes, $S_t(r_tA_{t-1}, w_tH_t)$, also leads to a kind of non-separability that may reduce the usefulness of the marginal utility of wealth constant functions, and the methods based on these functions. If one does not observe current interest incomes, the calculation of the after-tax marginal wage and interest rate requires observations on A_{t-1}, and we need panel data for (at least) two years. The calculation of the after-tax discount rate D_t, cf. equation (18), requires panel data for even more periods, and the after-tax wage and interest rate, and the user price of durables, are now endogenous. In order to simplify estimation, income taxes are often omitted. The lifetime wealth constraint then becomes separable in goods and prices across periods, and the only way a price change can influence demand in other periods, is through the wealth effect. According to Blomquist [4], the responses to price changes in other periods are then of a very special form.

The possibility of constraints in the credit market also complicates estimation of the Frisch demands considerably. Comparing equations (6) to (8) with (19) to (21), we notice that the substitution of the Euler equation (17) into the first order conditions introduces the borrowing constraint multipliers for all earlier periods into these conditions. Since these multipliers are latent, this substitution probably

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[4] Using estimates from participation equations only, according to these authors it is impossible to separate this kind of dependence from the dependence due to intertemporal non-separable preferences. Hotz, Kydland and Sedlacek [29] show that the separability properties of lifetime preferences can be studied by expressing the Euler equation in terms of consumption since consumption does not depend on the process generating the life cycle wages.

introduces more problems that it solves, and the specifications (6) to (8) seem to be more attractive.

In order to use the Frisch demands to explain differences in demand across households, the relation between λ_0 and all the exogenous variables must be determined. Substituting the λ-constant functions (22) to (24) into the lifetime wealth constraint in the case of no binding constraints in any market in any period, leads to

$$\begin{aligned} & d_T A_T - A_0 \\ = & \sum_{t=0}^{T} d_t \left\{ w_t H_t \left[(1+\rho)^t \lambda_0 D_t m_t \right] - S_{\lambda t} - p_t C_t \left[(1+\rho)^t \lambda_0 D_t p_t \right] \right. \\ - & \left. q_t \left[K_t \left[(1+\rho)^t \lambda_0 D_t q_{ut} \right] - (1-\delta_f) K_{t-1} \left[(1+\rho)^{t-1} \lambda_0 D_{t-1} q_{ut-1} \right] \right] \right\}, \quad (25) \end{aligned}$$

where $S_{\lambda t}$ denotes that the λ-constant functions are also substituted into the tax function. This function implicitly defines λ_0 as a function of the household's initial value of assets and durables, terminal value of assets, all prices for all periods, and the time preference rate and the other parameters determining lifetime preferences. If there are binding constraints in some markets, the shadow prices of these constraints are also included.

In most cases this equation cannot can be given a closed form solution with respect to λ_0. Even if we could, cf. section 3.3, the fact that existing panel data do not contain *complete lifetime* price paths, including the interest rate and the income tax rate paths, and seldom initial and terminal assets and equities, complicates estimation of the reduced form equation for λ_0. The fact that λ_0 is latent, complicates estimation further.

3 Estimation in the Perfect Certainty Case

While the aim of section 2 was to present some theoretical considerations that can be used for estimation of the life cycle model, we are now going to take a closer look at some important contributions and methods within this field. In this section we limit ourselves to methods particularly suited for the perfect certainty case.

3.1 MaCurdy's Fixed Effect Approach

The use of the marginal utility of wealth constant functions for estimation of the life cycle model can be associated with Thomas E. MaCurdy [34][5], and we will now study how he utilizes these functions to estimate an intertemporal model of labour supply.

3.1.1 Model specifications

Compared with our general model presented in the first section, MaCurdy disregards consumption of durables, personal income taxation and the possibility of credit market constraints. The lifetime wealth constraint is then

$$A_{i0} + \sum_{t=0}^{T} d_t w_{it} H_{it} = \sum_{t=0}^{T} d_t C_{it}, \tag{26}$$

where subscript i denotes person and terminal wealth, A_T, is assumed to be zero. The wage and interest rate, and the stock of assets, are now measured in real terms.

MaCurdy assumes that preferences can be described according to

$$\sum_{t=0}^{T}(1+\rho)^{-t}\left(\gamma_{1it} C_t^{\omega_1} - \gamma_{2it} H_t^{\omega_2}\right), \tag{27}$$

where ω_1 and ω_2 are time- and person-invariant modifiers of preferences, and γ_{1it} and γ_{2it} are person- and time-specific modifiers. The modifier of preferences for leisure, γ_{2it}, is assumed to be randomly distributed across the population according to $\ln \gamma_{2it} \equiv \sigma_i - \xi_t^*$, where σ_i is a non-stochastic parameter and ξ_t^* is a stochastic parameter. Since the consumer is assumed to have perfect knowledge of future prices and preferences, ξ_t^* is introduced in order to take account of the fact that the econometrician cannot observe all factors that influence preferences for work.

The taste modifier for leisure, γ_{1it}, is not specified further since MaCurdy focuses on labour supply and does not use the first order condition for consumption.

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[5] The original paper was part of his Ph.D. thesis, "Two Essays on the Life Cycle", University of Chicago, 1978.

Assuming the real rate of interest is constant over the life cycle and equal to r with the exception of for period zero where it is equal to ρ, the marginal utility of wealth constant labour supply function (24) now becomes

$$\ln H_{it} = F_{i0} + bt + a \ln w_{it} + \xi_{it}, \tag{28}$$

where $a \equiv 1/(\omega_2 - 1)$, $\xi_{it} \equiv a\xi^*_{it}$ and $F_{i0} \equiv [1/(\omega_2 - 1)](\ln \lambda_{i0} - \sigma_i - \ln \omega_2)$. This specification also requires that $\ln(1+r)$ and $\ln(1+\rho)$ can be approximated by r and ρ.

The assumptions about the interest rate imply that $(1+\rho)^t D_{it}$ in equation (24) is being reduced to bt, where $b \equiv a(\rho - r)$, and the interest rate becomes part of the b-parameter.

Since the specification of preferences means that it is impossible to find a reduced form solution for λ_0, cf. equation (25), MaCurdy assumes that "$\ln \lambda_{i0}$ can be approximated as a linear function of measured characteristics, Z_i, net natural log of wages at each age, initial wealth and an unobserved random variable, ζ_i, representing unmeasured characteristics". Hence

$$F_{i0} = Z_i \phi + \sum_{t=0}^{T^*} \gamma_t \ln w_{it} + A_{i0}\theta + \zeta_i, \tag{29}$$

where T^* denotes the age of retiring, γ_t and θ are scalars, and ϕ is a vector of constants that are assumed to be constant across consumers.

Estimation of this equation requires that the lifetime wage path and initial assets can be observed for each consumer. To observe the wage rates outside the sample period, and for the unemployed workers, MaCurdy assumes that wages follow a quadratic function in age,

$$\ln w_{it} = \pi_{0i} + t\pi_{1i} + t^2\pi_{2i} + \varsigma_{it}, \tag{30}$$

where ς_{it} is a random variable, and where the coefficients π_{0i}, π_{1i} and π_{2i} are linear functions of some age-invariant characteristics, M_i,

$$\pi_{ji} = M_i g_j, \; j = 0, 1, 2, \tag{31}$$

where g_0, g_1 and g_2 are vectors of parameters. The variables included in M, are education and age, and variables related to family background.

Most data set do not include even extensive measures of the consumers' current wealth, and to predict initial assets[6], MaCurdy assumes that the optimal lifetime income stream from buying assets, $Y_{it} = rA_{it}$, can be approximated by a quadratic function in age,

$$Y_{it} = \alpha_{0i} + t\alpha_{1i} + t^2\alpha_{2i} + \varrho_{it}, \tag{32}$$

where ϱ_{it} is a random variable, and where the parameters α_{0i}, α_{1i} and α_{2i} are linear functions of a vector of exogenous and time invariant determinants, J_i, of property incomes,

$$\alpha_{ji} = J_i q_j, \quad j = 0, 1, 2. \tag{33}$$

3.1.2 Estimation

MaCurdy does not estimate the parameters of the utility function, but applies equations (28) and (29) for prediction of labour supply.

The estimation of equation (28) takes into account that λ_{i0} is latent and correlated with the exogenous variables. Since F_{i0} is a linear transformation of λ_{i0} and some time invariant parameters, F_{i0} can be treated as a fixed effect. MaCurdy then estimates a and ρ, determined through $b \equiv a(\rho - r)$, by working on a first differenced version of the Frisch supply function (28),

$$\Delta \ln H_{ij} = b + a\Delta \ln w_{ij} + \Delta\xi_{ij}, \qquad \begin{array}{l} i = 1, 2, \cdots, n, \\ j = 2, 3, \cdots, \tau, \end{array} \tag{34}$$

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[6] Notice that MaCurdy's specification of the property income path seems to overdetermine the property income path. When consumption and labour supply are determined through the first order conditions, the period-specific wealth constraints can be utilized to derive saving in all periods. Given initial assets and a lifetime path for the interest rate we can then find assets and property incomes in all periods. Hence the a priori assumption of a particular lifetime property income path can lead to inconsistency problems.

where Δ is the difference operator, i.e. $\Delta X_t \equiv X_t - X_{t-1}$. Subscript j denotes sample period, τ is the number of sample periods, and n is the number of workers in the sample. The variables H_{ij} and w_{ij} then denote person i's labour supply and wage rate in sample period j.

