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Abstract:
This paper combines the theory of optimal extraction of exhaustible resources with the theory of greenhouse externalities, to analyse problems of global warming when the supply side is considered. The optimal carbon tax will initially rise but eventually fall when the externality is positively related to the stock of carbon in the atmosphere. It is shown that the tax will start falling before the stock of carbon in the atmosphere reaches its maximum. If, on the other hand, the greenhouse externality depends on the rate of change in the atmospheric stock of carbon, the evolution of the optimal carbon tax is more complex. It can even be optimal to subsidise carbon emissions to avoid future rapid changes in the stock of carbon, and therefore future damages. If the externality is related to the stock of carbon in the atmosphere and there exists a non-polluting backstop technology, it will be optimal to extract and consume fossil fuels even when the price of fossil fuels is equal to the price of the backstop. The total extraction is the same as when the externality is ignored, but in the presence of the greenhouse effect, it will be optimal to slow the extraction and spread it over a longer period.

Keywords: Global warming, exhaustible resources, carbon taxes, backstop technology

JEL classification: D62, Q38

Acknowledgement: I am indebted to Michael Hoel for helping me structuring the ideas and for his comments, to Kjell Arne Brekke for valuable discussion and comments, and to Knut Alfsen, Samuel Fankhauser, Petter Frenger and Rolf Golombek for comments on an earlier draft. I am also grateful to Atle Seierstad for helpful discussion. This paper was financed by Norges forskningsråd (Økonomi og Økologi), and is a part of Metodeprogrammet at Statistics Norway.

1. Introduction

One main environmental challenge in our time is to avoid or reduce the impacts of global warming. Carbon dioxide (CO_2) is the main greenhouse gas, and 70-75% of all CO_2 emissions is due to combustion of fossil fuels (see for example Halvorsen et al. (1989)). In most papers analysing the economics of global warming, the supply of fossil fuels is modelled like any other good, and the exhaustibility of these resources is not considered. There have been many studies on externalities from fossil fuel consumption and the depletion of exhaustible resources (see, e.g., Dasgupta and Heal (1979), Baumol and Oates (1988) and Pearce and Turner (1990)), but few authors have tried to combine these two theories. Among the first studies is Sinclair (1992) who analyses the impacts on consumer and producer prices of a constant tax rate on energy use besides the development of a carbon tax. He argues that in steady state, the ad valorem carbon tax should be falling over time. This is followed up in Ulph and Ulph (1992). The authors study the evolution of an optimal carbon tax using quadratic benefit and damage functions. They find that the carbon tax (in both specific and ad valorem terms) should initially rise but eventually fall. Sinclair's conclusion, they argue, is driven by very odd assumptions. Devarajan and Weiner (1991) use a two-period model to analyse the importance of international global warming agreements, assuming that the consumption of fossil fuels in period two is the difference between the initial stock of fossil fuels and period one consumption. Finally, Withagen (1993) compares the optimal rate of fossil fuel depletion without greenhouse externality with the case where this externality is present.

In this paper I combine the theory of greenhouse externalities with the theory of exhaustible resources using the framework of a simple model described in section 2. The first problem analysed is the design of the optimal policy response. Most studies analysing the damage from global warming, specify the damage as a function of the temperature level or alternatively the atmospheric concentration of greenhouse gases (GHGs). However, because the ecology can adapt to a certain change in the climate, given that the rate of change is not too high, it can be argued that what matters is the speed of climate change (e.g., the rate of change in temperature or atmospheric concentration) and not only the level of temperature itself. This argument is taken seriously by national governments, in the way that they have committed themselves to annual emissions restrictions of GHGs.[1] Thus, in section 3 the optimal carbon tax is derived and compared for two different specifications of the damage function; one in which the damage is due to the level of global warming, and the other where damage is related to the speed of climate change.

The optimal policy response is analysed without considering possible substitutes for fossil fuels. Even today there exist several alternatives to fossil fuels, however at higher costs. The traditional result from the theory of a competitive mining industry facing a backstop technology with a constant unit cost of extraction less than the choke price, is that the industry will deplete the resource until the price reaches the cost of the backstop. At this price, the resource is exhausted, and the consumers will switch immediately to the backstop. Introducing external greenhouse effects, will, however, give some new results described in section 4. I summarise the conclusions in section 5.

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[1] One example is the Norwegian Government which aims at stabilising the annual CO_2 emissions on 1989 level by 2000.

2. The Model

This section describes the basic model of the paper. I assume that the social planner maximises the present value of welfare to the global society given a competitive market of fossil fuels. That is, he seeks an extraction path of fossil fuels which will

$$
\text{(1)} \qquad \begin{aligned} & \textit{maximise} \int_0^\infty e^{-rt} \cdot [u(x_t) - c(A_t)x_t - D(S_t)]dt \\ & \quad s.t. \quad \dot{A}_t = x_t \\ & \qquad\quad \dot{S}_t = x_t - \delta S_t \\ & \qquad\quad x_t \geq 0 \end{aligned}
$$

where x is the extraction (and consumption) of all fossil fuels in carbon units, u(x) is the benefit of the society from fossil fuels consumption, c(A)x is the extraction cost, $A_t = \int_0^t x_t dt$ is accumulated extraction up to time t, S is the atmospheric stock of carbon in excess of the preindustrial stock and D(S) expresses the negative externality of carbon consumption. $r > 0$ is a discount rate and $\delta > 0$ represents the depreciation of carbon in the atmosphere. These parameters are fixed throughout the analysis. Further I set $A_0 = 0$ and $S_0 > 0$.

The benefits from fossil fuel consumption are assumed to increase in current consumption ($u'(x) > 0$), but the marginal utility is bounded above ($u'(0) < \infty$) and decreasing ($u''(x) < 0$ and $u'(\infty) = 0$). Define $u(x) = \int_0^x p(x)dx$, where p(x) is the consumer price. The utility function, u(x), is therefore similar to the consumer surplus, and the marginal utility equals the consumer price ($u'(x) = p$).

The total extraction cost, c(A)x, increases both with the current extraction rate ($c(A) > 0$ for all A) and the cumulative extraction up to date ($c'(A)x > 0$ for $x > 0$). Furthermore, the marginal extraction cost is constant, and I assume $c''(A)x > 0$ for $x > 0$. This means that the incremental cost due to cumulative extraction rises with the amount already extracted. No fixed quantity is assumed for the total availability of the resource. However, in line with Farzin (1992), only a limited total amount will be economically recoverable at any time. This is due to the assumption $c''(A)x > 0$ for $x > 0$, which means that increasingly large quantities of the fossil fuel resource can be exploited only at increasingly higher incremental costs. With $c(A) \to \infty$ as $A \to \infty$, it will be optimal to extract a finite amount of the resource since the marginal utility is bounded above. Thus, on the optimal path, the cumulative extraction reaches an upper limit, $\bar{A}$, as $t \to \infty$. This gives $\lim_{t \to \infty} c(A)x = 0$, i.e., $c(\bar{A}) < \infty$.

The damage from global warming is a function of the atmospheric stock of carbon.[2] I have not taken into account the lags between the atmospheric stock of carbon and the climate, a lagged adjustment process which is due to the thermal inertia of oceans (see Houghton et al. (1990, 1992)). The stock is increasing in fossil fuel combustion, but decays with the depreciation rate $\delta > 0$, according to the atmospheric lifetime of CO_2. This is of course a simplification. The depreciation rate is probably not constant, but will decrease with time since a possible saturation of the carbon-sink capacity of the oceans as they get warmer, will give a longer lifetime of CO_2 in the atmosphere (see Houghton et al. (1990, 1992)). However, I stick to this assumption for the time being, since it is widely employed in economic studies of global warming, see, e.g., Nordhaus (1991, 1993), Peck and Teisberg (1992), Ulph and Ulph (1992) and Withagen (1993). The preindustrial stock is assumed to be an equilibrium stock, meaning that the atmospheric stock will approach the preindustrial level in the long run ($S \to 0$) when fossil fuels are exhausted. I assume that the damage can be described by an increasing and convex function of the atmospheric stock (i.e., $D'(S) > 0$ and $D''(S) > 0$ for all $S > 0$), but that they are negligible for the initial stock increase ($D'(0) = 0$). There is no irreversible damage which means that $D(S) \to 0$ as $S \to 0$.

From the model, we find the consumer price as the sum of the producer price and a carbon tax, v (see Appendix A). The producer price is the sum of the marginal extraction cost, $c(A)$, and the scarcity rent π. Thus, introducing a carbon tax separates the consumer price and the producer price.

