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Discus

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Abstract:
Aslaksen *et al.* (1990) concluded that the petroleum wealth of Norway, and hence the permanent income from petroleum extraction, was as uncertain as the yearly oil revenues. Their conclusion was based on wealth estimates using official price projections, with no independent empirical analysis of the oil price process. In this paper the wealth estimates are based on an empirical analysis of the oil prices.

We find that the best estimate of the roots of the price process indicates a more stable wealth than the conclusions in Aslaksen *et al.* (1990) indicated. If we introduce a possible shift in the price process at the time of OPEC I in 1974, the price shift in OPEC II, has an indirect effect on petroleum wealth through its influence on the best parameter estimate. This indirect effect is considerable, and the main conclusions from Aslaksen *et al.* (1990) are maintained in spite of the low roots.

Keywords: Resource wealth, Parameter uncertainty.

JEL classification: H60, Q30.

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1 Introduction

Are the revenues from resource extraction income or disinvestment in natural capital? A natural answer to this question would be to calculate the value of the remaining reserves, the resource wealth, and define changes in this wealth as the part of the revenues that can be considered disinvestment in natural capital, and counting the returns to the wealth as income. With no uncertainty, a sustainable policy would require that only the return from the wealth is consumed, at least in the long run. With a reasonable stable wealth, a similar rule would apply even with uncertainty, as argued below. The nation would then have to save during good years when revenues are higher than the permanent income, and borrow in bad years.

An application of this approach to the Norwegian petroleum wealth in Aslaksen *et al.* (1990), revealed that the petroleum wealth was highly unstable, due to changes in the official price projections. The unexpected changes in wealth was estimated to be at the size of Norway's GDP at the highest. This finding undermines the normative relevance of the definition of income component of the petroleum revenues. Since the wealth was fluctuating with the oil prices, the wealth and hence the permanent income was high when the revenues were high. In this sense there were no 'good' or 'bad' years, and hence the policy indicated above would not stabilize the spendings. In other words, the 'permanent' income would not be permanent.

The estimates in Aslaksen *et al.* (1990) was based on official price projections, projections that were not intended for this use. Still these projection was the best avaialble information on the befliefs about future prices at each pint in time, hence to use these projections was a natural choice when considering the rationality of the policy given the beliefs. The importance of the changes in wealth reporten in Alskasen et al. (1990) is sufficient to justify an empirical analysis of the underlying oil price process. On the other hand, the official price projections are based on model analysis, and some of these model are based on a much more comprehensive econometric analysis than the one provided here. Still, these studies were undertaken to quantify parameters in a model, and not to investigate the time series properties of oil wealth.

Since the projections were not intended for wealth estimates, elements of the projections that are essential to wealth estimates, like the growth rate, may not have been carefully considered. If the projections were intended for the short or intermediate run, the difference between 0 and 2 % growth rate is negligible. For estimation of wealth, the growth rate is highly important. Moreover, different persons may have been responsible for the different projections, in which case much of the volatility of wealth estimates may be accidental. Thus an empirical investigation of the time series properties of the oil price process is required to evaluate the volatility of oil wealth. Such an investigation is the purpose of the present paper.

There is of course much both theoretical and empirical work on the oil prices. The simplest models prescribe prices to grow at a rate equal to the rate of interest, the Hotelling rule. While Hotellings (1931) seminal paper was the origin of this rule, he recognized that the simple rule was too simple to apply to real markets. For a recent discussion see Farzin (1992), and for an empirical evaluation see Miller and Upton (1985). Much work on optimal resource extraction has assumed that the prices follow a geometrical brownian motion, a stochastic variant of the Hotelling rule, but as demonstrated in Lund (1992) this process can hardly be the equilibrium price process in a resource market. Still the prevailing view in the market, as reported in Manne and Schrattenholtzer (1988), corresponds closely to the expectation paths that would prevail with prices following a geometrical brownian motion. The typical picture is that mean prices from different projections will start at the current price and grow at a constant rate. Many of these projections are based on much more data than used in the current empirical analysis. Studies analogous to the one reported here are found in Bernhardsen (1989) and espesially in Green et al (1993), who has also provided us with the datas used for this study. The present study differs from all of these studies in the focus on the parameter uncertainty and the effect of new information on the volatility of oil wealth.

