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Choosing Measures of Inequality for Empirical Applications

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Abstract:
This paper is concerned with the distribution of income and the problem of choosing summary measures of inequality for empirical applications. By introducing a simple transformation of the Lorenz curve one is led to three measures of inequality, which jointly prove to represent a fairly good approximation of the inequality in a distribution function and also yield essential information about the shape of the income distribution. The paper also demonstrates that this type of inequality measures have an explicit normative foundation as well as an attractive statistical/geometrical interpretation. Furthermore, it is shown that the measures' sensitivity to transfers depend on the shape of the income distribution.

Keywords: Measures of inequality, statistical and normative characterization of inequality.

JEL classification: D3, D63.

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1. Introduction

Empirical analyses of income distributions are conventionally concerned with the deviation of each individual's (or household's) income from that of an individual (household) living in a society of complete equality. The concept of inequality, defined as the deviation from the state of equality, is commonly assumed to satisfy the principle of scale invariance and the Pigou-Dalton transfer principle. The principle of scale invariance implies that inequality solely depends on relative incomes and not on levels of income. The principle of transfers imposes an important normative property on the concept of inequality by requiring that inequality is reduced if we transfer income from a richer to a poorer person without changing their relative positions.

The Lorenz curve, which was introduced by Lorenz (1905), has proved to be a useful device for analysing inequality in distributions of income. The Lorenz curve relates the cumulative proportion of income units to the cumulative proportion of income received when units are arranged in ascending order of their income, and it takes the form of a straight line if and only if all units in the population receive the same income. Thus, the Lorenz curve is concerned with shares of income rather than with relative income levels and differs in that respect from the particular decile-specific representation of income inequality which is frequently employed by Scandinavian national statistical bureaus. This method of representation provides decile-specific mean incomes and their corresponding fractions of the overall mean income and thus meets the requirement for an interpretation of inequality in terms of relative income levels. By introducing a simple transformation of the Lorenz curve this paper provides an alternative interpretation of the information content of the Lorenz curve which proves to be closely related to the decile-specific representation of income inequality.

The standard approach for ranking Lorenz curves is to apply the criterion of Lorenz-dominance which recognizes the higher of the Lorenz curves as preferable, see Kolm (1969) and Atkinson (1970). In practice, however, one often experiences that Lorenz curves intersect. In this case the criterion of Lorenz-dominance is useless. To achieve rankings of intersecting Lorenz curves there are two possible strategies. One aims at searching for weaker ranking criteria than the one based on the criterion of Lorenz-dominance. The other is to apply summary measures of inequality. The latter explains why numerous alternative measures of inequality are introduced in the literature. The most widely used measure of inequality is the Gini coefficient, which is equal to twice the area between the Lorenz curve and its equality reference. But since no single measure can reflect all aspects of inequality exhibited by the Lorenz curve the importance of using alternative measures to the Gini coefficient is universally acknowledged. The most popular approach combines the Gini coefficient with one or two measures of the Atkinson family (see Atkinson, 1970). This practice, however, suffers from certain weaknesses. First, as demonstrated by Newberry (1970), the Gini coefficient and members of the Atkinson family have distinct theoretical foundations which makes it difficult to evaluate their capacity as supplementary measures of inequality. Moreover, in contrast to the Gini coefficient the Atkinson measures cannot be expressed in a simple way by the Lorenz curve and they therefore do not admit a similar geometric interpretation either. Second, there exists no method for choosing a small number of measures from the Atkinson family. Similar criticism can be leveled against various alternative selections of inequality measures which appear in empirical applications. Thus, the choice of the measures mentioned above seem to rest on a rather weak theoretical basis.

To deal with the problem of choosing measures of inequality this paper relies on the practice of using the moments for summarizing probability distributions. Even though the Lorenz curve cannot be considered as a probability distribution it proves to satisfy the conditions of being a cumulative distribution function. Thus, the moments of the Lorenz curve may form the basis of describing and summarizing the Lorenz curve. Moreover, as will be demonstrated in Section 2 the moments of the Lorenz curve generates a convenient family of inequality measures. By drawing on standard statistical practice the first few moments emerges as the primary quantities for summarizing the Lorenz curve. However, for interpretational reasons Section 3 provides arguments for employing a simple transformation of the Lorenz curve, the M-curve, as a basis for analysing inequality. The moments of the M-curve show to form a family of inequality measures which except for the first moment turns out to coincide with the Lorenz moment-based inequality measures. Even though the development of these measures of inequality is based on descriptive arguments it is demonstrated in Section 4 that they can be given an explicit expression of social welfare.