(2) $$p_t = c(A_t) + \pi_t + v_t$$

In Appendix A, it is shown that the extracted amount of fossil fuels will fall over time in the absence of the externality ($D(S) = 0 \ \forall \ S$). With a constant carbon tax, v, the total extracted amount of fossil fuels will go down, but if the tax approaches zero in the long run ($v \to 0$), the accumulated extraction equals total extraction in the absence of a carbon tax.

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[2] Alternatively we could specify the damage as a function of the temperature level. However, I simplify the matters since the temperature is positively related to the atmospheric stock of carbon (see Houghton et al. (1990, 1992)).

3. The Optimal Policy Response to Global Warming

Most studies analysing the damage from global warming assume that the damage is related to the temperature level (see, e.g, Nordhaus (1991, 1993), Peck and Teisberg (1992) and Kverndokk (1994)). However, it can be argued that the damage will depend as much on the rate of temperature change as on the absolute value itself, because the ecology is able to adapt to a certain temperature change. There are however costs of adapting to a new stage (see for instance Tahvonen (1993)). If the rate of climate change is high, there may be a period of large damage until the original species have been replaced by more resistant ones. Agriculture is a sector highly dependent on the change in temperature. Another example is human beings, who are capable of adjusting to climatic variations, and can live under more or less every climatic condition existing on earth. However, rapid changes in climate have impacts on human amenity, morbidity and mortality (see Fankhauser (1992) and Cline (1992)). As the optimal policy response to global warming may depend on how we specify the damages, it is important to study the optimal carbon tax under the different specifications of the damage function. The damage from global warming probably depends on both the level of the climate as well as the rate of climate change. Therefore, damage functions taking into account only one of these elements must of course be considered as extreme cases. Analysing the two elements separately, however, points out some main features.

3.1 Damage Related to the Atmospheric Stock of Carbon

Consider first the case where damage is related to the atmospheric stock of carbon, as modelled in section 2. The optimisation problem is solved in Appendix B. The optimal solution can be implemented by a carbon tax ($\theta \geq 0$) representing the discounted future negative externalities due to accumulation of carbon in the atmosphere.

(3) $$\theta_t = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} D'(S_\tau) d\tau$$

The optimal carbon tax has the following properties (see Appendix B), where $\dot{\theta}$ is the time derivative of the tax:

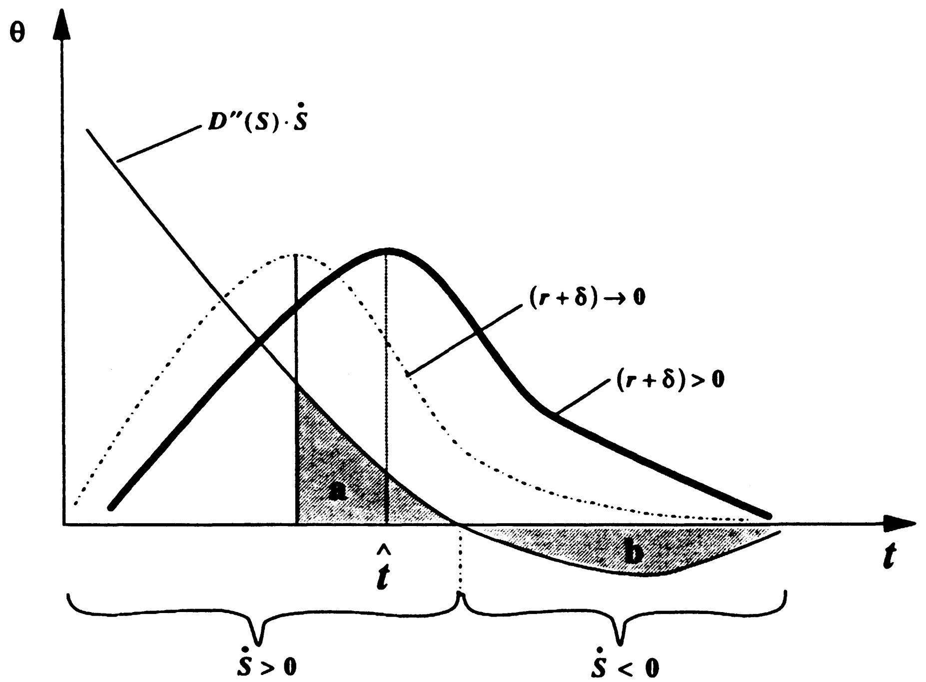
(4) $$\lim_{t\to\infty} \theta_t = 0$$

(5) $$\dot{\theta}_t = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} D''(S_\tau)\cdot\dot{S}_\tau d\tau$$

(6) $$\lim_{t\to\infty} \dot{\theta}_t = 0$$

From (3) we see that the carbon tax is positive as long as marginal damage is positive, that is as long as S > 0. It will smoothly approach zero in the long run as the stock of carbon in the atmosphere decays and reaches the equilibrium stock (see (4) and (6)). This means that the tax will be positive even if the atmospheric stock declines after a certain time due to low extraction and consumption of fossil fuels; the decay of carbon in the atmosphere is higher than the additional carbon from new extraction ($x < \delta S$).

Figure 1: The Optimal Carbon Tax when Damage is Related to the Atmospheric Stock of Carbon



$a = b$

$$\dot{\theta}_t = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} D''(S_\tau)\cdot\dot{S}_\tau\, d\tau$$

The behaviour of the carbon tax over time depends on whether the carbon stock is increasing or decreasing (see (5)). Assume that the accumulation of carbon in the atmosphere has a one-hump shape, i.e., the stock initially increases but eventually decreases. Actually, in Appendix B, it is shown that this will be the solution. Figure 1 shows the evolution of the optimal carbon tax over time. As long as $0 < (r + \delta) < \infty$, the optimal tax will decrease before the stock of carbon in the atmosphere decreases. This means that there exists a period with a falling carbon tax and increasing stock of carbon in the atmosphere. When the stock of carbon decreases ($\dot{S} < 0$), the carbon tax should definitely be falling.[3] The intuition behind this is as follows. We know that the marginal damage will be higher the larger is the stock of carbon in the atmosphere. Assume first that $r \to \infty$. Then, only damage in the current period counts, and the tax will therefore be equal to the current marginal damage. Thus, the tax will increase as long as $\dot{S} > 0$ and fall for $\dot{S} < 0$. For $r < \infty$ (and $\delta < \infty$), that is, the social planner is not totally myopic and the lifetime of CO_2 in the atmosphere is not zero, future damages will also count. Since the marginal damage will start falling in the future due to a decreasing carbon stock, a unit emitted immediately before S reaches its maximum creates more damage than a unit emitted when S is at its maximum point. On the other hand, a unit of carbon emitted in the beginning of the planning period stays in the atmosphere when the carbon stock is low. Thus, this unit creates less damage than a unit emitted when S is at its maximum. The optimal carbon tax, which reflects all damages made by a unit carbon emitted, will therefore reach its maximum between time 0 and the time giving the largest stock of atmospheric carbon. This is illustrated in Figure 1 where the carbon tax rises initially, while it falls from time $\hat{t}$ onwards when the stock of carbon is still increasing. The evolution of the carbon tax is consistent with the results of Ulph and Ulph (1992), who argue that the specific carbon tax initially rises but should eventually fall. However, they do not relate the behaviour of the optimal tax to the evolution of the stock of carbon in the atmosphere, but only claims that the carbon tax should definitely be falling once fossil fuels are exhausted.

[3] For $(r+\delta) = 0$, i.e., if there is no discounting and the lifetime of CO_2 in the atmosphere is infinite, equation (5) is no longer valid. In that case we can use the condition $\dot{\theta} = (r+\delta)\theta - D'(S)$, which is derived from equations (24) and (31) in Appendix B, and is valid for all values of $(r+\delta)$. For $(r+\delta) = 0$, we get $\dot{\theta} = -D'(S)$, that is, the carbon tax is always falling and, therefore, has its maximum value at $t = 0$. The intuition is the following. Since we have no discounting and the carbon will stay in the atmosphere forever, a unit of carbon emitted today is more damaging than a unit emitted in the future, simply because it stays around causing damage for a longer period.

3.2 Damage Related to the Rate of Change in the Atmospheric Stock of Carbon

As argued above, another alternative is to model the negative externality as a function of the time derivative and not the level of the atmospheric stock. Some first attempts in this direction have been made by Tahvonen (1993), Tahvonen et al. (1994) and Hoel and Isaksen (1993). These papers, however, do not take into account the exhaustibility of fossil fuels.