2 Uncertainty and the normative properties of wealth

A number of papers in Norway during the 1980s have focused on the vulnerability of an economy highly dependent on oil revenues. Much emphasis has been put on the correct measurement of national saving. The Norwegian National Accounts show an immense increase in national savings in the first half of the 1980s, as a result of the fast increase in petroleum extraction and the oil price shock at the start of the decade. The question then arose as to what the relevant measure of savings is, if changes in the wealth of petroleum are taken properly into account.

Skånland (1985) saw petroleum extraction as depletion of wealth only, and obtains "corrected" savings ratios simply by subtracting the annual resource rent from national income.[1] Strøm (1986) instead calculates savings ratios where changes in the wealth of petroleum reserves are accounted for. He defines petroleum wealth as current net price times reserves. This coincides with the expected net present value, if expected future net price of oil and gas increases at a rate equal to the discount rate (the Hotelling rule).[2]

In the aftermath of the 1986 turn-round, a government-appointed committee was established to review the economic prospects of Norway. The report delivered, NOU (1988), recommended that the wealth balances of oil revenue spending should be taken into account, suggesting that spending adjusted for wealth accounting had been too high. The committee chairman argued for a spending schedule based on permanent income, defined as the real return on the oil wealth. This preserves the oil wealth and gives each generation an equal share of the wealth. "Total savings were strongly overestimated (1980-86). Whether or not the politicians would have followed a less expansive economic policy, provided they had more correct information on national income and saving in these years, is a question we shall probably never know the answer to." (Steigum (1989).) According to Cappelen and Gjelsvik (1990), the reference here is probably to the computations in NOU (1988), in which the "petroleum-adjusted" savings ratio is 2-9 % lower than the ordinary savings ratio for the year 1981-85.

However, in Aslaksen *et al.* (1990) the authors claim that the conclusion is quite opposite.[3] For the period 1973 to 1989, the value of extracted petroleum have been lower than expected returns on the Norwegian petroleum wealth. The reason for the divergence, is that NOU (1988) compute the wealth as if prices and reserves were known in advance. For example, the wealth of petroleum in 1980 is calculated by assuming actual development of prices from 1980 till 1988, and then expected prices from 1988 and beyond. Thus, according to Aslaksen *et al.* (1990), these results cannot be used as a background for evaluation of the management of the wealth of petroleum, since they are based on information that was not available when the policy decisions were made: Recommendations based on these calculations will therefore

[1]However, as Strøm (1986) emphasizes, this is depletion of a non-existing wealth, if the wealth of petroleum is not included in the National Accounts. Skånland's savings ratio therefore turns out very low.

[2]Large fluctuations, especially in the price of petroleum, then also cause extreme fluctuations in the savings ratio (from 18 to 60 % in the period 1980-84).

[3]See also Brekke *et al.* (1989).

have a strong character of hindsight.

To avoid this hindsight, estimates of the wealth at time t has to be based on the information available at time t. As a consequence the wealth may be very unstable due to changes in the price prospect on oil. If the size of this unsability is as reported in Aslaksen *et al.*, the relevancy of wealth estimates as a basis for measuring the national saving may be questioned. .

The definition of petroleum wealth, used in the calculations in e.g. Aslaksen *et al.* (1990), is

$$W_t = \sum_{s=t}^{T} E^t \left[P_s x_s - c_s \right] (1+r)^{t-s}, \tag{1}$$

where W_t denotes the wealth, and $E^t[\cdot]$ denotes expectations given the information available at time t. P_s is the oil price in period s. $(x_t^t, x_{t+1}^t, \ldots)$ and $(c_t^t, c_{t+1}^t, \ldots)$ is the path of extraction and cost, respectively. These are the expected paths given the information at time t. Since new fields are discovered, and since the extraction path is reoptimized every period, c_s^t, and x_s^t, will vary with t. We have used the same cost and extraction series as in Aslaksen *et al.* (1990). See Børing (1994) for a discussion of these assumptions. T is the horizon, chosen long enough to include the date of depletion. Finally, r is the discount rate.