Since there may be omitted variables that are correlated with both the wage rate growth and the error term $\Delta\xi_{ij}$, the wage rate is treated as endogenous in the estimation. The parameters of equation (34) can then be (consistently) estimated by standard 2SLS[7].

An advantage of this approach is that the parameters determining the responses to current price changes along a given life cycle price path can be estimated from panel data for (at least) two periods for only the actual good in question (including the price), in addition to the relevant instrument variables. In MaCurdy's case this means that he can estimate the parameters related to labour supply without having data on total consumption. Since many data sets do not include observations of both labour supply and consumption including the consumption of durables, this fact is important.

A problem related to first differencing in particular, but also higher order differencing, is that it tends to accentuate measuring noise relative to the observed changes in the differenced variables, and the precision of the estimated parameters is reduced.

To estimate the reduced form equation for F_{i0}, MaCurdy uses the wage and property income path equations and obtains

$$F_{i0} = Z_i\phi + \pi_{0i}\overline{\gamma}_0 + \pi_{1i}\overline{\gamma}_1 + \pi_{2i}\overline{\gamma}_2 + \alpha_{0i}\overline{\theta} + \varepsilon_i, \tag{35}$$

where

$$\overline{\theta} = \theta/r, \qquad \overline{\gamma}_j = \sum_{t=0}^{T*} t^j\gamma_t, \quad j = 0, 1, 2,$$

and ε is the error term.

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[7] MaCurdy also considers another specification of equation (34) where the interest rate is allowed to vary over time.

The equations (31), (33) and (35) constitute a simultaneous equation system, that can be estimated if we have data for F_0, Z, the π's and α_0 for all consumers. The consumer characteristics, Z_i, are observed directly, and MaCurdy shows that unbiased estimates for all the π_i's and the α_i's can be obtained by making some particular transformations of equations (30) and (32). Estimates for all the F_{i0}'s are obtained from equation (28) inserted the estimates for b and a from the estimation of equation (34). Using these results, MaCurdy estimates the simultaneous equation system (31), (33) and (35) by ordinary 2SLS. The substitutions imply that the error term and the right hand side variables of the F_0-equation become correlated, but according to MaCurdy, the estimates are consistent for a "sufficiently large" number of observations per consumer.

The estimation of the reduced form equation for F_{i0} is very data demanding, since it requires life cycle data for each person. This means that if we want to estimate all aspects of demand, the fixed effect approach necessarily involves some kind of arbitrary assumptions about the lifetime price paths. In most cases estimation also requires approximations to the true relationship. This reduces the tight connection between theory and estimated regression function.

3.1.3 Restrictions on intra- and intertemporal preferences

MaCurdy's fixed effect approach requires that the marginal utility of wealth constant demand functions are linear functions of λ_0 or $\ln \lambda_0$. From the first order conditions (6) to (8) it should be evident that if the λ-constant demand functions are to be linear in λ_0, within-period preferences must be additive, cf. also Blundell and Walker [6]. This assumption may seem implausible, and to reduce restrictions as far as possible, Browning, Deaton and Irish [11] suggest to start out from the profit function instead of from the direct (within) utility function. This approach views the household's decision problem as a profit maximization problem, where the household produces utility, U, from consuming the consumption vector $\underline{C} = (C_1, C_2, ...)$. The household's profit is defined as the maximum profit attainable, $\pi(p^u, \underline{p})$, from selling utility to itself at a price p^u, subject to the technology of utility production, $U = v(\underline{C})$, that is,

$$\pi(p^u, \underline{p}) = \max_{u,\underline{c}} \left[p^u U - \underline{p}\underline{C}'; \; U = v(\underline{C}) \right], \tag{36}$$

where $\underline{p} = (p_1, p_2, \ldots)$ is the price vector corresponding to the input vector. Since $1/\lambda$ can be interpreted as the marginal cost of utility or the price of utility, $p^u \equiv 1/\lambda$. The profit function can be viewed as representing consumer preferences as a function of p^u and $\underline{p}$. It also appears that additive or block additive utility is equivalent to additive or block additive profits, and one of the consequences of intertemporal separability is then that the various periods can be treated separately.

Browning et al. find the most general profit function that satisfies the condition that the partial derivatives of the Frisch demands with respect to all prices, are independent of the marginal utility of wealth. Based on this function they conclude that "the treatment of p^u as additive in the hours and quantities demanded implies intraperiod quasi-homotheticity". In other words, viewing the households' decision problem as a profit maximization problem, the requirement that the Frisch demands are linear in p^u implies that demand is linearly related to within-period full income[8]. Blundell, Fry and Meghir [5] show an analogous result, that is, relaxing intraperiod additive separability leads to unitary within-period full income elasticities if the loglinear Frisch demands are to be loglinear in λ_0. Hence, the most common specifications of the Frisch demands introduce restrictions on intratemporal preferences.

Intuitively we would consider that the use of highly aggregated Hicks composite goods reduces the restrictiveness of assuming intraperiod additive separability. Intraperiod separability can then turn out to be a reasonable approximation for life cycle models of highly aggregated Hicks composite goods.

The assumption of intertemporal separability can also be questioned. According to Blundell [7], the indirect within-period utility corresponding to the profit function

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[8] **Full income is defined as the sum of interest incomes, the value of the household's time endowment and the asset decumulation.**

of Browning et al., is of the Gorman Polar form

$$G_t = F_t\left\{[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)\right\}, \tag{37}$$

where F_t is some concave monotonic transformation, $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ are particular concave linear homogeneous functions of the price vector $\underline{p}_t$ in period t, and Y_t is full income. Full income is defined as the sum of interest incomes, the value of the household's time endowment and the asset decumulation, and both $\underline{p}_t$ and Y_t are discounted to period zero.

Blundell uses this specification to discuss the restrictions on intertemporal substitution implied by the profit function of Browning, Deaton and Irish. He follows Browning [10] and assumes that the intertemporal substitution possibilities best can be measured by the intertemporal elasticity of substitution, $\phi = G_y/YG_{yy}$, where $G_y = \partial G/\partial Y$ and $G_{yy} = \partial G_y/\partial Y$. The utility index (37) implies

$$\phi_t = \frac{F_t'\{[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)\}b_t(\underline{p}_t)}{F_t''\{[Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)\}Y_t}, \tag{38}$$

where F_t' denotes partial derivative. Since $Y_t - a_t(\underline{p}_t)$ is dominated by Y_t, this means that $b_t(\underline{p}_t)$ represents the substitution possibilities for rich people. This result and the fact that the specification of the profit function implies that $b_t(\underline{p}_t)$ is a linear function of all the prices of $\underline{p}_t$, means that the use of linear Frisch demands constraints intertemporal substitution for the high income groups.

It can be argued that this matter is not very important since the substitution possibilities are also determined by the derivatives of the monotonic transformation F. Many works, however, let F be the identity transformation, and Blundell shows that the substitution elasticity for the loglinear and the exponential transformations approaches -1 and 0 as income increases. Linear or loglinear Frisch demands then also seem to restrict intertemporal substitution possibilities considerably.

3.1.4 Income taxation and MaCurdy's fixed effect approach

As mentioned MaCurdy disregarded income taxation and assumed that the interest rate was constant across periods and equal for all persons. In this case the Frisch demands only include current variables. Without these simplifications the Frisch labour supply function becomes

$$\ln H_{it} = F_{i0} + a \ln \left[D_{it}(1+\rho)^t m_{it}\right] + \xi_{it}, \tag{39}$$

where we recall that D_{it} is the after-tax marginal discount rate defined in equation (18), and m_{it} is the after-tax marginal wage rate. Since the discount rate D_{it} is endogenous, we can hardly assume it is constant during the life cycle, and in no way we can reasonably assume it is equal for all persons. Taking also into consideration that D_{it} depends on $R_{i0}, R_{i1}, \cdots, R_{it}$, the introduction of income taxes immediately seems to complicate estimation considerably. However, first differencing the Frisch demands yields

$$\Delta \ln H_{it} = a\rho + a[\Delta \ln m_{it} - \ln(1 + R_{it})] + \Delta\xi_{it}, \tag{40}$$

which illustrates that the method of differencing is even more attractive in this case. MaCurdy's fixed effect approach can then very well be used even in the case of personal income taxation.