It is reasonable to believe that there are costs of adapting to a colder ($\dot{S} < 0$) as well as a warmer climate ($\dot{S} > 0$), and that these costs increase the higher is the rate of climate change. Therefore, I assume that the damage is convex in the rate of atmospheric carbon accumulation. Further, there are no damage costs for a constant climate ($\dot{S} = 0$). Hence, if d(£) is the damage function, we have $d(\dot{S}) > 0$ for $\dot{S} \neq 0$, and $d(0) = 0$. I also assume that the damages are negligible for marginal changes in the stock under a constant climate ($\dot{S} = 0$), giving $d'(0) = 0$. The convexity of damages means:[4]

$$\text{(7)} \qquad \begin{gathered} d'(\dot{S}) > 0 \textit{ for } \dot{S} > 0 \;\; \wedge \;\; d'(\dot{S}) < 0 \textit{ for } \dot{S} < 0 \\ d''(\dot{S}) > 0 \textit{ for all } \dot{S} \end{gathered}$$

The new optimisation problem is thus:

$$\text{(8)} \qquad \begin{aligned} & \textit{maximise} \int_0^\infty e^{-rt} \cdot [u(x_t) - c(A_t)x_t - d(\dot{S}_t)]dt \\ & \quad s.t. \quad \dot{A}_t = x_t \\ & \qquad\quad\;\; \dot{S}_t = x_t - \delta S_t \\ & \qquad\quad\;\; x_t \geq 0 \end{aligned}$$

[4] Alternatively, we could argue that small changes in the climate do not make any damage. This can be specified as $d(\dot{S}) = d'(\dot{S}) = 0$ for $|\dot{S}| \leq K$, where K is a constant. If $K \geq \delta S_t$ for all t, the decline in the atmospheric concentration of CO_2 will never be large enough to make $d'(\dot{S})$ negative (as $|\dot{S}| = |\delta S| \leq K$ for $x = 0$). This will simplify some of the results below, since in this case, the shadow price of the atmospheric stock, γ, will always be non-negative (see (10)). However, for $\dot{S} > K$, we will still have the two contradictory effects described below (see (9)). In paragraph 3.1 above, we could in a similar way argue that $D(S) = D'(S) = 0$ for $S \leq M$, where M is a constant. This would not change the main results, however, the carbon tax would reach zero in finite time when $S_t \leq M$ for all t.

This model is solved in Appendix C where it is shown that the optimal carbon tax, σ, can be expressed as:

(9) $$\sigma_t = d'(\dot{S}_t) - \gamma_t$$

where γ_t is the shadow price associated with accumulated atmospheric stock up to t.

(10) $$\gamma_t = \delta \int_t^{\infty} e^{-(r+\delta)(\tau - t)} d'(\dot{S}_\tau) d\tau$$

Consider first the situation with an increasing stock of carbon in the atmosphere ($\dot{S} > 0$). Fossil fuel consumption (and extraction) increases the damage from global warming via accelerated buildup of the atmospheric stock (represented by $d'(\cdot)$ in equation (9)), but on the other hand, this leads to a larger stock in the atmosphere and therefore higher decay in the future. A high decay will reduce the rate of change in the atmospheric stock, and hence the damage from global warming. Note therefore, that while the shadow price of accumulated atmospheric stock is negative in the first model where damage is determined by the stock level ($\mu = -\theta < 0$; see Appendix B), it is positive in the second model ($\gamma > 0$) if $\dot{S} > 0$ for a sufficiently long period.[5] This is also shown in Tahvonen (1993). Thus, a larger stock of carbon in the atmosphere represents a cost if the damage is positively related to the level of this stock, while it represents a benefit if the damage is positively related to the rate of change in the stock as long as this stock is increasing for a sufficiently long period.

For $\dot{S} < 0$, an increase in fossil fuel consumption (and extraction) will reduce the absolute value of $\dot{S}$, $|\dot{S}|$, and therefore the adaption costs. This effect gives a lower optimal carbon tax, and is represented by $d'(\dot{S}) < 0$ in (9). However, increasing fossil fuel consumption gives a larger stock of carbon in the atmosphere, and therefore a larger decay of this stock in the future. This leads to even lower values of $\dot{S}$, and higher adaption costs, in the future. Thus, while the shadow price of accumulated atmospheric stock may be positive for $\dot{S} > 0$, it is negative when $\dot{S} < 0$.

[5] Typically, the stock of carbon in the atmosphere will initially increase but eventually decrease (see Appendices A and B). Therefore, the shadow price associated with accumulated atmospheric stock, γ, consists of both positive and negative elements as $d'(\cdot) > 0$ for $\dot{S} > 0$ and $d'(\cdot) < 0$ for $\dot{S} < 0$ (see equation (10)). This means that $\dot{S}$ has to be positive for a sufficiently long period for γ to be positive.

Increasing fossil fuel consumption may therefore give two contradictory effects. From (9) we see that the carbon tax can be negative or positive depending on which effect is the strongest. This is also a different result compared to the model expressed in (1), where the optimal carbon tax is always non-negative.

To study the evolution of the carbon tax, we first note the properties of γ derived in Appendix C, where $\dot{\gamma}$ is the time derivative of γ

(11) $$\lim_{t\to\infty} \gamma_t = 0$$

(12) $$\dot{\gamma}_t = \delta \int_t^{\infty} e^{-(r+\delta)(\tau-t)} d''(\dot{S}_\tau)\cdot(\dot{x}_\tau - \delta \dot{S}_\tau) d\tau$$

(13) $$\lim_{t\to\infty} \dot{\gamma}_t = 0$$

Using these properties and defining $\dot{\sigma}$ as the time derivative of σ, we find that

(14) $$\lim_{t\to\infty} \sigma_t = 0$$

(15) $$\dot{\sigma}_t = d''(\dot{S}_t)\cdot(\dot{x}_t - \delta \dot{S}_t) - \dot{\gamma}_t$$

(16) $$\lim_{t\to\infty} \dot{\sigma}_t = 0$$

As seen from (12) and (15), the behaviour of the optimal carbon tax depends on the evolution of the marginal damage over time (which is $\partial d'(\dot{S})/\partial t = d''(\dot{S})\cdot(\dot{x}-\delta\dot{S})$). However, the behaviour of σ is rather complex due to the two contradictory effects described above. To get a further insight, I study the three different cases; $\dot{S} = 0$, $\dot{S} > 0$ and $\dot{S} < 0$.

***a)* $\dot{S} = 0$**

Due to the assumption $d'(0) = 0$, we see from (9) and (10) that $\sigma = -\gamma$ for $\dot{S} = 0$. This means that the optimal carbon tax should be equal to the shadow price of the atmospheric stock, γ, but with opposite sign. γ can be negative or positive as discussed above. Even if the current emissions make no change in the current atmospheric stock and therefore give no current damage, it will be optimal to increase the emissions if the future atmospheric stock is increasing for a sufficiently long period of time. Increased emissions will give a higher future decay and reduce future damage. In this case it will therefore be optimal to subsidise emissions. If the future stock is decreasing, we get the opposite conclusion.

b) $\dot{S} > 0$

If $\dot{S} > 0$ for a sufficiently long period of time, we see from (9) and (10) that σ is positive if and only if

$$d'(\dot{S}_t) > \gamma_t \tag{17}$$

which means that the direct effect of an increase in the atmospheric stock of carbon is larger than the indirect effect of increased future decay of this stock. However, even if the optimal tax is positive, it is difficult to say anything about its evolution. From (15) we see that the condition for a rising carbon tax is

$$d''(\dot{S}_t)\cdot(\dot{x}_t - \delta\dot{S}_t) - \dot{\gamma}_t > 0 \tag{18}$$