Introducing $\pi_t = P_t x_t - c_t$, actual changes in wealth, $\Delta W_t = W_{t+1} - W_t$, can be decomposed as

$$\begin{aligned} \Delta W_t &= W_{t+1} - (1+r)(W_t - \pi_t) \\ &\quad + r(W_t - \pi_t) \\ &\quad - \pi_t \\ &= \sum_{s=t+1}^{T} (E^{t+1}[\pi_s] - E^t[\pi_s])(1+r)^{t+1-s} \\ &\quad + r \sum_{s=t+1}^{T} E^t[\pi_s](1+r)^{t-s} \\ &\quad - \pi_t. \end{aligned} \tag{2}$$

The change in wealth is here decomposed into three terms. The first one expresses changes in expectations, ΔW_t^e. The second term is the expected return from the wealth remaining after x_t is extracted, ΔW_t^{rw}. Finally the last term is the revenues from extraction at time t, ΔW_t^p. Expected return may be higher than the current value of extraction. This will typically be the case if π_t is small compared to $E^t[\pi_s]$ for sufficiently many $s > t$.

Aslaksen *et al.* (1990) found that the wealth of petroleum has fluctuated between 413 and 2273 bill Nkr, 1986 prices, during the 1980's. The reason for these large changes in wealth, is mainly price fluctuations. From 1981 to 1987, a steady reduction in price expectations caused a reduction in the wealth of petroleum of 2278 bill Nkr. This is about four times the size of the yearly Norwegian GDP for the same period. Unexpected changes in wealth of this size causes obvious problems for the management of petroleum wealth.

According to the Hicksian definition of income, the income part of the revenue is the amount the nation could consume without impoverishing itself. With no uncertainty, constant interest rate and constant prices, this would imply a constant wealth. We note that if the first term above (related to changes in expectations) is zero, the wealth could be kept constant if $\pi_t = \frac{r}{1+r} W_t$, which is then the permanent income from the wealth. For the Norwegian petroleum wealth this was calculated to be two to three times the actual revenues at least until OPEC III in 1986. Spending this amount annually would have been a very bad policy for Norway, due to the huge unexpected changes (reductions) in wealth. But all this is, according to the calculations in Aslaksen *et al.* (1990), based on governmental price projections. With alternative assumptions about the future oil prices the picture could be different.

Consider the case when the oil prices are behaving like purely white noise, i.e. π_s is independent of π_t for $t \neq s$. Then $E^t[\pi_s] = E^{t'}[\pi_s]$ for all $t, t' < s$. Thus the only possible unexpected change in the wealth from one year to the next, is the value of the next years revenues. Expected future revenues are unchanged. The other extreme case is the random walk, where $P_t = P_{t-1} + u_t$, with $E[u_t] = 0$. In this case $E^t[P_s x_s] = P_t E[x_s]$, if x and P are independent. Thus a shift in the price from one year to the next would change the expected value of all future revenues. Thus the time series properties of the oil price is important to the size of the unexpected changes in wealth. We now turn to the study of these properties.

3 Unit roots and volatility

Consider a stochastic process X_t of the form[4]

$$X_t = AX_{t-1} + u_t.$$

The roots of this process is the eigenvalues of the matrix A. The process has a unit root if one of the eigenvalues λ of A is unitary, i.e. $|\lambda| = 1$. If the eigenvalues are distinct, A can be written as $A = U'\Lambda U$ where U is a unitary matrix, and $\Lambda = diag(\lambda_1, \ldots, \lambda_n)$. In this case

$$E\left[X_s | X_t\right] = U'\Lambda^{s-t}UX_t,$$

where $\Lambda^{s-t} = diag(\lambda_1^{s-t}, \ldots, \lambda_n^{s-t})$. If $|\lambda_i[< 1$ for all i, the effect of a shift in X_t will be dampened over time, Λ approaches the zero matrix in the long run. If one of the eigenvalues is unitary, a shift in X_t will have a permanent effect on the future expectations.

A simple model that would contain both the extreme cases mentioned in the introduction is an AR(1) process. In the simplest form we may assume that

$$P_t = aP_{t-1} + u_t.$$

In this case the root of the process is a. The process has a unit root if $|a| = 1$, but in this particular case only $a = 1$ is relevant. In the case $a = 1$ the process will be the random walk considered above, while the process is white noise if $a = 0$. The governmental price projections typically state a constant growth rate, an assumption that would correspond to a random walk on logarithmic form. For this reason we have chosen to work with the logarithmic form. We slightly extend the class of processing by adding a constant.