3.2 A Fixed Effect Tobit Model

Heckman and MaCurdy [27] extend MaCurdy's fixed effect approach by implementing a bivariate fixed effect Tobit model for married females' labour supply. This work uses the utility function

$$\sum_{t=0}^{T} \frac{1}{(1+\rho)^t} \left[B_{ht}(\overline{L} - H_t)^\alpha + B_{ct}C_t^\gamma\right], \tag{41}$$

where $\overline{L}$ is total time available in the period, and α and γ are unknown coefficients. The age- and person-specific modifier of tastes for leisure is assumed to be related to a set of observed consumer characteristics, Z, and a stochastic component, ϵ_{1t}, according to $B_{ht} \equiv \exp(Z_t\phi + \epsilon_{1t})$. The taste modifier for consumption, B_{ct}, is not specified further.

The wage rate equation is

$$\ln w_t = X_t\beta + \epsilon_{2t}, \tag{42}$$

where X is a vector of observed consumer characteristics, β is the corresponding vector of coefficients, and ϵ_{2t} is a stochastic component. Identification of α, ϕ and $(\rho - r)$ requires that X includes at least one variable that is not included in Z, cf. equation (42) and (46).

It is also assumed that the disturbance vector $(\epsilon_{1t}, \epsilon_{2t})$ follows a components of variance scheme,

$$\epsilon_{jt} = \eta_j + \mu_{jt}, \quad j = 1, 2, \tag{43}$$

where the vector (μ_{1t}, μ_{2t}) is allowed to be correlated within-periods, but is assumed to be serially uncorrelated and generated by a bivariate normal distribution. The components η_1 and η_2 are left freely correlated. Thus, the econometric model has the structure of a bivariate Tobit model, cf. also [13]

Assuming $\ln[(1+\rho)/(1+r)]$ can be approximated by $\rho - r$, the labour supply function corresponding to equation (13) now becomes

$$\ln(\overline{L} - H_t) = \begin{cases} f + Y_t + V_{1t} & \text{if } \; V_{1t} \leq -f - Y_t + \ln \overline{L} \\ \ln \overline{L} & \text{if } \; V_{1t} > -f - Y_t + \ln \overline{L}, \end{cases} \tag{44}$$

where

$$f = \frac{1}{\alpha - 1}(\ln \lambda_0 - \ln \alpha - \eta_1 + \eta_2), \quad V_{1t} = \frac{1}{\alpha - 1}(\mu_{2t} - \mu_{1t}) \tag{45}$$

and

$$Y_t = \frac{\rho - r}{\alpha - 1} t - Z_t \frac{\phi}{\alpha - 1} + X_t \frac{\beta}{\alpha - 1}. \tag{46}$$

This approach then takes into consideration that labour supply has two different dimensions, cf. the discussion of equation (13), and the simultaneous likelihood function for the model consisting of the labour supply function (44) and the wage rate equation can be found. Since f includes the marginal utility of wealth, f is correlated with all the exogenous variables of the model, and to overcome this problem, it is treated as a fixed effect to be estimated.

Again there is no reduced form equation for λ_0 or f. Heckman and MaCurdy approximate, and regress estimated fixed effects, f, (obtained from the ML-estimation) on education, average household income, future fertility plans, premarital work experience and realized fertility measures. The effects of lifetime prices and initial wealth as determinants of demand, are ignored.

Compared with the estimation strategy found in MaCurdy [34], this estimation method reduces the possible selection bias from using a subsample of unconstrained persons. Regarding the estimation of the responses to current price changes along a given life cycle price path, both methods have in common that they only require observations of variables related to the actual good in question. Both methods typically require approximations to the true relationship for the reduced form equation for the marginal utility of wealth. The way they estimate the fixed effects (F_0 and f) is different. While MaCurdy's approach involves a quite cumbersome procedure, f is estimated simultaneously with all the other parameters in the approach of Heckman and MaCurdy. Both methods require panel data (Heckman and MaCurdy use panel data for eight years) for estimation of these effects. Depending on the parameter specifications, Heckman and MaCurdy can estimate the parameters determining the responses to current price changes along a given life cycle price path, from cross section data. In contrast, MaCurdy's fixed effect approach requires panel data for at least two periods.

3.3 Reduced form estimation

In this section we review some works where it is possible to find the explicit solution for the household's optimization problem.

Bover [8] recognizes that when preferences can be described by a Stone-Geary function, it is possible to find the reduced form equation for λ_0. This equation can then be substituted into the Frisch demands.

Assuming the Stone-Geary utility function

$$\sum_{t=0}^{T}(1+\rho)^{-t}[\beta_{1it}\ln(\gamma_h - H_{it}) + \beta_{2it}\ln(C_{it} - \gamma_c)], \tag{47}$$

no income taxation and the lifetime wealth constraint (26), the reduced form for λ_0 is

$$\lambda_{i0} = \frac{\sum_{t=0}^{T}(1+\rho)^{-t}(\beta_{1it} + \beta_{2it})}{A_{i0} + \sum_{t=0}^{T}(1+r)^{-t}(\gamma_h w_{it} - \gamma_c)}. \tag{48}$$

Here γ_h, γ_c and ρ are unknown coefficients, β_{1it} and β_{2it} are taste modifiers, and A_{i0} is the initial stock of assets measured in real terms.

As we would expect from the discussion of equation (25), λ_0 is a function of preferences, initial wealth, and the lifetime price paths, here represented by the (real) interest, r, and wage rate, w_{it}. Since no data sets include all these variables, estimation requires further assumptions. Bover assumes that $\beta_{1it} = 1 - \beta_{2it} = \beta_{it}$, that the time horizon is infinite and that $\rho = r$, where r is not observed. She also specifies the wage equation

$$w_{it} = l_{0i} + l_{1i}t + \nu_{it}, \tag{49}$$

where l_{0i} and l_{1i} are "unspecified functions of time invariant determinants of wages".

The earnings function now becomes

$$w_{it}H_{it} = \gamma_h w_{it} - \beta_{it}\left[\gamma_h \hat{l}_{0i} + \frac{\gamma_h}{r}\hat{l}_{1i} + \frac{r}{1+r}A_{i0} - \gamma_c\right], \tag{50}$$

where $\hat{l}_{0i}$ and $\hat{l}_{1i}$ are unbiased estimates of l_{0i} and l_{1i} obtained from a separate estimation of the wage equation. Assuming further that β_{it} can be related to observable variables and a random error, Bover uses this specification as basis for estimating a linear earnings equation on w_{it}, $\hat{l}_{0i}$, $\hat{l}_{1i}$, A_{i0}, individual characteristics, and interactions of characteristics with $\hat{l}_{0i}$, $\hat{l}_{1i}$ and A_{i0}, by minimizing a generalized instrumental variables criterion function. From this estimation she obtains estimates of all the parameters of the utility function, including the time preference rate $\rho = r$.

Regarding the properties of this approach, we notice that it is based on the relevance of the Stone-Geary utility index. To find the reduced form expression for

λ_0, one has to assume that there are no binding constraints in any market in any period. Estimation requires observations of variables related to the actual good, initial assets and demographic variables, but since most data sets do not include observations of initial assets, estimation can be more complicated than it may look like at first glance. Bover also disregards income taxation, and this decision simplifies estimation and reduces the data requirement.

An explicit solution of the optimization problem is also obtained in [3], where Biørn estimates a Stone-Geary utility index with the household's aggregate consumption as the only argument in the utility index. By splitting income at the beginning of the planning horizon into permanent and transitory income, he relates consumption to permanent and transitory incomes.

In [9], Bover modifies her Stone-Geary specification described above, to allow for habits;

$$\sum_{k=t}^{T}(1+\rho)^{t-k}\left[\beta_{1ik}\ln(\gamma_h+\phi H_{ik-1}-H_{ik})+\beta_{2ik}\ln(C_{ik}-\gamma_c)\right]. \tag{51}$$

This specification implies that the utility from supplying H_k hours of labour depends on H_{k-1}, and the ϕ-parameter reflects to what extent the household considers last year's labour supply in its determination of current labour supply.