c) $\dot{S} < 0$

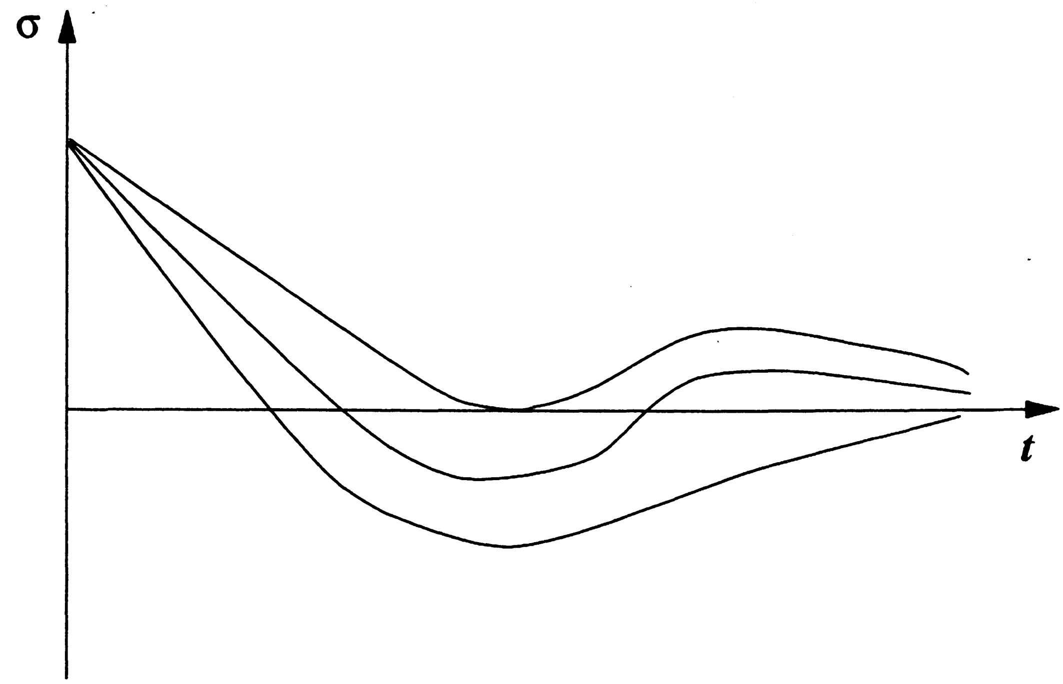
For $\dot{S} < 0$, an increase in extraction means reduced adaption costs ($d'(\cdot) < 0$). This goes in the direction of a lower carbon tax. On the other hand, increasing extraction means larger stock and therefore higher decay in the future leading to higher adaption costs ($\gamma < 0$). Thus, the optimal tax should be positive if the second effect dominates the first, i.e.,

$$|d'(\dot{S}_t)| < |\gamma_t| \tag{19}$$

Actually, for $d'(\cdot) < 0$ and $\gamma < 0$, this gives condition (17). The condition for an increasing carbon tax is again given by (18).

For a decline in the atmospheric stock of carbon, it can therefore be optimal to have a positive carbon tax. This confirms the result derived from model (1), but while the result in that model rests on the assumption that stock levels larger than the initial stock give an external effect, the result in model (8) is due to the assumption of positive adaption costs even for reductions in the stock.

Figure 2: Possible Optimal Carbon Tax Paths when Damage is Related to the Rate of Change in the Atmospheric Stock of Carbon



In Figure 2, some possible time paths for the optimal carbon tax are illustrated. The behaviour of the optimal carbon tax in this model is more complex than in model (1). If S_0 were equal to zero, initially the tax would be positive since we move from a situation with no external effects to a situation with external effects. This would then reflect the preindustrial situation. However, as $S_0 > 0$, the optimal carbon tax can be negative as well as positive in the beginning of the planning horizon. The discount rate, r, and the lifetime of CO_2 (reflected in δ) are important for the evolution of the optimal carbon tax. If the discount rate is high, and the lifetime of CO_2 is long (δ is low), the future indirect effects may be negligible. In that case, the optimal carbon tax is positive for $\dot{S} > 0$, while it is negative for $\dot{S} < 0$. In the long run, the carbon tax will steadily approach zero (see (14) and (16)).

In both models, the optimal carbon tax will approach zero as time goes to infinity, see (4) and (14). As shown in Appendix A, this means that the optimal extraction will approach zero as time goes to infinity. It is also shown that even if the damage from global warming is specified differently, the total extraction is the same.

4. Optimal Depletion with a Non-polluting Backstop Technology

So far, I have not explicitly considered the existence of substitutes for fossil fuels. However, substitutes such as synthetic fuels, nuclear power, hydro power, biomass, solar and wind power already exist. Assume that there exists a non-polluting perfect substitute for fossil fuels, y, with an unlimited stock and a constant unit cost, $\bar{c}$. In a competitive market, the price will be equal to this cost since there are no stock constraints. By definition, a backstop source is available in unlimited quantities at a constant marginal cost. The traditional result from the theory of a competitive mining industry facing a backstop technology with the constant unit cost less than the choke price, but higher than the initial marginal extraction cost, is that the industry depletes the resource until the price reaches the cost of the backstop. At this price, the consumers will switch immediately to the backstop (see Appendix D). This result is also derived in for example Dasgupta and Heal (1979), but they assume a fixed quantity of the exhaustible resource. The extraction path will be different in the presence of external greenhouse effects. By introducing the non-polluting backstop into the model from section 2, the social planner seeks to

$$
\begin{aligned}
& \textit{maximise} \int_0^{\infty} e^{-rt}\cdot[u(x_t+y_t)-c(A_t)x_t-\bar{c}y_t-D(S_t)]dt \\
& \qquad s.t. \quad \dot{A}_t = x_t \\
(20) \qquad & \qquad\qquad \dot{S}_t = x_t - \delta S_t \\
& \qquad\qquad x_t \geq 0 \\
& \qquad\qquad y_t \geq 0 \\
& \qquad\qquad c(0) < \bar{c} < u'(0)
\end{aligned}
$$

The necessary conditions for an interior optimum are given in Appendix D.

In accordance with section 2, I define $u(z) = \int_0^z p(z)dz$, $z = x+y$, where $p(z)$ is the consumer price. Then, $u'(x+y) = p$. In Appendix D, it is shown that on the optimal path, the consumer price has to satisfy the following conditions:

(21) $$p(x_t + y_t) = c(A_t) + \pi_t + \theta_t, \quad x_t > 0$$

(22) $$p(x_t + y_t) = \bar{c}, \quad y_t > 0$$

As the negative externality is a function of the atmospheric stock of carbon, the optimal carbon tax, θ, has the characteristics derived in paragraph 3.1. In Appendix A, I show the properties of the scarcity rent, π. Note that the evolution of this rent is given by:

(23) $$\frac{\partial(c(A_t) + \pi_t)}{\partial t} = r\pi_t$$

Taking time derivatives of (21) and (22), and using (23), we find the evolution of the consumer price:

(24) $$\dot{p}_t = r\pi_t + \dot{\theta}_t, \quad x_t > 0$$

(25) $$\dot{p}_t = 0, \quad y_t > 0$$

For $c(0) < \bar{c}$, the consumer price will initially be lower than the price of the backstop. This gives $x > 0$ and $y = 0$ since $p < \bar{c}$. In other words, only fossil fuels will be consumed initially. In the absence of a backstop, the consumer price would increase and approach $u'(0)$ since $p = u'(x)$. But since $\bar{c} < u'(0)$, the price will at some time reach $\bar{c}$. At this price, we can have $x > 0$ and $y > 0$ at the same time (see (23) and (24)). The condition for this is:

(26) $$\dot{\theta}_t = -r\pi_t$$

that is, the optimal tax is falling with the same rate as the producer price is increasing, namely the discount rate times the scarcity rent.

If the extraction of fossil fuels is so high that the consumer price rises above the price of the backstop, it will not be optimal to consume fossil fuels due to the cheaper substitute. But if we stop depleting fossil fuels when $p = \bar{c}$, the stock of carbon in the atmosphere will decrease ($\dot{S} = -\delta S < 0$), and the optimal carbon tax will fall (see (5)). This means that the consumer price of fossil fuels will fall below $\bar{c}$, which again makes fossil fuels economically viable. Therefore, it will not be optimal to stop the extraction of fossil fuels. The optimal extraction path of fossil fuels when $p = \bar{c}$ is determined by:

(27) $$\bar{c} = c(A_t) + \pi_t + \theta_t$$

When the consumer price of fossil fuels equals the unit cost of the backstop, it will be optimal to consume both fossil fuels and the backstop. Fossil fuels will be extracted so that the consumer price remains constant and equal to this unit cost.

The optimal consumer price path is illustrated in Figure 3a. The consumer price of fossil fuels is constant and equal to the consumer price of the backstop technology (which is the unit cost) from time t_a onwards (see Figure 3a).