Let p_t denote the log of the price process, $p_t = \ln(P_t)$. We assume that the log of the price process is a linear AR(1) process of the form

(3) $$p_t = a_0 + a_1 p_{t-1} + u_t,$$

where $u_t \sim IIN(0, \sigma^2)$. Recursive use of (3) yield

[4]Note that a process with longer lags may also be written on this form. E.g. the AR(2) process $P_t = a_1 P_{t-1} + a_2 P_{t-2} + u_t$ will be of this form with $X_t' = [P_t, P_{t-1}]$ and

$$X_t = \begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix} X_{t-1} + \begin{bmatrix} u_t \\ 0 \end{bmatrix}$$

$$(4) \qquad p_s = a_0 A_{s,t} + a_1^{s-t} p_t + \sum_{i=0}^{s-t-1} a_1^i u_{s-i},$$

where $A_{s,t} = \sum_{i=0}^{s-t-1} a_1^i$. Thus P_s is lognormal distributed given the observation P_t for $s > t$, and

$$(5) \qquad E[P_s|P_t] = \exp\left(a_0 A_{s,t} + \frac{1}{2}\sigma^2 A_{s,t}^2\right) P_t^{a_1^{s-t}},$$

where $A_{s,t}^2 = \sum_{i=0}^{s-t-1} a_1^{2i}$.

The oil wealth at time t, defined as the expected present value of future revenues, is:

$$(6) \qquad W_t = \sum_{s=t}^{T} (E[P_s|P_t]x_s - c_s)(1+r)^{t-s} = X_t - C_t,$$

where $X_t = \sum_{s=t}^{T} E[P_s|P_t]x_s(1+r)^{t-s}$ is the present value of the production, and $C_t = \sum_{s=t}^{T} c_s(1+r)^{t-s}$ is the present value of the cost. Let $\bar{P}_s = E[P_s|P_t]$ denote the expected price, given the current price and the parameters a_0, a_1 and σ. $\bar{X}_t$ denote the corresponding value of X_t. We want to consider how sensitive the wealth is to changes in the price P_t and the parameters. Suppose that the price at time t is given a proportional shift, to the new price ξP_t. We note that C_t is as before, while by (5)

$$(7) \qquad X_t = \sum_{s=t}^{T} \bar{P}_s \xi^{a_1^{s-t}} x_s (1+r)^{t-s}.$$

The effect of this shift is determined by the parameter a_1. If $a_1 = 1$, then $\xi^{a_1^{s-t}} = \xi$ for all s. Thus $X_t = \xi \bar{X}_t$. Since the cost is unchanged, the effect on the petroleum wealth is more than proportional to the shift in the oil price. Thus with $a_1 = 1$, the uncertainty of the petroleum wealth is even higher than the uncertainty of the oil prices. On the other hand, if $a_1 < 1$, then $a_1^{s-t} \to 0$ as $s \to \infty$, and $\xi^{a_1^{s-t}} \to 1$. The rate of convergence depends upon the size of a_1. The effect of a change in prices on the wealth depends on the extraction path, the relative size of X and C, and finally on the parameter a_1. To give some flavor to (7), we compute the changes in wealth corresponding to an shift in the oil price when $X = 1.5 \cdot C$, $r = 0.07$, and the baseline value of extraction path is constant ($\bar{P}_s x_s$ is independent of s) and infinite. Let the elasticity of wealth, with respect to oil prices, denote the percentage change in wealth when current price increases 1 %, $\xi = 0.01$. This elasticity is given in the following table, for different values of a_1:

a_1	0.2	0.5	0.7	0.9	0.95
elasticity	0.24	0.37	0.57	1.23	1.75

Due to the cost-term, the elasticity is higher than one, even when the process has no unit root. As a benchmark, note that if the cost is constant over time, then $P_t x_t = 1.5 \cdot c_t$, given the assumptions above. Thus the oil revenues would increase by 3 % if the oil price shift 1 %, but the wealth is less volatile than the oil revenues $(P_t x_t - c_t)$, except for $a_1 = 1$. As pointed out in the introduction, the volatility of oil wealth is important to the management of the wealth.

In the case $a_1 = 1$, any shift in the price P_t will have a permanent effect on future prices, in the sense that expected prices at all future points in time will shift proportionally. For $a_1 < 1$, the shift in expected future prices are less than proportional. However, this requires that the parameters are stable.