Since we do not observe $(H_k-\phi H_{k-1})$ (or ϕH_{k-1}), it seems difficult to estimate this specification, but rewriting the wealth constraint in terms of $H_k^+ \equiv H_k-\phi H_{k-1}$, the Frisch labour supply and consumption demand functions become

$$H_{it}^+ = \gamma_h - \frac{\beta_{1it}}{w_{it}^+}\lambda_{0i}^{-1} \quad \text{and} \quad C_{it}=\gamma_c+\beta_{2it}\lambda_{0i}^{-1}, \tag{52}$$

where

$$w_{ik}^+ = \sum_{j=0}^{T-k}\left(\frac{\phi}{1+r}\right)^j w_{ik+j}. \tag{53}$$

Bover now allows for wage replanning[9]. The specification of the wage equation is similar to equation (49), but it is now assumed that l_0 and l_1 are both time- and

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[9] This assumption introduces a particular kind of uncertainty, but according to Bover there is no obvious extension to allow for general uncertainty.

person-specific. It is still assumed that $\beta_{2ik} = 1 - \beta_{1ik}$, and that the taste modifier β_{1ik} can be related to observable consumer characteristics, Z_{ik}, and a random error. Using a plausible approximation to the reduced form for λ_t, the earnings function corresponding to equation (50) becomes

$$\begin{aligned} w_{it}H_{it} &= \gamma_h w_{it} + \phi w_{it}H_{it-1} + a_1 Z_{it} + a_2 Z_{it}\hat{w}_{it} + a_3 Z_{it}\hat{l}_{1it} \\ &+ a_4 Z_{it}A_{it} + a_5 H_{it-1}Z_{it}\hat{w}_{it} + a_6 H_{it-1}Z_{it}\hat{l}_{1it}, \end{aligned} \tag{54}$$

where a_j, $j = 1, 2, \cdots, 6$, are nonlinear functions of the structural parameters, and $\hat{w}_t$ is the predicted wage rate from the estimated wage equation. Notice that the habit parameter ϕ from the utility function, is the coefficient on the variable $w_{it}H_{it-1}$.

Estimation of this equation allows for identification of all the structural parameters. Despite the fact that this equation does only include current variables and variables lagged one period, estimation requires panel data for more periods in order to estimate the wage equation. (Bover uses 10 years of panel data from the Michigan Panel of Income Dynamics.) Notice also that in contrast to specification (50), that includes initial assets, this specification includes current assets, a great advantage from an econometric point of view. With the exception that this specification considers habits, the approach is quite similar to Bover's approach discussed above, and the properties of these two approaches are quite similar.

3.4 Two-Stage Budgeting

According to Blundell [7], empirical specifications that are based on parametrization of the direct utility, can often be shown to be unnecessarily restrictive compared to dual representations. Blundell [7], and Blundell and Walker [6], therefore suggest to use the two-stage budgeting theory for estimation. This approach is particularly relevant for estimation of the parameters of within-period preferences. At the end of this section we show that this approach can be combined with a particular use of the Euler equation in order to estimate the parameters of the monotonic transformation of within-period preferences. For simplicity we assume that there are no binding constraints in any market.

Assuming intertemporal additive separability, the life cycle theory can be viewed as a two-stage budgeting process. In the first step the household adjusts saving according to the Euler equation (9). We recall that full income, Y_t, is defined as the sum of interest incomes, the value of the household's time endowment, and asset decumulation, and when the household has decided how much to save, full income is determined. From equation (25) it is clear that the distribution of full income depends on all lifetime price paths, initial and terminal wealth, and the parameters determining lifetime preferences. Estimation of this relationship is then rather data demanding.

Conditional on full income, the household in the next step determines the allocation of full income on all goods. In order to simplify notation, assume $\underline{C}_t$ is the vector of consumption goods with corresponding price vector $\underline{p}_t$, and that C_t is an arbitrary good among these goods with corresponding price p_t. If there are no binding constraints in any market, the within-period demands are

$$\underline{C}_t = \underline{f}_t(\underline{p}_t, Y_t), \tag{55}$$

where $\underline{f}_t$ is the vector of demand functions.

These functions are labelled y-conditional demands since the consumption vector is chosen conditional on full income. They are homogenous of degree zero in prices, $\underline{p}_t$, and full income, Y_t, and the functional form varies across goods. According to Blundell and Walker [6], the demand function for all goods will generally change if there are binding constraints in any market in period t. Binding constraints in the future only change the value of Y. The assumption of intertemporal additive separability then allows for decentralization over time.

Although we are assuming perfect certainty, we notice that the introduction of uncertainty about future prices and preferences leaves the two-stage budgeting model almost unchanged, since all uncertainty is captured through the distribution of full income across the life cycle.

Compared with the Frisch demands, the y-conditional demands only include variables that can be observed, and they can be observed within the current period. Estimation of the demand function for a particular good requires observations of the demand for the good, the price vector $\underline{p}_t$ and full income. Since many surveys are either income or expenditure surveys, it can be hard to find data sets that include all these variables.

In order to get more insight in the y-conditional approach, assume within-period utility is of the general Gorman Polar form (37). While the fixed effect approach requires a particular specification of the price indices $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$, the y-conditional approach does only require that $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ are concave linear homogenous functions of $\underline{p}_t$, cf. Blundell [7]. Blundell and Walker [6] discuss particular forms of these functions, with emphasis on their flexibility and how they can be related to demographic variables, but we do not go further into these items here.

Using Roy's identity, the within-period demand for a particular good in the vector $\underline{C}_t$ is given by

$$C_t = a_t'(\underline{p}_t) + \frac{b_t'(\underline{p}_t)\left[Y_t - a_t(\underline{p}_t)\right]}{b_t(\underline{p}_t)}, \tag{56}$$

where $a_t'(\underline{p}_t)$ and $b_t'(\underline{p}_t)$ are the derivatives of $a_t(\underline{p}_t)$ and $b_t(\underline{p}_t)$ with respect to the price of the good, p_t.

While this specification is linear in Y_t, estimation of the y-conditional demands does not require that. In contrast to the Frisch demands, which include the partial derivative of the monotonic transformation of the within-period utilities, the specification of the y-conditional demands is independent of the choice of this transformation. This means that for general choices of the monotonic transformation, it is impossible to derive estimates of the parameters of this function from estimating y-conditional demands.

The two-stage procedure implies that Y_t is predetermined. Despite this fact, Blundell claims there is no reason to assume that Y_t is exogenous from an econometric point of view. Since we are assuming perfect certainty, the reason must be that there are variables that are known for the consumer, but unobserved for the econometrician. If these variables are autocorrelated, Y_t will be correlated with the error term of the y-conditional demands, and estimation requires an instrument variable method. Blundell suggests a method for estimation, and testing the exogeneity assumption, proposed by Hausman [23].

Blundell also shows that if there are no binding constraints in the credit market, the parameters of the monotonic transformation of within-period utility can be estimated by utilizing the Euler equation (9), cf. also Hall [20]. To clarify this method, assume the indirect within-period utility is $F_t[U_t(Y_t, \underline{p}_t)]$. The marginal utility of wealth is now $\lambda_t^* = F'_{tu} U'_{ty}$, where F'_{tu} and U'_{ty} denote partial derivatives with respect to $U_t(Y_t, \underline{p}_t)$ and Y_t. Substituting this result into the Euler equation leads to

$$U'_{ty} = (1+\rho)\frac{F'_{t-1u}}{F'_{tu}}\left[\frac{1}{1+R_t}U'_{t-1y}\right], \tag{57}$$

where the partial derivatives can be estimated by using the parameter estimates from the estimation of the y-conditional demands, and observations of Y_t and $\underline{p}_t$. Depending on the parametrization of the F-function, we can use panel data for two or more periods to estimate ρ and the parameters of the F-function. In section 4.3 we show how this approach modifies in the uncertainty case.

Blundell uses this approach for a particular specification of the Gorman Polar form (37), that is

$$F_t[U_t(Y_t, \underline{p}_t)] = \frac{\alpha_t (Y_t^*)^\beta}{\beta}, \tag{58}$$

where $Y_t^* = [Y_t - a_t(\underline{p}_t)]/b_t(\underline{p}_t)$ is "real supernumary" outlay, and α_t and β are coefficients.

Equation (57) now becomes

$$\Delta \ln Y_t^* \equiv \sigma \Delta \ln \alpha_t - \sigma \Delta \ln b_t(\underline{p}_t) + \sigma \ln\left(\frac{1+R_t}{1+\rho}\right), \tag{59}$$

where $\sigma \equiv 1/(1-\beta)$. Assuming α_t is normalized such that $\alpha_0 = 1$, and that the time preference rate ρ is known, we can estimate β and α_t, for $t = 1, 2, ..., T$, if we have panel data for full income, the price vector $\underline{p}_t$ and R_t, for all the periods $t = 0, 1, 2 \ldots T$.

While the fixed effect approach restricts the specification of the monotonic transformation of within-period utilities, cf. section 3.1.3, there is no such a priori restrictions in the present method. It therefore seems to allow for more flexible preferences, cf. Blundell [7]. However, estimation of a rich specification of the F-function tends to require panel data for many years, cf. specification (59), and if we do not have access to all these data, the theoretical advantage can be reduced in empirical applications.

4 Estimation in the Uncertainty Case

This section presents the main approaches for estimating life cycle models in the case that the household is uncertain about future prices and variables influencing future preferences. To get a basic understanding of the various estimation strategies, we first study the modifications of the optimization problem, and the interpretation of the first order conditions, compared to the perfect certainty case.