Figure 3: Optimal Extraction and Consumer Prices with a Backstop Technology

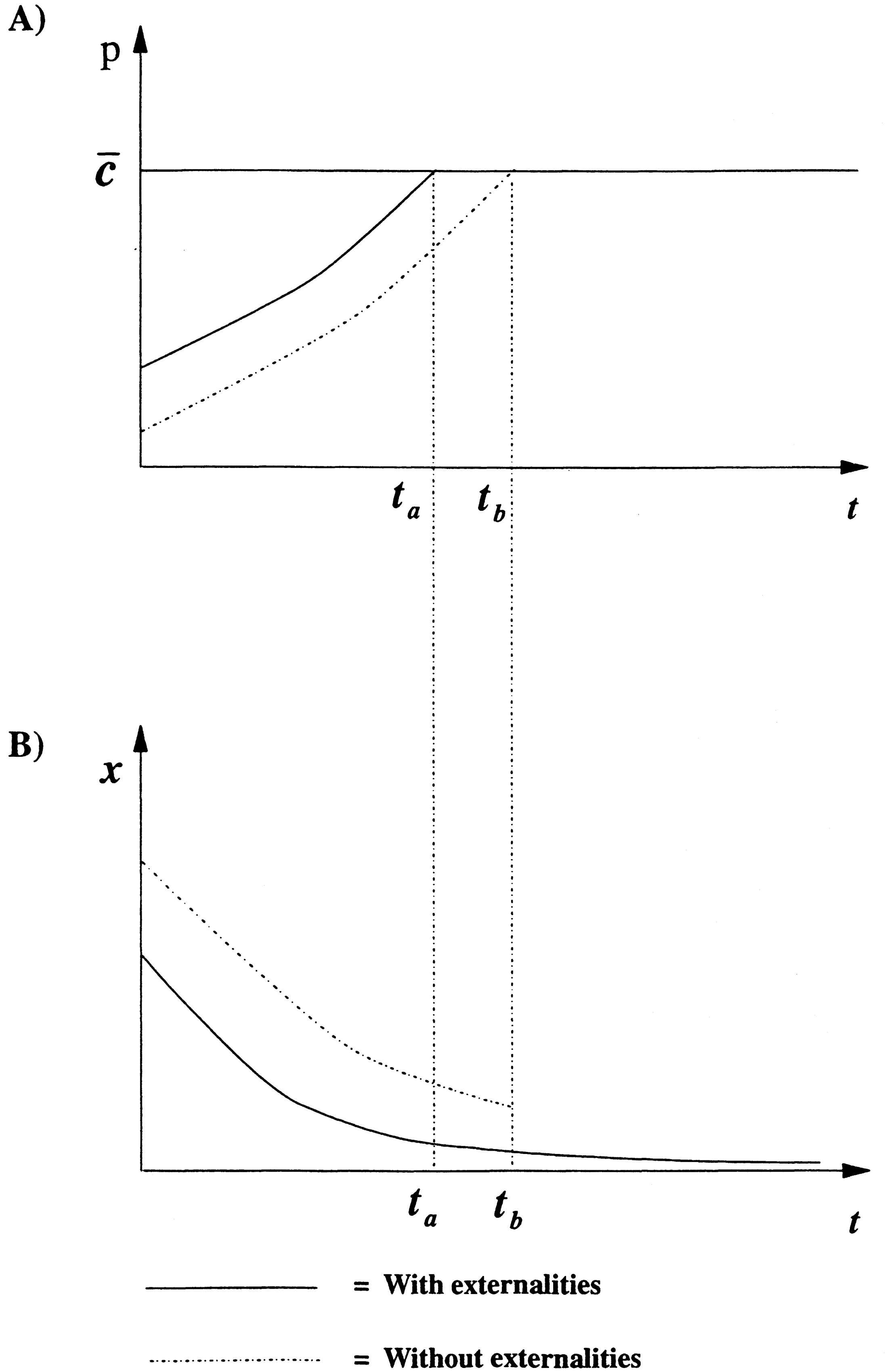


Figure 3b illustrates the optimal extraction path of fossil fuels. With the greenhouse externality, both fossil fuels and the backstop technology will be consumed from time t_a onwards. The advantage of consuming fossil fuels instead of the backstop is the lower production costs. The disadvantage is, however, the external effects. As the optimal tax approaches zero as time goes to infinity (see (4)), the optimal fossil fuel extraction will also fall and approach zero in the long run (see Appendix A). Thus, the extraction of fossil fuels will fall over the entire time horizon.

In Figure 3, I have also compared the extraction path and the consumer price with the corresponding paths in the absence of external effects. The total extraction in the presence of the greenhouse externality will actually be the same as accumulated extraction without externalities. In the latter case, the consumer price will reach $\bar{c}$ at time t_b (see Figure 3a). This means

(28) $$\lim_{t \to t_b} \{c(A_t) + \pi_t\} = \bar{c}$$

In the presence of external effects, the following condition must be satisfied:

(29) $$\bar{c} = \lim_{t \to \infty} \{c(A_t) + \pi_t + \theta_t\}$$

Since $\pi \to 0$ as $x \to 0$ (see Appendix A) and $\theta \to 0$ as $t \to \infty$ (see (4)), the accumulated extraction in absence of external effects is given by $c(A_b) = \bar{c}$, while in the presence of external effects we get $c(A_a) = \bar{c}$. Hence, $A_a = A_b$, meaning that accumulated extraction is the same in both models. But while fossil fuels will be depleted in finite time in the absence of externalities, greenhouse externalities makes it optimal to deplete fossil fuels in infinite time since the optimal carbon tax converges at zero for $t \to \infty$ (see Appendix A). Thus, the greenhouse effect makes it optimal to disperse the depletion over a longer period.

As the total extraction is the same with the external effect as without, and the consumer price reaches $\bar{c}$ in finite time (as $\bar{c} < u'(0)$), the total extraction is higher in the absence of external effects than with the external effects at time t_b. Therefore, the extraction path without externalities will be higher than the path were greenhouse externalities are present (see Figure 3b). In the same way, in Figure 3a it is seen that the price path without externalities will be lower than the corresponding path with externalities.

5. Conclusions

Most papers on the economics of global warming concentrate on the external effects from fossil fuels combustion without taking into account the exhaustibility of these resources. This paper combines the theories of greenhouse externalities and non-renewable resources, to analyse several aspects of global warming.

The basic model presented in section 2, defines the negative greenhouse externalities as positively related to the stock of carbon in the atmosphere. The exhaustibility of fossil fuels is modelled by increasing extraction costs in accumulated extraction. A carbon tax is used to implement the optimal solution to this model. This tax should initially be increasing but eventually fall and approach zero as time goes to infinity. It should start decreasing before the stock of carbon in the atmosphere reaches its maximum point.

Changing the specification of the externalities to depend on the rate of change in the atmospheric stock of carbon, completely changes the model. While the shadow price of the atmospheric stock was negative in the basic model, indicating a cost of increasing the stock, it is positive in this new model if the stock of carbon rises over a sufficiently long period. This is due to an increase in the depreciation of carbon in the atmosphere when the stock of carbon increases, which gives a lower rate of change in the future stock. Actually, this effect can make the optimal carbon tax negative even for high concentrations of carbon in the atmosphere.

The last problem analysed is the depletion of fossil fuels if there exists a non-polluting backstop technology. If we ignore the external effects, the traditional theory gives the result that the resource should be depleted until the price reaches the cost of the backstop. At this price, consumers will switch immediately to the backstop. Taking into account the greenhouse effect will give different time paths for prices and extraction. When the consumer price of fossil fuels reaches the price of the backstop, it will still be optimal with fossil fuel consumption, that is, both the backstop and fossil fuels will be consumed. This is due to a falling carbon tax of fossil fuels, and therefore a fall in the consumer price if fossil fuels are not consumed. Total extraction will be the same as for no external effects, but the greenhouse effect makes it optimal to slow down the extraction and spread it over a longer period.

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Appendix[6]

A. The Competitive Equilibrium when there is an Arbitrary Tax on Fossil Fuels

Let v_t be an arbitrary tax on fossil fuels at time t (measured in carbon units). The optimal consumption/extraction path under a competitive equilibrium is then derived from the maximisation problem below:

$$\text{(1)} \qquad \begin{aligned} &maximise \int_0^\infty e^{-rt}\cdot[u(x_t) - c(A_t)x_t - v_t x_t]dt \\ &s.t. \quad \dot{A}_t = x_t \\ &\qquad\quad x_t \geq 0 \end{aligned}$$

The *existence of an optimal solution* can be shown as follows. Assume $c(0) > 0$ and $u'(\infty) = 0$. It is easily seen that it never pays to have $u'(x) - c(0) < 0$ anywhere along a path, since decreasing x on such intervals would then increase the integrand in the criterion, not only on this interval but from then on and to infinity. Hence $u'(x) - c(0) \geq 0$. Choose a $\bar{x}$ such that $u'(\bar{x}) - c(0) < 0$. Then we can safely argue that $x \in [0,\bar{x}]$, and in fact that no optimal $x_t = \bar{x}$. By Theorem 15, chapter 3, in Seierstad and Sydsæter (1987), an optimal control exists. This proof is also valid for the model presented in Appendix B.

The current value Hamiltonian for problem (1):

$$\text{(2)} \qquad H = u(x_t) - c(A_t)x_t - v_t x_t + \lambda_t x_t$$

The necessary conditions for an interior optimum:

$$\text{(3)} \qquad \frac{\partial H}{\partial x_t} = u'(x_t) - c(A_t) - v_t + \lambda_t \leq 0, \quad (= 0 \ \ for\ x_t > 0)$$

[6] The symbols and the concavity/convexity characteristics of the functions are explained in the main text.