The oil market has been very difficult to model. Partly due to the unstable political state in the Middle East, the oil market has experienced several shocks since 1973. If these shocks are interpreted as permanent changes in the market structure, the parameters of the price process should change. Let us first consider the shift in expected prices when a_0 is shifted. Note that $A_{s,t}$ start at the value 1 for $s = t+1$ increasing to $1/(1-a_1)$ as $s \to \infty$. Thus the shift in expected prices will be increasing in s. On the other hand there is always a cause for a shift in parameters.

Suppose that a shift ξ in the price, at time t, is explained as a shift in the long run equilibrium path. Then

$$\ln(\xi) = \Delta p_t = \frac{\Delta a_0}{1-a_1}.$$

Note that (5) can be rewritten as

$$E[P_s|P_t] = \exp\left(a_0 A_{s,t} + \frac{1}{2}(\sigma^2 A_{s,t}^2) + a_1^{s-t} p_t\right). \tag{8}$$

Moreover, since

$$A_{s,t} + \frac{a_1^{s-t}}{1-a_1} = \frac{1}{1-a_1},$$

we see that $E[P_s|P_t] = \xi \bar{P}_s$. Thus even if a_1 is small, the effect of a price shift is permanent if it is interpreted as a shift in parameters. Thus if OPEC I in 1973 is interpreted as a shift in market structure, and represented as a dummy in the estimation, the whole path of expected future prices will shift, and the oil wealth will be very sensitive to such chocks. This is quite intuitive. Less obvious is the effect on the volatility of oil wealth in the period after the shock.

Consider OPEC I in 1973 interpreted as a permanent shock. Using yearly prices, the first observation that give any information on the shift of Δa_0 would be the price in 1974. This is obviously a meager information to estimate the size of the shift. Thus every new observation would contribute significantly to the knowledge about this shift. Thus even if a_1 is very low, and even if we assume no shift between 1974 and 1975, the price changes from 1974 to 1975 will have permanent effect on expected future prices, since Δa_0 is very sensitive to the 1975 observation.

We conclude that it is not sufficient to test for unit roots to evaluate the volatility of oil wealth, even in the years between the structural shifts. To estimate volatility, the wealth should be computed each year, given the information available at that year. An empirical study along these lines is presented below.

4 Empirical result

Aslaksen *et al.* (1990) have recovered the "projections" about the oil price from governmental White Papers, especially national budgets and long-term programs. These projections typically start at the current price level, and grow steadily at an annual rate between 0 and 2 %. Interpreting these projections as expectations, they corresponds to the expectations given in the model above, with $a_1 = 1$, and a_0 varying from 0 to 2 %. The effect of a price shift on the wealth is somewhat ambiguous. Since $a_1 = 1$, a price shift will have a permanent effect on the future expected prices. On the other hand, a simultaneous shift in a_0 may either dampen or reinforce this effect. Both effects are present in the official projections. When prices increased in 1980, there was a modest optimism, reflected in $a_0 = 1$ %. Even though the prices fell from 1980 to 1981, the optimism appears to have grown, since in 1981 the growth rate in prices was 2 %. This shift dominates the effect of the price fall, and unexpected changes in wealth is actually positive. But the prices continued to fall during the 1980's, and the optimism decreased, reflected in a declining growth rate, eventually in 1985, reaching $a_0 = 0$ %. These shifts reinforced the effects of falling prices, and the wealth fell fast in spite of rather stable prices.

As pointed out in the introduction, we should be careful in interpreting time series properties of the governmental view on the oil price process from such projections. An optimistic projection may be used strategically to support an expansive policy, or the optimists within the Ministry of Finance may be strengthened from the increasing prices. The projection may thus reflect nobody's view on the price process. Finally, the governmental beliefs may be inconsistent with data. Thus it is interesting to study the empirical properties of the price process.

As a point of departure, we estimate the parameters in (3) on yearly data on the oil price from 1908, using a time series on oil prices constructed by Green *et al.* (1993). The parameters are reestimated for each year after 1973. The petroleum wealth is then computed for each year, given the parameters that are estimated given the oil prices until and including this year. Thus the wealth estimate for year t is based on the available information at time t, but not on information that was not yet available. The estimate of a_1 is about 0.74 which is high, but significantly less than 1. Since this estimation is based on about 100 observations, an additional observation will only have a minor effect on the parameters, and hence the parameters are rather stable over the period from 1973 to 1989.