4.1 Consumption and Labour Supply Behaviour

We assume that the household is uncertain about future prices and variables influencing future preferences. In contrast, current exogenous (pre-tax) prices as well as variables influencing current preferences, are realized at the beginning of the period, and they are then known. At each age t the household[10] maximizes the expected value of the time-preference-discounted sum of total utility,

$$U_t(C_t,\ K_t,\ H_t) + \frac{1}{1+\rho}\mathrm{E}_t \left\{ \sum_{k=t+1}^{T} (1+\rho)^{t+1-k} U_k(C_k,\ K_k,\ H_k) \right\}, \qquad (60)$$

with respect to $C_{t'}$, $K_{t'}$ and $H_{t'}$, for $t' = t, t+1, \cdots, T$, subject to the within-period wealth constraints, the borrowing constraints and the labour supply constraints pre-

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[10] For simplicity we assume a single person household.

sented in section 2.1, and given values of A_{t-1}, $(1-\delta_f)K_{t-1}$ and A_T. Here p_t, q_t, w_t, r_t and all the parameters of the tax function S_t as well as the value of current taste modifier variables are known in period t, while $p_{t'}$, $q_{t'}$, $w_{t'}$, $r_{t'}$ and all the parameters of the tax function $S_{t'}$, for $t' > t$, as well as the value of future taste modifier variables, are not known. The expectation operator E_t indicates that the household accounts for all information available in period t, in the calculation of expected values. Since households are assumed to have rational[11] expectations, E_t denotes both the household's subjective expectation and the mathematical conditional expectation as of period t.

To study this optimization problem, we can use a dynamic programming formulation. According to Rust [41], our (discrete time) continuous choice process with exogenous state variables can be viewed as a controlled Markov process satisfying certain conditions. Assume then that the state variable $s_t = (y_t, z_t)$ can be divided into endogenous state variables, y_t, such as the after-tax marginal interest and wage rates and the marginal utility of wealth, and exogenous state variables, z_t, such as consumption prices and exogenous variables influencing preferences. We also require that the law of motion, $\Upsilon(dz_{t+1}|z_t)$, for the exogenous state variables is independent of the household's decisions c_t, that is (C_t, H_t, K_t), and that the realized values of the endogenous state variables depend on household's decisions according to $y_{t+1} = \Psi(y_t, c_t, z_{t+1})$ with probability one. Here Ψ is a continuously differentiable function of (y, c). The approach also requires that the transition probability density for the state variable $\{s_t\}$, $p(ds_{t+1}|s_t, c_t)$, factors as

$$\begin{aligned} p(ds_{t+1}|s_t, c_t) &= p(dy_{t+1}, dz_{t+1}|y_t, z_t, c_t) \\ &= I\left[y_{t+1} = \Psi(y_t, c_t, z_{t+1})\right] \Upsilon(dz_{t+1}|z_t), \end{aligned} \tag{61}$$

where $I = 1$ if $y_{t+1} = \Psi(y_t, c_t, z_{t+1})$ and 0 otherwise.

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[11] Begg [2] defines rational expectations as "The hypothesis of Rational Expectations asserts that the unobservable subjective expectations of individuals are exactly the true mathematical conditional expectations implied by the model itself".

In order to find Bellman's equation corresponding to the maximization problem (60), let

$$V_{t+1} \equiv V(A_t, (1-\delta_f)K_t, z_{t+1}) = \max_{C,K,H} \left\{ \mathrm{E}_{t+1} \left[\sum_{k=t+1}^{T} (1+\rho)^{t+1-k} U_k(C_k,\, K_k,\, H_k) \right] \right\} \tag{62}$$

be the value function at age $t+1$. This function shows the household's *maximum* expected lifetime utility at age $t+1$ when all the wealth, the labour supply and the borrowing constraints are satisfied, the household is endowed with initial wealth equal to A_t and $(1-\delta_f)K_t$, and the value of A_T is given. Since V_{t+1} is a function of all the parameters generating future preferences and prices[12], and the household is uncertain about their realized values as of age t, V_{t+1} is uncertain at age t.

Bellman's equation corresponding to the household's optimization problem (60) now becomes

$$\begin{aligned} & V(A_{t-1}, (1-\delta_f)K_{t-1}, z_t) \\ = \; & \max_{C,K,H,A} \left\{ U_t(C_t,\, K_t,\, H_t) + \frac{1}{1+\rho} \int V(A_t, (1-\delta_f)K_t, z_{t+1}) \Upsilon(dz_{t+1}|z_t) \right\} \\ - \; & \lambda_t \left[A_t - (1+r_t)A_{t-1} - w_t H_t + S_t(r_t A_{t-1}, w_t H_t) + p_t C_t + q_t \left[K_t - (1-\delta_f)K_{t-1} \right] \right] \\ + \; & \gamma_t (A_t + \kappa q_t K_t) + \alpha_t H_t. \end{aligned} \tag{63}$$

The first order conditions are

$$\frac{\partial U_t}{\partial C_t} = \lambda_t p_t, \tag{64}$$

$$\frac{\partial U_t}{\partial K_t} = \lambda_t q_t - \frac{1}{1+\rho} \int \frac{\partial V(A_t, (1-\delta_f)K_t, z_{t+1})}{\partial[(1-\delta_f)K_t]} (1-\delta_f) \Upsilon(dz_{t+1}|z_t) - \gamma_t \kappa q_t, \tag{65}$$

$$-\frac{\partial U_t}{\partial H_t} = \lambda_t m_t + \alpha_t \tag{66}$$

and

$$\frac{1}{1+\rho} \int \frac{\partial V(A_t, (1-\delta_f)K_t, z_{t+1})}{\partial A_t} \Upsilon(dz_{t+1}|z_t) - \lambda_t + \gamma_t = 0. \tag{67}$$

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[12] We suppress these arguments for simplicity.

The Envelope Theorem implies

$$\frac{\partial V(A_{t-1},(1-\delta_f)K_{t-1},z_t)}{\partial[(1-\delta_f)K_{t-1}]}(1-\delta_f) = (1-\delta_f)\lambda_t q_t$$

and

$$\frac{\partial V(A_{t-1},(1-\delta_f)K_{t-1},z_t)}{\partial A_{t-1}} = (1+R_t)\lambda_t.$$

Thus, the first order condition for durables, and the updating equation for the marginal utility of wealth, can be rewritten as

$$\frac{\partial U_t}{\partial K_t} = \lambda_t q_t - \frac{1-\delta_f}{1+\rho}\mathrm{E}_t[\lambda_{t+1}q_{t+1}] - \gamma_t \kappa q_t \tag{68}$$

and

$$\lambda_t = \frac{1}{1+\rho}\mathrm{E}_t[(1+R_{t+1})\lambda_{t+1}] + \gamma_t. \tag{69}$$

The first order conditions are quite similar to the conditions in the perfect certainty case, cf. section 2. The main difference is that in an environment of uncertainty, the marginal utility of wealth, λ_{t+1}, and the after-tax interest rate, R_{t+1}, become stochastic. In the case of no current binding constraint in the credit market, the Euler equation (69) says that saving should be adjusted until the marginal utility of wealth in the current period, equals *expected* marginal utility of using the same resources next year added after-tax interest incomes. If the household is constrained in the credit market, the marginal utility is greater than the expected marginal utility.

If there are no binding constraints in any market, demand is determined by substituting the value of λ_t into the demand functions. If the good is a durable good, the value of $\mathrm{E}_t[\lambda_{t+1}q_{t+1}]$ must also be substituted into the demand function. To complete this view of behaviour, we also assume that the household at the beginning of the planning period calculates λ_t and $\mathrm{E}_t[\lambda_{t+1}q_{t+1}]$, and that it uses all available information about future prices and variables influencing future preferences in this calculation. As time passes by, the household continuously acquires new, unanticipated information about these variables. When the forecasting errors are realized, the household (continuously) revises the value of $\mathrm{E}_t[(1+R_{t+1})\lambda_{t+1}]$ and

$E_t[\lambda_{t+1} q_{t+1}]$, and simultaneously adjusts demand according to the unanticipated elements. The effects of unanticipated price changes are then taken account of only through the changes in $E_t[(1+R_{t+1})\lambda_{t+1}]$ and $E_t[\lambda_{t+1} q_{t+1}]$, while the responses to anticipated price changes can be measured by the coefficients on the (current) price variables, cf. equation (75). The importance of distinguishing between these two aspects has been recognized since the works of Heckman [24, 25], and Ghez and Becker [18].

To improve the knowledge about the stochastic process generating the marginal utility of wealth, notice that we can always express the actual value of $\ln \lambda_t$ as the sum of its expected value viewed from period $t-1$, and a one period forecast error ϵ_t^* that represents unanticipated realizations of the stochastic variables in period t. It can be shown, cf. MaCurdy [36], that this decomposition implies

$$\ln \lambda_t = b_t^* + \ln \lambda_{t-1} + \epsilon_t^*, \tag{70}$$

where

$$b_t^* \equiv \ln \left[\frac{1+\rho}{1+R_t} \right] - \ln \left[E_{t-1}(e^{\epsilon_t^*}) \right]. \tag{71}$$

Since R_t is not stochastic, b_t^* is deterministic.