(4) $$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t$$

(5) $$\dot{A}_t = x_t$$

(6) $$\lim_{t\to\infty} e^{-rt}\lambda_t A_t = 0$$

Due to the assumption $u'(0) < \infty$, we know that on the optimal path A_t will converge (see section 2), i.e.,

(7) $$0 < \lim_{t\to\infty} A_t = \hat{A} < \infty$$

Therefore, from (6) and (7) we get:

(8) $$\lim_{t\to\infty} e^{-rt}\lambda_t = 0$$

λ_t (< 0) is the shadow cost associated with cumulative extraction up to t. Thus, the scarcity rent at time t, π_t, is defined as

(9) $$\pi_t = -\lambda_t$$

To show the characteristics of the scarcity rent, I first use (4) and (8) to derive λ_t, thereafter use (9) to find:

(10) $$\pi_t = \int_t^\infty e^{-r(\tau-t)}c'(A_\tau)x_\tau d\tau$$

and

(11) $$\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \int_t^\infty e^{-r\tau}c'(A_\tau)x_\tau d\tau / e^{-rt}$$

applying L'Hopital's rule and notifying that by (7) $x_t \to 0$ as $t \to \infty$, we get:

(12) $$\lim_{t\to\infty} \pi_t = \lim_{t\to\infty} \frac{c'(A_t)x_t}{r} = 0$$

According to (12), the scarcity rent will converge at zero for $t \to \infty$. This is due to the increasing marginal extraction cost; The costs will be so high that an additional unit of fossil fuels extracted will not add to the welfare.

To find the optimal extraction path, consider (3). For $x_t > 0$, this condition is fulfilled with equality. Then, applying $u'(x_t) = p_t$ (see section 2), we find:

(13) $$p_t = c(A_t) + \pi_t + v_t, \quad x_t > 0$$

Thus, the consumer price is separated from the producer price via the carbon tax. Further, the producer price is the sum of marginal extraction costs and the scarcity rent, i.e., $c(A_t) + \pi_t$. The behaviour of the consumer price over time is given by

(14) $$\dot{p}_t = c'(A_t)x_t + \dot{\pi}_t + \dot{v}_t, \quad x_t > 0$$

using (4), (9) and $\dot{p}_t = u''(x_t)\dot{x}_t$, we find

(15) $$\dot{p}_t = u''(x_t)\dot{x}_t = r\pi_t + \dot{v}_t, \quad x_t > 0$$

a) Assume $v_t = \dot{v}_t = 0$ for all t. Since $\pi_t > 0$ for $x_t > 0$ (see (10)) and $u''(x_t) < 0$, we see that $\dot{x} < 0$. Thus, *in the absence of a carbon tax the extraction of fossil fuels is falling and the consumer price is rising over time.*

b) Assume $\dot{v}_t > 0$. Thus, we see from (15) that $\dot{x} < 0$ as long as $x_t > 0$. Hence, *with an increasing carbon tax, the extraction of fossil fuels is falling and the consumer price is rising over time.*

c) Assume $\dot{v}_t < 0$. We see from (15) that $\dot{x} < 0$ as long as $x_t > 0$ if and only if

$$|\dot{v}_t| < r\pi_t \tag{16}$$

Hence, *with a falling carbon tax, the extraction of fossil fuels may rise and the consumer price may fall over a limited period of time*. Since accumulated extraction of fossil fuels will converge, extraction of fossil fuels cannot increase forever.

As $\dot{S}_t = x_t - \delta S_t$, initially $\dot{S} > 0$ if $x > \delta S_0$. But eventually $\dot{S} < 0$ as x_t decreases and S_t increases over time. In the long run $S \rightarrow 0$ since $x_t \rightarrow 0$ when $t \rightarrow \infty$. *Hence, if $\dot{x}_t \leq 0$ for $\forall$ t, the atmospheric stock of carbon will initially increase but eventually decrease if S_0 is not too high.*

As $A \rightarrow \grave{A}$, the extraction will fall and reach or converge at zero. If x_t converges at zero in infinite time, we have $x_t > 0$ for all t, and equation (3) is fulfilled with equality. Assume $x_t \rightarrow 0$ as $t \rightarrow \bar{t}$, where $\bar{t} = \min\{t: x_t = 0\} < \infty$, and consider the case where we have no carbon tax. By taking limit of (3) as $t \rightarrow \bar{t}$, and using $\pi_{\bar{t}} = 0$ (since $x_t = 0$ for $t \geq \bar{t}$, see equation (10)), we find,

$$u'(0) \leq c(\grave{A}), \quad x_t = 0 \tag{17}$$

However, (17) has to be fulfilled with equality (see also the proof for an optimal solution above). If not, the last unit of fossil fuels extracted and consumed reduces the welfare since it gives a higher marginal cost ($c(\grave{A})$) than marginal utility ($u'(0)$). Thus, $\grave{A}$ is determined by:

$$u'(0) = c(\grave{A}) \tag{18}$$

With constant $v > 0$, and by taking the limit of (3) as $t \rightarrow \infty$, we get

$$u'(0) = c(\bar{A}) + v \tag{19}$$

By the same argument as above, (19) is fulfilled with equality. Thus, we see that *with a constant carbon tax, accumulated extraction of fossil fuels is less than without a carbon tax,* i.e., $\mathbf{A} < \grave{A}$.

If $v_t \rightarrow 0$ as $t \rightarrow \infty$, and by taking the limit of (3) as $t \rightarrow \infty$, we are back to (18). Thus, *if the carbon tax approaches zero in the long run, the accumulated extraction of fossil fuels equals the extraction in the absence of a carbon tax.* However, as long as v_t decreases, the producer price has to increase, giving a positive extraction of fossil fuels. This is seen from equation (15), which is fulfilled with equality according to the argument above. Hence, it will never be optimal to stop depleting as long as the carbon tax is falling.

B. Damage as a Function of the Atmospheric Stock

Assume that there is a negative externality related to the atmospheric stock of carbon. If the social planner seeks to maximise the present value of the welfare to the society, we get the following optimisation problem:

$$\text{(20)} \qquad \begin{aligned} & \textit{maximise} \int_0^\infty e^{-rt} \cdot [u(x_t) - c(A_t)x_t - D(S_t)]dt \\ & s.t. \quad \dot{A}_t = x_t \\ & \qquad\;\; \dot{S}_t = x_t - \delta S_t \\ & \qquad\;\; x_t \geq 0 \end{aligned}$$

The current value Hamiltonian to the problem:

$$\text{(21)} \qquad H = u(x_t) - c(A_t)x_t - D(S_t) + \lambda_t x_t + \mu_t \cdot (x_t - \delta S_t)$$

The necessary conditions for an interior optimum:

$$\text{(22)} \qquad \frac{\partial H}{\partial x_t} = u'(x_t) - c(A_t) + \lambda_t + \mu_t = 0$$

(23) $$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t$$

(24) $$\dot{\mu}_t - r\mu_t = -\frac{\partial H}{\partial S_t} = D'(S_t) + \delta\mu_t$$

(25) $$\dot{A}_t = x_t$$

(26) $$\dot{S}_t = x_t - \delta S_t$$

(27) $$\lim_{t\to\infty} e^{-rt}\lambda_t A_t = 0$$

(28) $$\lim_{t\to\infty} e^{-rt}\mu_t S_t = 0$$

Equation (22) is fulfilled with equality in accordance with Appendix A. Since A_t is bounded and will converge on the optimal path (see section 2), we find from (27):

(29) $$\lim_{t\to\infty} e^{-rt}\lambda_t = 0 \ \wedge \ 0 < \lim_{t\to\infty} A_t = \bar{A} < \infty$$

As $x_t \to 0$ when $t \to \infty$, we know that $S \to 0$. S_t is bounded since A is bounded, which means that Theorem 16, chapter 3, with Notes 20 and 21 in Seierstad and Sydsæter (1987) applies. According to this Theorem:

(30) $$\lim_{t\to\infty} e^{-rt}\mu_t = 0$$

The characteristics of λ_t, the shadow cost associated with cumulative extraction, are expressed in (8)-(12).