2. The moments of the Lorenz curve

Since Lorenz curves may intersect the criterion of Lorenz-dominance does not apply in many practical situations. The standard approach for ranking Lorenz curves in such cases is to employ a few summary measures of inequality, one of which usually appears to be the Gini coefficient. By making explicitly use of the Lorenz curve Mehran (1976), Kakwani (1980), Donaldson and Weymark (1980, 1983), Weymark (1981) and Yitzaki (1983) introduce "generalized" Gini families of inequality measures, but none of these authors provide sufficient theoretical arguments to support the choice of measures for the purpose of application.

Measures of inequality aim at summarizing the Lorenz curve. In this respect they correspond to the moments of a probability distribution. However, from observing that the Lorenz curve satisfies the conditions for being a cumulative distribution function we can exploit this parallel and employ the moments of the Lorenz curve to summarize the Lorenz curve.

Now, let X be an income variable with cumulative distribution function $F(\cdot)$ and mean μ. Let $[0,\infty>$ be the domain of F where $F^{-1}(0)\equiv 0$. The Lorenz curve $L(\cdot)$ for F is defined by

$$L(u)=\frac{1}{\mu}\int_0^u F^{-1}(t)\,dt, \quad 0\le u\le 1, \tag{2.1}$$

where F^{-1} is the left inverse of F.

The k^{th} order LC-moment (of the Lorenz curve L) for income distribution F, $D_k(F)$, is defined by

$$\tilde{D}_k(F)=\int_0^1 u^k\,dL(u). \tag{2.2}$$

Note that $\tilde{D}_k$ for each k satisfies the principles of transfers and scale invariance. Thus, the LC-moments $\{\tilde{D}_k : k=1,2,\ldots\}$ constitute a family of inequality measures. As is evident from definition (2.2), the range of $\tilde{D}_k$ varies with k which means that the $\tilde{D}_k$-measures have different range. This drawback can, however, be removed by replacing $\tilde{D}_k$ by the following linear transformation of $\tilde{D}_k$,

$$D_k(F)=\frac{k+1}{k}\tilde{D}_k(F)-\frac{1}{k}. \tag{2.3}$$

Thus, $\{D_k : k=1,2,\ldots\}$ is a family of inequality measures with range [0,1] that is uniquely determined by the LC-moments. Since the Lorenz curve L can be considered as a distribution function defined on a bounded interval it follows (see e.g. Kendall and Stuart, 1958) that L is uniquely determined by its moments. Thus, without loss of generality we can restrict the examination of inequality in F to the family $\{D_k : k=1,2,...\}$ of inequality measures. Moreover, by noting that a probability distribution is uniquely determined by its mean and Lorenz curve this result demonstrates that a probability distribution F defined on $\mathbf{R}_+$ is completely characterized by its mean μ and family $\{D_k : k=1,2,...\}$ of inequality measures, provided that the mean exist. Note that this is not true for the conventional moments. Thus, a probability distribution may be specified by its mean and normalized LC-moments even if some of the conventional moments do not exist. As will be demonstrated below this characterization turns out to be closely related to the characterization provided by Chan (1967) and

Konheim (1971). Their results, which are valid for probability distributions defined on **R**, shows that a distribution with finite mean is characterized by $\{\theta_k : k=1,2,...\}$ where θ_k is defined by

(2.4)
$$\theta_k = \int x \, dF^k(x).$$

Note that $\theta_k = EX_{k:k}$, where $X_{k:k}$ is the largest observation of a random sample of size k drawn from F. Now, inserting for (2.1) and (2.4) in (2.2) it follows immediately that

(2.5)
$$\tilde{D}_k(F) = \frac{\theta_{k+1}}{(k+1)\theta_1},$$

where $\theta_1=\mu$. By equation (2.5), the LC-moment of order k is uniquely determined by θ_{k+1}/θ_1, which means that $\tilde{D}_k$ (and D_k), k=1,2,..., exist even for distributions defined on **R**, provided that the mean is finite. Consequently, we have justified the following theorem,

Theorem 1. Let F be a distribution function with finite mean μ and Lorenz curve L. Then
i) the normalized LC-moments D_k, k=1,2,..., exist,
ii) L is characterized by its normalized moments D_k, k=1,2,...,
iii) F is characterized by its mean μ and normalized LC-moments D_k, k=1,2,... .