The petroleum wealth, with the estimates of a_0, a_1 and q, is given in table 1. These wealth estimates are in general much lower that the corresponding in Aslaksen *et al.* (1990). This reflect that the model of the oil market represented by (3) is very different from the view represented by governmental projections. Most students of the oil market believed that the OPEC I had shifted the market structure. The analysis in Green *et al.* (1993) based on the same data shows that a series of shift can be identified. While OPEC I was identified as a shift in their analysis, they used oil prices until 1989 to test this hypothesis. But what could we know about the market structure with the data available in the 1970's? To consider this we estimate a model with a OPEC I-dummy.

The modeling of OPEC I is not straightforward. The obvious first step is to introduce a dummy d in (3) with value 1 from 1974 and zero before,

$$p_t = a_0 + qd_t + a_1 p_{t-1} + u_t. \tag{9}$$

The estimated parameters for this model is presented in table 2, and the corresponding wealth estimates in table 3.

We immediately notice that the wealth in 1974 is completely out of scale. At a closer look we note that the long-term equilibrium price, which is approximately $\exp\left(\frac{a_0+q}{1-a_1}\right)$, is of size 1000 dollar per barrel. We easily recognise why the model produces such results. In

Year	W_t	ΔW_t^e	ΔW_t^{rw}	ΔW_t^p
1973	-32	100	-2	-4
1974	70	-23	6	-12
1975	64	50	5	-12
1976	132	34	10	-7
1977	183	35	13	-3
1978	233	-7	16	10
1979	232	364	15	24
1980	587	278	37	55
1981	848	75	55	59
1982	920	-14	61	51
1983	915	7	60	59
1984	923	254	60	62
1985	1176	-975	78	56
1986	223	70	14	16
1987	291	-98	19	21
1988	192	40	12	20
1989	223	-	14	30

Table 1: Petroleum wealth; AR(1) model without OPEC dummy.

Year	a_0	a_1	q
1973	0.563	0.741	0.000
1974	0.563	0.741	1.231
1975	0.888	0.593	0.771
1976	0.939	0.570	0.678
1977	0.967	0.557	0.632
1978	0.990	0.547	0.597
1979	0.993	0.545	0.591
1980	0.961	0.560	0.625
1981	0.935	0.572	0.628
1982	0.923	0.577	0.629
1983	0.935	0.572	0.628
1984	0.950	0.565	0.625
1985	0.957	0.562	0.622
1986	0.997	0.543	0.585
1987	0.982	0.550	0.571
1988	0.941	0.569	0.520
1989	0.920	0.579	0.502

Table 2: Parameter estimates; AR(1) model with OPEC dummy.

Year	W_t	ΔW_t^e	ΔW_t^{rw}	ΔW_t^p
1973	-32	22640	-2	-4
1974	22609	-22948	1583	-12
1975	1257	-347	89	-12
1976	1011	-209	71	-7
1977	880	-44	62	-3
1978	900	-90	62	10
1979	863	83	59	24
1980	981	173	65	55
1981	1164	370	77	59
1982	1553	346	105	51
1983	1952	220	133	59
1984	2246	-354	153	62
1985	1983	-530	135	56
1986	1532	-356	106	16
1987	1267	-16	87	21
1988	1316	-235	91	20
1989	1152	-	79	30

Table 3: Petroleum wealth; AR(1) model with OPEC dummy.

1974, there is only one observation on which to base the estimate on the one parameter q. Thus the model will fit this observation perfectly, i.e. $u_t = 0$ and hence

$$p_t = a_0 + q + a_1 p_{t-1},$$

which implies that

$$p_t - p_{t-1} = \left(\frac{a_0 + q}{1 - a_1} - p_{t-1}\right)(1 - a_1).$$

Thus the shift from p_{t-1} to p_t is interpreted as only a share $(1 - a_1)$ of the underlying long-term shift to the new long-term equilibrium. With the estimated a_1, this shift is only about 26 % of the total. Thus while the price increased with a factor of almost 3, this is interpreted as the first 26 % of the total shift *in a logarithmic scale.* It seems fair to say that this model does not reflect the prevailing view of the market in 1974.