μ_t (< 0) is the shadow cost associated with accumulated atmospheric stock up to t. The optimal carbon tax at time t, θ_t, is defined as:

(31) $$\theta_t = -\mu_t$$

Using the same techniques as under the derivation of the scarcity rent in Appendix A (replace $c'(\cdot)x_t$ by $D'(\cdot)$ and r by $(r+\delta)$), and given (30) and $D'(0) = 0$, we find the characteristics of the optimal carbon tax:

(32) $$\theta_t = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} D'(S_\tau)\,d\tau$$

(33) $$\lim_{t\to\infty} \theta_t = \lim_{t\to\infty} \frac{D'(S_t)}{(r+\delta)} = 0$$

According to (33), the optimal carbon tax converges at 0 for $t \to \infty$. The intuition behind this is as follows. As $S \to 0$, there will be no cost associated with a marginal increase in the atmospheric stock when $t \to \infty$, thus $\mu \to 0$. As the shadow cost reflects the optimal carbon tax, this will converge at zero for $t \to \infty$.

Consider (32). Assuming $(r+\delta) \neq 0$ and integrating by parts yields, after some manipulation:

(34) $$\theta_t = \frac{D'(S_t)}{(r+\delta)} + \frac{1}{(r+\delta)}\int_t^{\infty} e^{-(r+\delta)(\tau-t)} \frac{\partial D'(S_\tau)}{\partial \tau}\,d\tau$$

Applying (31) and substituting (34) into (24) gives

(35) $$\dot{\theta}_t = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} \frac{\partial D'(S_\tau)}{\partial \tau}\,d\tau = \int_t^{\infty} e^{-(r+\delta)(\tau-t)} D''(S_\tau)\cdot\dot{S}_\tau\,d\tau$$

Then, substituting (31) and (34) into (24), and applying (33) and $S \to 0$, we get

(36) $$\lim_{t\to\infty} \dot{\theta}_t = \lim_{t\to\infty} (r+\delta)\theta_t - \lim_{t\to\infty} D'(S_t) = 0$$

To study the evolution of the optimal carbon tax, consider (35). Initially $\dot{S} > 0$ (if S_0 is not too high) since extraction and consumption of the fossil fuel give an initial buildup of the atmospheric stock of carbon. But eventually $\dot{S} < 0$ due to the exhaustibility of fossil fuels (see also Appendix A). Thus, from (35) we see that θ_t will initially rise but eventually fall as $D''(S_t)$ is always positive. Is it possible that the carbon tax, θ_t, can increase and fall several times? For $\dot{\theta}$ to be negative at some time t_1, we need $\dot{S} < 0$ for some period of time after t_1. Then, for θ_t to first decrease and later increase, S_t has to first decrease and later increase. This means that $\dot{x}$ has to be positive in given time periods as $\dot{S}$ can only change from being negative to positive if there is an increase in x_t. As was shown in Appendix A, the extraction can increase if the fall in the tax is sharp enough. But for a rising tax, we know that the extraction falls ($\dot{x} < 0$).

(37) $$\dot{\theta}_t = (r+\delta)\theta_t - D'(S_t)$$

To further study the evolution of the tax, consider the condition
which is derived from equations (24) and (31). Suppose that there exist $t_2 < t_3$ (see Figure A1) such that:

(38) $$\dot{\theta}_t = 0 \quad for\ t = t_2 \wedge t = t_3$$
$$\wedge$$
$$\dot{\theta}_t > 0 \quad for\ t_2 < t < t_3$$

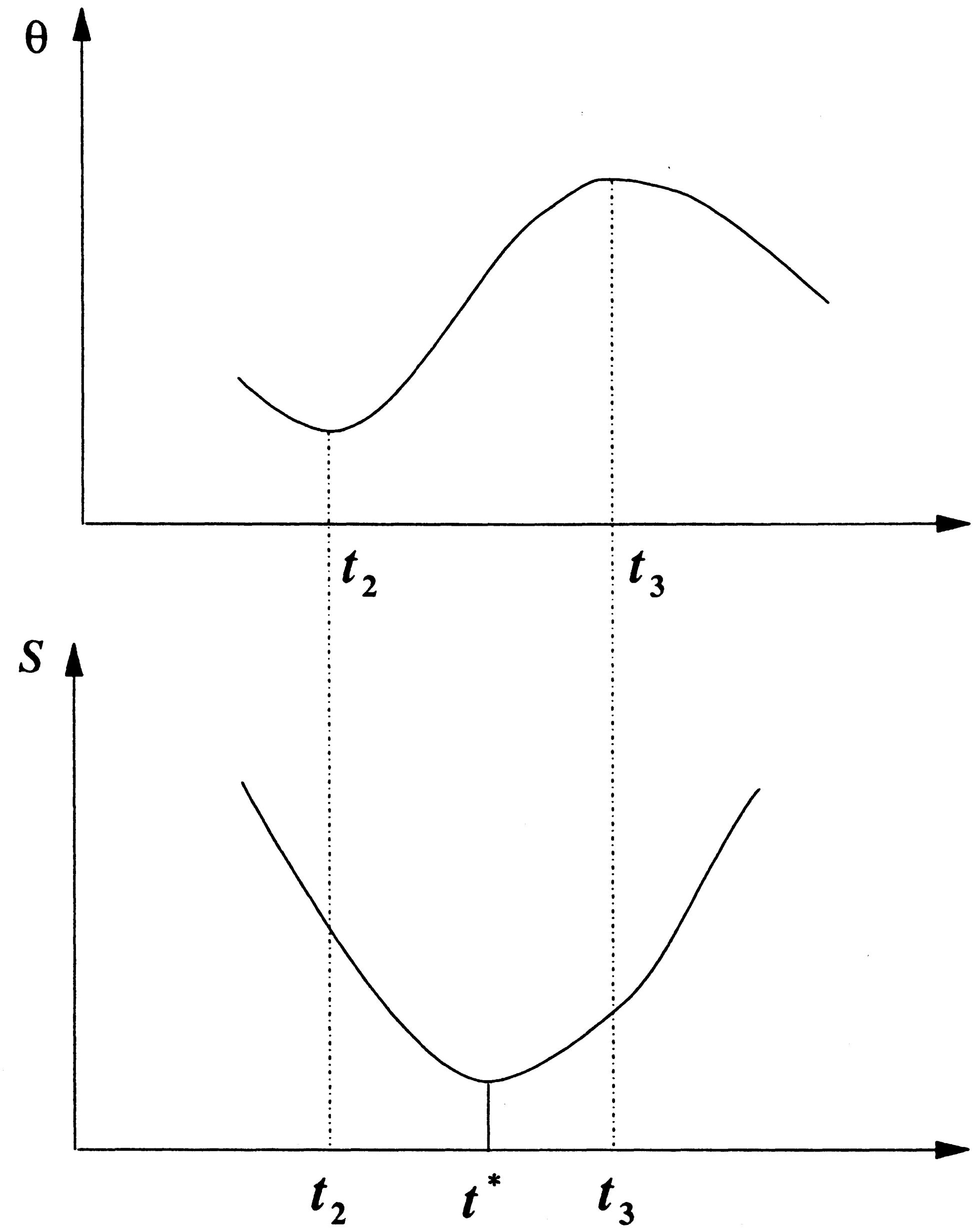
Thus $\dot{\theta}$ is changing from negative to positive at t_2 and back to negative at t_3 since $\dot{\theta}$ is continuous. This means that the tax has a local minimum at t_2 and a local maximum at t_3. Let $\ddot{\theta}$ be the time derivative of $\dot{\theta}$. From (37) we find:

(39) $$\ddot{\theta}_t = (r+\delta)\dot{\theta}_t - D''(S_t)\dot{S}_t$$

As $D''(S_t) > 0$ for all t, and $\ddot{\theta}_{t2} > 0$ we see from (39) that $\dot{S}_{t2} < 0$. We also see that $\ddot{\theta}_{t3} < 0$ gives $\dot{S}_{t3} > 0$. Assuming $\dot{S}$ is continuous, there is a t^* such that $t_2 < t^* < t_3$ and $\dot{S}_{t*} = 0$. Thus, S has a local minimum at t^*. Since $S_{t3} > S_{t*}$ and $\dot{S}_{t3} > \dot{S}_{t*}$, we must have $x_{t3} > x_{t*}$. Thus $\dot{x}_t > 0$ for at least some value of $t \in [t_2,t_3]$. This is a contradiction to the result in Appendix A that

$\dot{x}_t < 0$ for $\ddot{\theta}_t > 0$. Therefore, the optimal carbon tax cannot first decrease and later increase. This rules out the possibility that the graph can have two humps or more. *The optimal carbon tax will therefore initially increase and eventually decrease and approach zero.* This also proves that even if the extraction of fossil fuels may increase for a given period of time, *$\dot{S}$ will be negative only at the end of the planning horizon.* Therefore, the accumulation of carbon in the atmosphere and the optimal carbon tax will only have one-hump shapes when the negative externality is related to the atmospheric stock of carbon.