Repeated substitution gives

$$\ln \lambda_t = \sum_{j=0}^{t} b_j^* + \ln \lambda_0 + \sum_{j=1}^{t} \epsilon_j^*, \tag{72}$$

and in order to adjust for the mean value of $\ln(1+R_t)$ across periods, we define $b_t^{**} = b_t^* - \overline{b^*}$, where $\overline{b^*} = (1/t)\sum_{j=0}^{t} b_j^*$. We then find

$$\ln \lambda_t = \ln \lambda_0 + \overline{b^*} t + \sum_{j=0}^{t} b_j^{**} + \sum_{j=1}^{t} \epsilon_j^*, \tag{73}$$

which indicates that in an environment where the household continuously revises its predictions of the marginal utility of wealth to account for new information, $\ln \lambda_t$ follows a stochastic process resembling a random walk with drift. Substituting this result into the first order conditions, implies that consumption and labour supply follow a nonstationary stochastic process over life cycle.

We end this section by noticing that the introduction of uncertainty seems to complicate the estimation of preferences for durables. In the perfect certainty case with no binding constraints in the credit market, we can use the Euler equation to express the first order condition for durables in terms of λ_t only, and not both λ_t and λ_{t+1}, cf. equation (23). In the uncertainty case this is more problematic, since the Euler equation includes $\mathrm{E}_t[(1+R_{t+1})\lambda_{t+1}]$, while the first order condition for durables includes $\mathrm{E}_t[\lambda_{t+1}q_{t+1}]$, cf. equations (69) and (68). Since the expectation of a product in general cannot be rewritten as the product of the expectation of the two components, we must assume that both the after-tax interest rate, and the purchase price of durables, are non-stochastic if we want to follow the strategy from the perfect certainty case.

4.2 MaCurdy's Fixed Effect Approach with Uncertainty

We are now going to study how MaCurdy [36][13] suggests to utilize the Frisch demand functions for estimation of a life cycle model of labour supply in the case of uncertainty. This paper generalizes MaCurdy [34] by allowing for uncertainty, and since the perfect certainty case is treated thoroughly in section 3.1, we focus only on the estimation problems particular to the uncertainty case.

Assume the lifetime utility index

$$\sum_{t=0}^{T}(1+\rho)^{-t}\,(\,\Gamma_{1it}C_{it}^{\omega_1}-\Gamma_{2it}H_{it}^{\omega_2}\,), \tag{74}$$

where $\Gamma_{2it}=\exp\left(-X_{it}\psi^*-\nu_{it}^*\right)$. The vector X_{it} represents measured characteristics, ψ^* is a vector of parameters and ν_{it}^* reflects unmeasured characteristics. MaCurdy disregards income taxation and the possibility of binding constraints in the credit and labour markets, and the marginal utility of wealth constant labour supply function now becomes

$$\ln H_{it}=F_{it}+a\ln w_{it}+X_{it}\psi+\nu_{it}, \tag{75}$$

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[13] He did not estimate the model in this paper.

where $a \equiv 1/(\omega_2 - 1)$, $F_{it} \equiv a[\ln \lambda_{it} - \ln \omega_2]$, $\psi \equiv a\psi^*$ and $\nu_{it} \equiv a\nu^*_{it}$. We recall that w and A (equation (77)) are measured in real terms.

As shown in section 2, F_{it} is latent and correlated with all the exogenous variables in the model. In [34] MaCurdy solved this problem by expressing the Frisch demands in terms of λ_{i0} before he applied first differencing to eliminate it. Using the same approach here, we get

$$\ln H_{it} = F_{i0} + a \sum_{j=0}^{t} b^*_{ij} + a \ln w_{it} + X_{it}\psi + \eta_{it}, \tag{76}$$

where $\eta_{it} \equiv \nu_{it} + a \sum_{j=0}^{t} \epsilon^*_{ij}$ is the disturbance term, and where b^*_{ij} is defined in equation (71). The disturbance term represents the effects from omitted variables and the accumulation of all past forecast errors.

Differencing this equation eliminates F_{i0}, but we are left with the unobservable variable b^*_{it}, that is both household- and time-specific. Since b^*_{it} includes the after-tax interest rate, it is correlated with all the exogenous variables in this equation, and the error term η_{it}. If the distribution generating the forecast errors is such that $\mathrm{E}_{t-1}(e^{\epsilon^*_{it}})$ varies with changes in household characteristics, b^*_{it} is also correlated with X_{it} and η_{it} even in the absence of income taxation. This means that without further assumptions, the first differencing approach cannot be used for estimation of this equation.

If we can assume that b^*_{ij} is constant across both households and periods, the method of first differencing can be used. The introduction of uncertainty gives an additional argument for treating the wage rate (and the wage rate growth) as well as R_t as endogenous in the estimation. Since the wage rate and the after-tax interest rate include unanticipated components, $\ln w_{it}$ and R_t are correlated with current and past values of ϵ^*_{it}. A similar argument can be applied on X_{it}, that is, for X_{it} being exogenous in the empirical analysis, it must both be known prior to period one and uncorrelated with the unobserved taste factors ν_{it}. This fact is also of significance for the choice of instrument variables, in that, for a variable to be a valid instrument, it must be uncorrelated with all unanticipated components.

We then conclude that if we can assume that b_{ij}^* is constant across both consumers and time, the parameters determining the responses to anticipated price changes can be estimated from panel data for at least two periods for the actual good in question. In most cases, however, this assumption seems quite unreasonable.

Regarding the estimation of the reduced form equation for the marginal utility of wealth, MaCurdy suggests to approximate the relationship corresponding to equation (29), by

$$F_{it} = Z_{it}\phi_t + \sum_{j=t}^{T} \gamma_{tj} \mathrm{E}_t \ln w_{ij} + A_{it}\theta_t + a_{it}, \tag{77}$$

where subscript t on all the parameters and the variables indicates that they are observed or anticipated at age t. MaCurdy interprets A_{it} as the stock of assets at the beginning of period t, and it is then exogenous for the household as of period t.

Since the household is assumed to continuously replan as it acquires new information, there is one (different) equation for each year. Subscript t is introduced on all parameters to take account of that the parameter values may change systematically with age. The reason is that, as the household updates expectations about the distribution of future preferences and prices, this may well change the mathematical form of the reduced form expression for the marginal utility of wealth. Hence, the (mathematical) form of this equation may change across periods, but in order to simplify, MaCurdy assumes that only the parameter values are changed.

Apart from the huge problems related to obtaining information about the households' expectations about future wages and other exogenous variables, the fact that F_{it} varies across both households and periods, makes it hard to think of how we can estimate these equations without further assumptions. Recall also that even for the most simple models, it is impossible to find the reduced form expressions for λ_{it}. We then conclude that MaCurdy's fixed effect approach cannot be used for estimation of life cycle models under uncertainty.

4.3 Marginal Rate of Substitution Functions

This section focuses on how the marginal rate of substitution functions[14] can be utilized for estimation of within-period preferences, cf. Altonji [1] and MaCurdy [35]. We also show how this approach can be combined with a particular use of the Euler equation to estimate the parameters of the monotonic transformation of within-period preferences, cf. also the perfect certainty case in section 3.4.

When we wish to estimate the first order equations (64), (66) and (68), it is a problem that λ_t is latent[15]. These equations can, however, be viewed as (three) separate expressions that can be used for elimination of λ_t. Using the first order conditions for consumption and labour supply, we find

$$\frac{-\alpha_t - \partial U_t/\partial H_t}{\partial U_t/\partial C_t} = \frac{m_t}{p_t}. \tag{78}$$

Here $-(\partial U_t/\partial H_t)/(\partial U_t/\partial C_t)$ is the marginal rate of substitution between leisure and non-durables, and if the consumer is not constrained in the labour market, the equation implies that, in equilibrium the person sets the marginal rate of substitution between leisure and non-durables equal to the real marginal wage rate.

Choosing a particular specification of the utility function, equation (78) does only include variables that can be observed, the unknown parameters of the within-period preferences, and the Lagrange multipliers of eventual binding constraints. Using a subsample of unconstrained households, the marginal rate of substitution functions can then be used for estimation of the parameters of within-period preferences[16].

If preferences are separable within periods (in addition to intertemporal separable), estimation requires cross section observations of prices and consumption of at least two goods, and the taste shifter variables. We also need a set of instrument

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[14] According to Rust [41] this method can be thought of as a special case of the method of moments, cf. the next section, but with a different orthogonality condition.

[15] In the estimation of equation (68) it is also a problem that $E_t[\lambda_{t+1} q_{t+1}]$ is latent.

[16] Since the various parameters of the marginal rate of substitution equations may be functions of two or more parameters from the utility function, we may, however, be confronted with an identification problem.

variables since the marginal rate of substitution functions generally will be (nonlinear) simultaneous equations. If within-period preferences are non-separable, we must observe the consumption of all goods, but we can still use cross section data.