Note that the following alternative expression can be used for D_k,

(2.6)
$$D_k(F) = (k+1)\int_0^1 u^{k-1}\left(u - L(u)\right) du.$$

which demonstrates that D_1 is equal to the Gini coefficient and that D_k, for k>1, adds up weighted differences between the Lorenz curve and its egalitarian line. Note that $\{D_k : k = 1,2,...\}$ is a subfamily of a family of inequality measures that was introduced by Mehran (1976).

Now, inserting (2.1) for L in (2.6) and changing the order of integration yield the following alternative expression for D_k,

(2.7)
$$D_k(F) = \frac{1}{\mu}\int_0^\infty F(x)\left(1 - F^k(x)\right) dx.$$

Since D_1, D_2 and D_3 are uniquely determined by the first, second and third order moment of the Lorenz curve, they jointly may make up a fairly good summarization of the Lorenz curve. Note that the integrands in the expressions (2.7) for D_1, D_2 and D_3 are non-negative and convex functions of F which are equal to 0 when F is equal to 0 or 1. While the integrand of the Gini coefficient (D_1) is symmetric and takes its maximum value when F is equal to 0.5, D_2 and D_3 are asymmetric and take their maximum values at the upper part of the income distribution F. This means that G focus on the central part of F whereas D_2 and D_3 focus on the upper part of F, and D_3 more strongly than D_2. The reason why neither of the moments of the Lorenz curve L focus on the lower part of F is related to the fact that L is a convex distribution function, which means that L is always skew to the left. Thus, even though D_1, D_2 and D_3 in many cases jointly provide a sufficiently good description of the Lorenz curve, it would, both for informational and interpretational reasons, be favourable to base the measurement of inequality on three inequality measures that supplement each other with regard to focus on the lower, the central and the upper part of the income distribution.

3. The M-curve and its moments

As suggested in Section 1 it appears particular interesting to reinterprete the information content of the Lorenz curve in terms of ratios of income levels. Thus, a natural point of departure is the conditional mean function defined by

(3.1)
$$E\left[X \mid X \le F^{-1}(u)\right] = \begin{cases} \frac{1}{u}\int_0^u F^{-1}(t)\,dt, & 0<u\le 1 \\ 0, & u=0. \end{cases}$$

It follows immediately from (2.1) and (3.1) that the ratio between the conditional mean function and the overall mean, called the M-curve, forms an alternative representation of the inequality information provided by the Lorenz curve,

(3.2)
$$M(u) = \frac{E\left[X \mid X \le F^{-1}(u)\right]}{EX}, \quad 0 \le u \le 1.$$

Now inserting for (3.1) and (2.1) in (3.2) the following simple relationship between the M-curve and the Lorenz curve emerges,

(3.3)
$$M(u) = \begin{cases} \frac{L(u)}{u}, & 0<u\le 1 \\ 0, & u=0, \end{cases}$$

where $M(1)=1$ and $\lim_{u\to 0}(L(u)/u) = M(0)$.