Figure A1: An Impossible Evolution of the Optimal Carbon Tax and the Atmospheric Stock of Carbon



C. Damage as a Function of the Rate of Change in the Atmospheric Stock

The optimisation problem when the negative externality is related to the time derivative of the atmospheric stock is as follows:

(40)
$$\begin{aligned} &maximise \int_0^\infty e^{-rt}\cdot[u(x_t) - c(A_t)x_t - d(\dot{S}_t)]dt \\ &s.t.\ \dot{A}_t = x_t \\ &\dot{S}_t = x_t - \delta S_t \\ &x_t \geq 0 \end{aligned}$$

The current value Hamiltonian:

(41)
$$H = u(x_t) - c(A_t)x_t - d(x_t - \delta S_t) + \lambda_t x_t + \gamma_t \cdot (x_t - \delta S_t)$$

The necessary conditions for an interior optimum are:

(42)
$$\frac{\partial H}{\partial x_t} = u'(x_t) - c(A_t) - d'(\dot{S}_t) + \lambda_t + \gamma_t = 0$$

(43)
$$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t$$

(44)
$$\dot{\gamma}_t - r\gamma_t = -\frac{\partial H}{\partial S_t} = -\delta d'(\dot{S}_t) + \delta \gamma_t$$

(45)
$$\dot{A}_t = x_t$$

(46)
$$\dot{S}_t = x_t - \delta S_t$$

(47)
$$\lim_{t \to \infty} e^{-rt}\lambda_t A_t = 0$$

(48) $$\lim_{t\to\infty} e^{-rt}\gamma_t S_t = 0$$

where equation (42) is fulfilled with equality in accordance with Appendix A. In same way as in Appendices A and B, we find from (47) and (48):

(49) $$\lim_{t\to\infty} e^{-rt}\lambda_t = 0 \;\;\wedge\;\; 0 < \lim_{t\to\infty} A_t = \hat{A} < \infty$$

(50) $$\lim_{t\to\infty} e^{-rt}\gamma_t = 0 \;\;\wedge\;\; \lim_{t\to\infty} S_t = 0$$

The characteristics of λ_t are expressed in (8)-(12). γ_t is the shadow price associated with accumulated atmospheric stock up to t. γ_t has the characteristics described below, using the same derivation techniques as under Appendices A and B, and given (50), $\dot{S}_t \to 0$ when $t \to \infty$ and $d'(0) = 0$:

(51) $$\gamma_t = \delta\int_t^{\infty} e^{-(r+\delta)(\tau-t)} d'(\dot{S}_\tau)\,d\tau$$

(52) $$\lim_{t\to\infty}\gamma_t = \lim_{t\to\infty}\frac{\delta d'(\dot{S}_t)}{(r+\delta)} = 0$$

(53) $$\dot{\gamma}_t = \delta\int_t^{\infty} e^{-(r+\delta)(\tau-t)}\frac{\partial d'(\dot{S}_\tau)}{\partial\tau}d\tau = \delta\int_t^{\infty} e^{-(r+\delta)(\tau-t)} d''(\dot{S}_\tau)(\dot{x}_\tau - \delta\dot{S}_\tau)\,d\tau$$

(54) $$\lim_{t\to\infty}\dot{\gamma}_t = \lim_{t\to\infty}(r+\delta)\gamma_t - \lim_{t\to\infty}\delta d'(\dot{S}_t) = 0$$

According to (52), $\gamma_t \to 0$ as $t \to \infty$. As γ_t represents the increase in welfare of a marginal increase in the atmospheric stock of carbon at time t, quite intuitively $\gamma \to 0$ since $S_t \to 0$ when $t \to \infty$. γ_t can be positive as well as negative since $d'(\dot{S}) > 0$ for $\dot{S} > 0$ and $d'(\dot{S}) < 0$ for $\dot{S} < 0$.

Using $u'(x_t) = p_t$, we find from (42):

$$p_t = c(A_t) - \lambda_t + d'(\dot{S}_t) - \gamma_t \tag{55}$$

Since p_t is the consumer price and $c(A_t) - \lambda_t$ is the producer price (see Appendix A), the optimal carbon tax at time t, σ_t, is

$$\sigma_t = d'(\dot{S}_t) - \gamma_t \tag{56}$$

It has the following properties,

$$\lim_{t\to\infty} \sigma_t = \lim_{t\to\infty} d'(\dot{S}_t) - \lim_{t\to\infty} \gamma_t = 0 \tag{57}$$

$$\dot{\sigma}_t = d''(\dot{S}_t)\cdot(\dot{x}_t - \delta\dot{S}_t) - \dot{\gamma}_t \tag{58}$$

$$\lim_{t\to\infty} \dot{\sigma}_t = \lim_{t\to\infty} d''(\dot{S}_t)\cdot(\dot{x}_t - \delta\dot{S}_t) - \lim_{t\to\infty} \dot{\gamma}_t = 0 \tag{59}$$

D. Extraction of Fossil Fuels given a Non-polluting Backstop Technology

In the presence of a non-polluting backstop, y, the optimisation problem of the social planner can be described as below. In this specification I use the basic model. Thus, I define the negative externality to be a function of the atmospheric stock of carbon.

$$maximise \int_0^{\infty} e^{-rt}\cdot[u(x_t+y_t) - c(A_t)x_t - \bar{c}y_t - D(S_t)]dt$$

(60)
$$\begin{aligned} s.t.\ \dot{A}_t &= x_t \\ \dot{S}_t &= x_t - \delta S_t \\ x_t &\geq 0 \\ y_t &\geq 0 \\ c(0) &< \bar{c} < u'(0) \end{aligned}$$

The current value Hamiltonian:

(61) $$H = u(x_t+y_t) - c(A_t)x_t - \bar{c}y_t - D(S_t) + \lambda_t x_t + \mu_t\cdot(x_t - \delta S_t)$$

The necessary conditions for an interior optimum:

(62) $$\frac{\partial H}{\partial x_t} = u'(x_t+y_t) - c(A_t) + \lambda_t + \mu_t \leq 0 \quad (=0 \ \ for\ x_t > 0)$$

(63) $$\frac{\partial H}{\partial y_t} = u'(x_t+y_t) - \bar{c} \leq 0 \quad (=0 \ \ for\ y_t > 0)$$

(64) $$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial A_t} = c'(A_t)x_t$$

(65) $$\dot{\mu}_t - r\mu_t = -\frac{\partial H}{\partial S_t} = D'(S_t) + \delta\mu_t$$

(66) $$\dot{A}_t = x_t$$

(67) $$\dot{S}_t = x_t - \delta S_t$$

(68) $$\lim_{t\to\infty} e^{-rt}\lambda_t A_t = 0$$

(69) $$\lim_{t\to\infty} e^{-rt}\mu_t S_t = 0$$

In accordance with Appendices A and B, we find:

(70) $$\lim_{t\to\infty} e^{-rt}\lambda_t = 0 \;\wedge\; 0 < \lim_{t\to\infty} A_t = A^* < \infty$$

(71) $$\lim_{t\to\infty} e^{-rt}\mu_t = 0 \;\wedge\; \lim_{t\to\infty} S_t = 0$$

The scarcity rent, π_t, and the optimal carbon tax, θ_t, have the characteristics derived in Appendices A and B. I define $u(z) = \int_0^z p(z)dz$, $z = x+y$, where $p(z)$ is the consumer price. Then, in accordance with section 2, $u'(x+y) = p$. Combining this with (9), (31), (62) and (63) we get:

(72) $$p(x_t+y_t) = c(A_t) + \pi_t + \theta_t, \quad x_t > 0$$

(73) $$p(x_t+y_t) = \bar{c}, \quad y_t > 0$$

Taking time derivatives and using (9) and (64) we find:

(74) $$\dot{p}_t = r\pi_t + \dot{\theta}_t, \quad x_t > 0$$

(75) $$\dot{p}_t = 0, \quad y_t > 0$$

If there were *no external effects*, i.e., $\theta_t = 0$ for all t, we see that $\dot{p} > 0$ for $x_t > 0$ (as $\pi_t > 0$) and $\dot{p} = 0$ for $y_t > 0$. This means that *x and y cannot be positive at the same time*. For $c(0) < \bar{c}$, it will be cheaper to extract fossil fuels initially than to produce the backstop. Thus, $x_t > 0$ and $y_t = 0$ in the beginning of the planning period. But since $\dot{p} > 0$ for $x_t > 0$ and $\bar{c} < u'(0)$, the price will increase and eventually reach $\bar{c}$. Further extraction of fossil fuels will increase the consumer price of fossil fuels above $\bar{c}$. This will not be optimal since the backstop can be produced at this price. Therefore, from the time p_t equals $\bar{c}$ onwards, we have $x_t = 0$ and $y_t > 0$.

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