A solution to this problem would be to adjust the model to

$$p_t = a_0 + q d_{t-1} + a_1 p_{t-1} + u_t. \tag{10}$$

The lag in the dummy implies that the dummy first apply to the price changes from 1974 to 1975. The idea of this model is that the long run equilibrium shifted in 1974, but that the change from 1973 to 1974 was extraordinary and hence (9) does not apply to 1974. We believe that (10) corresponds better to the prevailing view. This model will be very sensitive to the 1975 price, like the previous was sensitive to the 1974 price, but since there is not as dramatic changes from 1974 to 1975, the estimates will be less extreme. On the other hand, the value of q in 1974 is still a problem. Either the shift has not yet been recognized and $q = 0$, or the shift has been recognized, but there is no observations on which to base the estimate of q.

If the belief was that the new price was approximately the new long run equilibrium, then $p_t = (a_0 + q)/(1 - a_1)$, and hence

$$q = p_{1974}(1 - a_1) - a_0.$$

While this assumption would be purely ad hoc, such leaps of faith would be necessary with this model. Each new observation would considerably improve the knowledge about the shift q, and as noted in the previous section, the effect on the oil wealth is approximately like the effect of a high a_1. If a_1 is high too, the wealth will be very sensitive to new prices.

Year	a_0	a_1	q
1973	0.563	0.741	0.000
1974	0.690	0.692	0.352
1975	0.690	0.692	0.205
1976	0.695	0.690	0.270
1977	0.695	0.690	0.285
1978	0.695	0.690	0.278
1979	0.698	0.688	0.303
1980	0.685	0.694	0.370
1981	0.652	0.709	0.375
1982	0.637	0.716	0.376
1983	0.646	0.712	0.375
1984	0.656	0.707	0.372
1985	0.658	0.706	0.370
1986	0.674	0.699	0.320
1987	0.693	0.691	0.335
1988	0.636	0.717	0.279
1989	0.638	0.716	0.281

Table 4: Parameter estimates; AR(1) model with lagged dummy.

Year	W_t	ΔW_t^e	ΔW_t^{rw}	ΔW_t^p
1973	-32	571	-2	-4
1974	541	-362	39	-12
1975	229	174	17	-12
1976	432	34	31	-7
1977	503	34	35	-3
1978	576	-18	40	10
1979	588	229	39	24
1980	832	247	54	55
1981	1078	313	71	59
1982	1404	288	95	51
1983	1736	181	117	59
1984	1975	-229	134	62
1985	1818	-739	123	56
1986	1147	-180	79	16
1987	1029	-159	71	21
1988	920	-132	63	20
1989	831	-	56	30

Table 5: Petroleum wealth; AR(1) model with lagged dummy.

The estimated parameters and the corresponding wealth is presented in tables 4 and 5, respectively. The ad-hoc value of q in 1974 is about 0.35, compared to $a_0 = 0.69$. The first estimated value is 0.20, increasing to 0.27 in 1978. The high prices after OPEC II shifts the estimated value of q to 0.37 in 1980, and the value is quite stable after that. Thus the price shift due to OPEC II has two effects on the wealth. Since $a_1 > 0$, there is a direct effect on expected future prices. With the rather low value of $a_1 = 0.7$, 90 % of the direct effect has vanished for prices 5 years ahead. Thus the direct effect on the wealth is rather modest. If we only considered the time series properties of the price process, we would be mislead to conclude that the petroleum wealth should be stable, but then we would miss the indirect effect of the price shift, through parameter values, which is the most important. The shift in the constant $a_0 + q$ is from 0.97 to 1.05. Simultaneously, a_1 increases, and the combined effect is that the long-term equilibrium price increases from 23 dollar per barrel to 37 dollar per barrel. This shift has a permanent effect on the expected prices, in the sense that all the expected future prices, even far ahead, will be shifted.

Note that this shift is caused by OPEC II, while we have only introduced a dummy for OPEC I. If an additional dummy for OPEC II is introduced, the model will be even more sensitive to new information.

A possible problem with the analysis in this paper is the use of yearly prices. Monthly, weekly or even daily prices are available, and with say monthly prices, we would have 36 observation after three years, and not just three as in the previous analysis. On the other hand, a lag lenght of one year would require an AR(12) model with monthly data. Thus there would be more parameters to estimate, which counters the effect of more observations. The robustness of our conclusions to the use of yearly or monthly datas, is thus an empirical question for future investigation.