Equation (78) cannot be used for estimation of the monotonic transformation of within-period preferences. Assume that the within-period utility is

$$U_t = F_t\left[U_t(C_t K_t,\, H_t)\right], \tag{79}$$

where F_t is a monotonically increasing function. The equilibrium condition corresponding to equation (78) is now identical with equation (78), and it is impossible to identify the parameters of the F-transformation.

Since the Euler equation determines the household's relative preferences for consumption and leisure in the various periods, it seems natural to use this equation for estimation of the F-transformation. Utilizing also the first order condition for consumption, we obtain

$$\frac{F_t' \partial U_t/\partial C_t}{p_t} = \frac{1}{1+\rho}\mathrm{E}_t\left[(1+R_{t+1})\frac{F_{t+1}' \partial U_{t+1}/\partial C_{t+1}}{p_{t+1}}\right] + \gamma_t, \tag{80}$$

where F_t' is the partial derivative of F_t with respect to U_t. Given this expression we can also write

$$\frac{F_{t+1}' \partial U_{t+1}/\partial C_{t+1}}{p_{t+1}} = \frac{1+\rho}{1+R_{t+1}}\left[\frac{F_t' \partial U_t/\partial C_t}{p_t} - \gamma_t\right](1+e_{t+1}), \tag{81}$$

where e_{t+1} is the forecast error.

If the household has rational expectations, e_{t+1} is uncorrelated with

$$\frac{1+\rho}{1+R_{t+1}}\left[\frac{F_t'}{p_t}\frac{\partial U_t}{\partial C_t}\right],$$

and specification (81) can be used for estimation of the parameters of the F-function, and the time preference rate ρ, cf. the discussion of equation (57). Notice that even if within-period preferences are additive, this specification requires panel data (for at least two periods) for *all* goods influencing preferences. The reason is that even though $\partial U_t/\partial C_t$ does only depend on C_t, the partial derivative of F depends on all goods. Under particular circumstances, among them that the parameters of the F-function is independent of household, estimation may also be possible with time series data.

Estimation requires information about eventual binding constraints in the credit market since the Lagrange parameter γ_t equals zero only if the household is unconstrained in the credit market. If F_t includes a stochastic component, the right hand side variables of equation (81) may be correlated with the error term. Assume

$$F_t\left[U_t(C_t,\, K_t,\, H_t)\right] = \Omega_t \frac{[U_t(C_t,\, K_t,\, H_t) + \delta]^{\sigma} - 1}{\sigma}, \tag{82}$$

where Ω_t is related to household characteristics, X_t, and a stochastic component, φ_t, according to $\Omega_t = \exp\{X_t\phi + \varphi_t\}$. The error term of equation (81) now becomes $\varphi_t - \varphi_{t+1} + \ln[1 + e_{t+1}]$. Since it includes φ_t, the right hand side variables can be correlated with the error term, and they must be treated as endogenous in the empirical analysis. If, however, F_t is deterministic, the error term only includes the prediction error, and the right hand side variables can be considered as predetermined in the estimation.

According to MaCurdy, this estimation method is sensitive to measurement errors. If the error term of equation (78) includes measurement errors, the predictions of the variables in equation (81) are biased. Since the measurement error does not satisfy $X_t = X_t^* + \vartheta$, where X_t^* is the true value and ϑ is an error term that is randomly distributed and independent of all instrument variables, the parameter estimates will be inconsistent.

The approach discussed above allows for a rich class of utility functions, since we do not need to find the closed form solution for the demand functions. We do not either need making any assumptions regarding the households' anticipations about future prices including the interest and the tax rates, and the variables influencing future preferences. As it is presented here, it is also assumed that we can ignore the possible selection problem from using a subsample of unconstrained households in the estimation. In empirical applications of this approach it may be a problem that it is difficult to find good instruments for the right-hand-side-variables of the marginal rate of substitution functions. Estimation of the transformation function F is also data demanding.

4.4 The Generalized Method of Moments

In this section we review Hansen and Singleton's [22] generalized method of moments (GMM). A recursive application of this approach has been used in [29], where Hotz, Kydland and Sedlacek allow for a particular kind of intertemporal non-separability in an analysis of labour supply. The estimation method in MaCurdy [35] can also be viewed as a variant of this approach, and the approach is also described in Rust [41].

The basic idea underlying the generalized method of moments, is that economic models typically yield a set of stochastic Euler equations that characterize optimum. These equations imply a set of population orthogonality conditions that depend on the parameters of the objective/utility function and some observable variables. By setting the sample analogies equal zero (according to a certain metric), we can estimate the parameters of the utility function (and subsets of the parameters determining the law of motion of the state variables).

In order to clarify this approach, notice that in the case of no binding restrictions in any market, the Euler equation (69) and the Frisch labour supply function (66) imply

$$\mathrm{E}_t \left[\frac{1}{m_t} \frac{\partial U_t}{\partial H_t} - \frac{1 + R_{t+1}}{1 + \rho} \frac{1}{m_{t+1}} \frac{\partial U_{t+1}}{\partial H_{t+1}} \right] = 0. \tag{83}$$

That is, the conditional expectation evaluated at the true parameter values equals zero.

Depending on what variables are observed and for what periods, various combinations of the first order conditions can be substituted into the Euler equation. Hansen and Singleton assume a representative consumer, and use time series data (national accounts), but this approach can also be applied to cross section or panel data sets where the orthogonality conditions are based on averages for the consumers. In this case consistency of the estimated parameters requires that the state and control variables of the Euler equation are uncorrelated across consumers.

The orthogonality equation (83) does only include variables that can be observed, and the parameters of the utility function. To simplify notation, write the set of orthogonality conditions that are to be estimated as

$$\mathrm{E}_t g(x_t, \tilde{x}_{t+1}, \theta^*) = 0, \tag{84}$$

where $x_t = (y_t, z_t, c_t)$ is defined in section 4.1, $\tilde{z}_{t+1}$, $\tilde{y}_{t+1}$ and $\tilde{c}_{t+1}$ are the corresponding random variables with respect to the household's information set in period t, and θ^* is the vector of the true parameters of the utility function. The value of $\tilde{y}_{t+1}$ evaluated at the true parameter $\theta = \theta^*$ is given by $\tilde{y}_{t+1} = \Psi(y_t, c_t, \tilde{z}_{t+1}, \theta^*)$ with probability one, and $\tilde{c}_{t+1} = \delta^*(\tilde{y}_{t+1}, \tilde{z}_{t+1})$, where δ^* is the optimal decision rule.

Notice that $g(x_t, \tilde{x}_{t+1}, \theta^*)$ can be interpreted as the disturbance vector, u_t, in the estimation, and that the orthogonality conditions must be chosen such that the matrix $u_t u_t'$ has full rank.

Using the law of iterated expectations, equation (84) implies

$$\mathrm{E} g(x_t, \tilde{x}_{t+1}, \theta^*) = \mathrm{E}\left[\mathrm{E}_t g(x_t, \tilde{x}_{t+1}, \theta^*)\right] = 0, \tag{85}$$

that is, the unconditional expectations also equal zero. According to Dudley [16], if the unknown parameters are independent of time t, and if $\{x_t\}$ is a strictly stationary and ergodic[17] process, the time averages of functions of x_t converge to their unconditional expectations with probability one,

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} g(x_t, \tilde{x}_{t+1}, \theta^*) = \mathrm{E}\left[g(x_t, \tilde{x}_{t+1}, \theta^*)\right] = 0. \tag{86}$$

By analogy we estimate θ^* by that value of $\hat{\theta}$ that makes the sample average $\sum_{t=1}^{T} g(x_t, \tilde{x}_{t+1}, \hat{\theta})$ close to zero.

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[17] Confer Cox and Miller [14] page 92-93 for a definition.

If the number of unknown parameters is greater than the number of orthogonality conditions, we cannot identify the parameters. Hansen and Singleton then propose to use a vector of instrument variables, v_t, that can be any function of the household's information at age t, provided it is observed by the econometrician. Apart from this, the only requirement is that v_t is predetermined as of time t, and current and lagged values of y_t can then be used. In this case the Kronecker product $\mathrm{E}_t g(x_t, \tilde{x}_{t+1}, \theta^*) \otimes v_t \equiv f(x_t, \tilde{x}_{t+1}, v_t, \theta^*)$ equals zero, since v_t can be treated as a constant in the calculation of the expectations. By analogous reasoning that lead to equation (86), we find that

$$G_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} f(x_t, \tilde{x}_{t+1}, v_t, \theta), \tag{87}$$

evaluated at the true value of $\theta = \theta^*$, should approximate zero for large values of T. Assuming $\{x_t, v_t\}$ is a stationary, ergodic process, Hansen and Singleton define the GMM estimator $\hat{\theta}$ as that value of θ that minimizes

$$J_T(\theta) = G_T(\theta)' W_T G_T(\theta), \tag{88}$$

where W_T is a symmetric, positive definite weighting matrix. This estimator is consistent and asymptotic normally distributed under mild regularity assumptions, cf. Hansen [21]. According to Hansen and Singleton, it is also consistent even in the case that the error terms of the orthogonality conditions (84) are serially correlated and the instruments are not exogenous, but only predetermined.