5 Conclusion

We have demonstrated that it is not sufficient to consider only the time series properties of the oil price process to evaluate the stability of the petroleum wealth. Even if the process has modest roots, shift in oil prices may have significant long-term effects through the new information that these prices provides. The driving force of this result is the introduction of a dummy for OPEC I. Observations of the price for the next ten years turned out to have considerable impact on the best estimate of this shift.

Figure 1 summarizes the findings. The wealth path calculated in Aslaksen *et al.* (1990) is included as a benchmark. Compared to those estimates, the wealth derived from the AR(1) process with no dummy would be much lower. The relative changes in wealth are still huge with the wealth increasing almost by a factor of four from 1979 to 1983 compated to a factor of three in the lagged dummy model. On the other hand, the scale is lower. The fluctuation of wealth due to changing prices is thus lower, at least in an absolute sence. The explanation of this is simply that the period of low prices prior to 1973, dominates the data set and hence the parameter estimates. Once we allow for a shift in the price in 1973, the picture changes. First the post 1973 prices are now believed to be higher than the previous one, and hence the level of the wealth increases. At the same time each new observation contributes significantly to the available information and thus to the estimates. Thus the wealth is more sensitive to price changes. We note that while the shift after OPEC II takes longer time, the wealth eventually reaches the same level as the one reported in Aslaksen *et al.* (1990). In this sense this study confirms the volatility of oil wealth.

The introduction of a dummy, is admittedly ad-hoc. A satisfactory study would require a complete model with stable parameters. In particular we would like to have a model that could account for the political shifts like the OPEC I-III episodes. This has been very difficult to obtain for the oil market. However, even if we would be able to construct such a model, the conclusions of this paper may still be true. Suppose that the price process is a mixture of an AR(1) process and a Poisson-process, with sudden shifts at random years. Even if we could find stable parameters of the distribution of the size and permanence of these shift, the actual shifts would not be directly observable. In that case the observation following a shift would provide essential new information, and thus be highly influencial on the estimates of the petroleum wealth.

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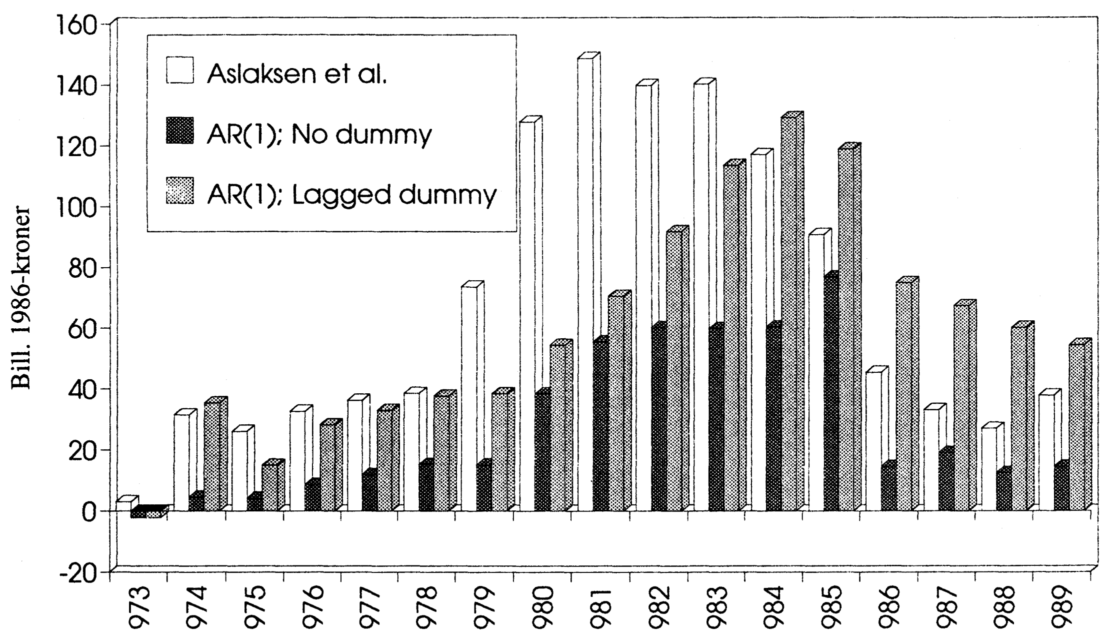

Figure 1: Permanent income from petroleum wealth 1973-89

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