Hansen and Singleton explain how to choose W_T optimally, in order to minimize the asymptotic covariance matrix of the estimator. The estimation method then becomes a two-stage procedure: In the *first* stage, estimate θ by equation (88) using an arbitrary weighting matrix. Calculate the optimal weighting matrix W_T^* according to

$$W_T^* = \left[\hat{R}_0 + \sum_{j=1}^{m} \left(1 - \frac{j}{m+1} \right) \left(\hat{R}_j + \hat{R}_j' \right) \right]^{-1}, \tag{89}$$

where

$$\hat{R}_j = \frac{1}{T} \sum_{t=j+1}^{T} f(x_t, \tilde{x}_{t+1}, v_t, \hat{\theta}) f(x_{t-j}, \tilde{x}_{t+1-j}, v_{t-j}, \hat{\theta})', \tag{90}$$

and where m equals the number of non-zero autocorrelations in $f(x_t, \tilde{x}_{t+1}, v_t, \hat{\theta})$. Then, in the *second* stage, estimate $\hat{\theta}$ by equation (88) using the optimal weighting matrix.

Hansen [21] shows that T times the minimized value of equation (88) evaluated at the optimal weighting matrix,

$$TG_T(\hat{\theta})'W_T^*G_T(\hat{\theta}), \tag{91}$$

is Chi-square distributed with degrees of freedom equal the number of orthogonality conditions, O, minus the number of parameters, P. This result can be used for testing model specifications: In the estimation, P of the orthogonality conditions are set close to zero in order to estimate the P unknown parameters. If the number of orthogonality conditions exceeds the number of unknown parameters, $O - P$ of the orthogonality conditions are not used in the estimation, and the model is said to be overidentified. However, if the model specifications are true, these orthogonality conditions should also be close to zero. This means that if the test statistics (91) is "large", there is reason to believe that there is something wrong about our model specifications including the choice of the instrument variables. Thus, the general method of moments also provides a test for the validity of the model specifications.

4.4.1 Advantages and limitations of Euler equation methods

The main advantages of this approach are:

- Only minimal assumptions about the distribution of the unobservable exogenous state variables are required.
- GMM can be applied on time series, cross section or panel data sets. Depending on the separability property of preferences, it is possible to estimate the preferences for a subgroup of goods from time series or cross section data for only that subgroup. Repeating this approach for all subgroups, it is possible to estimate a complete life cycle model from a combination of various data sets. GMM is then quite flexible with regard to data set specifications.

- The instrument variables need only be predetermined and not exogenous, and the approach gives considerable latitude in selecting instruments.
- We do not need to explicitly solve for the value function or the optimal decision rule. Instead we can use the Euler equation and the first order conditions.
- Standard gradient algorithms can be used for estimation.
- The approach yields a useful diagnostic test statistic as a by-product.

According to Rust [41], GMM is not applicable for the following model spesifications:

- Specifications with binding constraints in the goods and/or the credit markets, cf. equations (4) and (5), and the possibility of institutional constraints in the labour market. In this case the orthogonality conditions will be inequalities and not close to zero.
- Most specifications where the unobserved state variables enter the Euler equation directly, that is, not implicitly as functions of other observed state and control variables. For instance, unobserved components of the exogenous variables cannot influence preferences directly. These variables are typically correlated with the forecast errors that arise from unanticipated realizations of prices and variables influencing preferences, and they are then correlated with the price variables and the taste modifier variables in the Euler equation. Hence they cannot be treated as a part of the error term.
- Specifications where the Euler equation contains state or control variables that reflect macroeconomic shocks that are correlated across consumers. In this case the data might not satisfy the ergodicity conditions, and the result (86) may not be valid.
- Specifications where the transition probabilities, $p(ds'|s,c)$, depend on the control c. In this case the first order conditions include the value function

V, cf. equation (62), and it seems to be no way to substitute it out of the problem. The orthogonality conditions corresponding to equation (84), now require explicit solutions to Bellman's equation, and we need explicit parametric specifications of the law of motion for all the state variables.

Regarding the point of binding constraints, in some cases we can also think of specifications where the econometrician makes use of that he knows what households will be unconstrained in the future. However, as Pakes [40] points out, "If we simply select out those observations for which it ends up being true, we will be selecting the sample on the basis of behaviour determined by information not available at date t, and any selection procedure based on such information will generate an inconsistency in the estimation procedure.".

Rust also comments on a problem with the use of the test statistic (91), that is,

- The unconditional expectation of the Euler equation can be close to zero even though the conditional expectation may be non-zero for some histories.

The point here is that while the conditional expectation of the Euler equation is zero for every history given that our model specifications are true, the criterion function (87) attempts to set the average of expectations, that is, the unconditional expectation, to zero. This means that if our model is misspecified such that the conditional expectations are non-zero, but on average equal zero, the test statistic (91) fails to reject the model specifications. See also West [43] for an interesting illustration of this problem.

5 Summary

From the previous sections it should be apparent that a great deal of effort has been spent trying to estimate structural life cycle models on micro data. Despite this effort most empirical analyses make use of strong simplifications. Most works are incomplete in the sense that they do not account for the simultaneity in the determination of labour supply and consumption, and in particular the consumption of durables. In addition most works disregard habits.

While these simplifications concern the specification of preferences, the specification of the wealth constraint is also simplified. Many analyses treat income taxes as well as the existence of bequests and inheritances superficially. Most analyses also disregard the many possibilities of constraints in the various markets, and even if they are taken into consideration, the model is usually estimated from a subsample of unconstrained households without trying to correct for the possible estimation bias.

There are many reasons for these simplifications, the most important being that, ideally, estimation of the full life cycle model requires complete life cycle data for variables such as female and male labour supply, consumption of non-durables and durables, the stock of assets and all the matching prices; including the interest and income tax rates. Since households adjust according to anticipated prices, and the expectations are continuously being updated as the household acquires new information, ideally, these expectations should be observed in each period. To allow for heterogenous households, we also need observations of taste shifter variables, and the estimation of the wage equation requires variables such as female and male education and work experience. Depending on the econometric specification, we may also need further instrument variables, and the estimation of a complete life cycle model is then very data demanding.

Estimation of the preferences for durables involves a particular problem. Economic theory is weak with respect to how to measure the stock or consumption of durables, and without a precise definition, the price is also undefined. This means that the standard first order conditions are not particularly suitable for estimation of the preferences for durables.

Estimation is considerably simplified if one is willing to assume intertemporal separability. Within-period preferences can then be estimated using only variables currently observed, cf. the equations based on the marginal rate of substitution functions or the two-stage budgeting theory.

If we can also assume intraperiod separability, it is possible to estimate the parameters determining the responses to current price changes along a given life cycle price path, using panel data for only the actual good in question, cf. MaCurdy's fixed effect approach. Since many cross section data do not include observations of both labour supply and consumption, this property is of importance.

In order to analyse the effects of shifts in the price paths, MaCurdy suggests to estimate the reduced form equation for the marginal utility of wealth. This estimation typically requires approximations to the true relationship, and a priori assumptions about lifetime price paths. A considerable weakness of this approach is that it cannot be used if the consumers are uncertain about future prices and variables influencing future preferences. It also restricts preferences more than the approaches based on the marginal rate of substitution functions and the two-stage budgeting theory.

While the equations based on the marginal rate of substitution functions and the y-conditional demands can be used for estimation of within-period preferences, they cannot be used for estimation of the monotonic transformation of within-period preferences. It is then suggested to combine these approaches with a particular use of the Euler equation. In the two-stage budgeting theory case this method requires panel data for full income and all prices, in addition to observations of the after-tax interest rate for at least the current period. On the contrary, the estimation based on the substitution functions requires panel data for all goods and one price in addition to observations of the after-tax interest rate, and this result is independent of the separability property of within-period utility. Since most data sets do no include all these variables, and since the number of years covered by panel data is typically quite short, most analyses do not estimate this transformation.

We end this paper noticing that common to all the surveyed estimation strategies is that they use the first order conditions as basis for deriving the specifications used for estimation. Compared to the real world, the first order conditions are restrictive in the sense that the household is to compare marginal utility with the marginal cost of buying the good. Concerning the labour market decisions for instance, it might be the case that the job searchers only can choose between a limited number of packages that consist of wage rate, hours of work, fringe benefits and other working conditions, cf. Dagsvik and Strøm [15]. The job searchers must then make utility comparisons between these packages, and the first order conditions do not hold. This argument is also relevant for some durables, and it implies that the predictions based on estimations of the first order conditions will be biased no matter how sophisticated the estimation procedure really is. Considering also all the simplifications pointed out at the beginning of this section, we conclude that we cannot be satisfied with the current stage of development and knowledge of the empirical life cycle model of labour supply and consumption.

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