Figure 1. Lorentz curves for distributions of average annual earnings in Norway

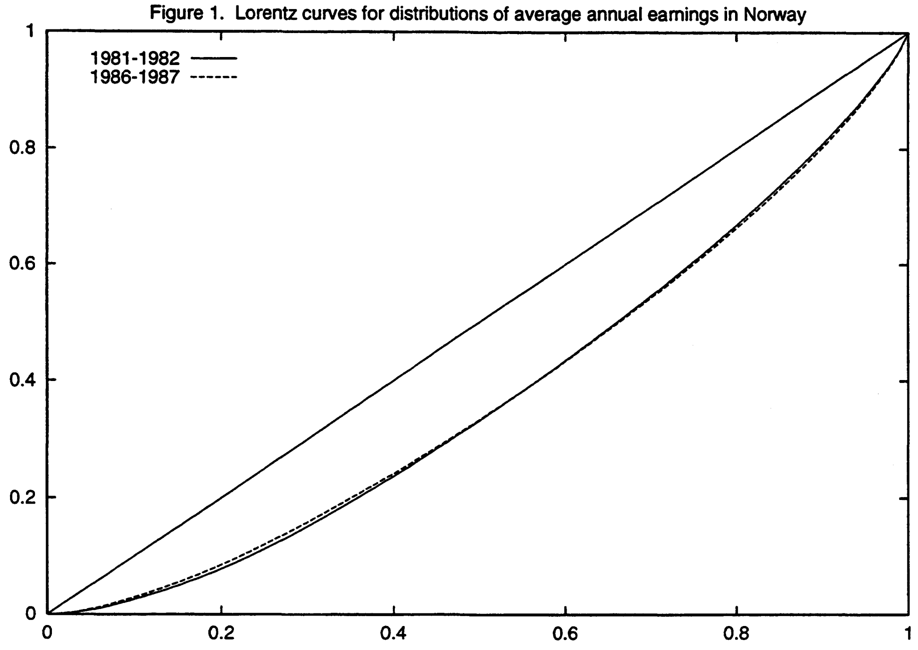
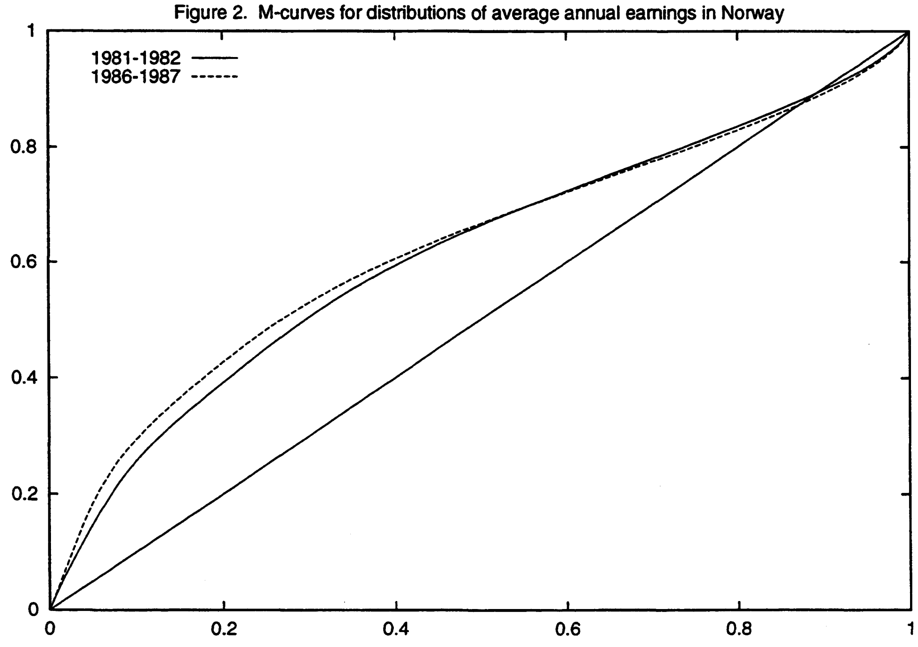


Figure 2. M-curves for distributions of average annual earnings in Norway



The M-curve possesses several attractive properties. First, it provides a convenient alternative interpretation of the information content of the Lorenz curve. For a fixed u, M(u) is the ratio between the mean income of the poorest 100u per cent of the population and the overall mean. Second, the M-curve of a uniform (0,a) distribution proves to be the diagonal line joining the points (0,0) and (1,1). and thus represents a useful reference line. The egalitarian line, coincides with the horizontal line joining the points (0,1) and (1,1). At the other extreme, when one person holds all income, the M-curve coincides with the horizontal axis except for u=1. Third, the family of M-curves is bounded by the unit square. Therefore visually, there is a sharper distinction between two different M-curves than between the two corresponding Lorenz curves. As an illustration Figures 1 and 2 give the M-curves and the Lorenz curves of the distributions[1] of average annual earnings in Norway for the periods 1981-1982 and 1986-1987.

As can be seen from the M-curves there may be differences in inequality between the lower tails of two distribution functions which may appear negligible when comparing the plots of the corresponding Lorenz curves.

Like the Lorenz curve the M-curve satisfies the conditions for being a cumulative distribution function. As opposed to the Lorenz curve, however, the M-curve is not restricted by a convex functional form. On the contrary, the functional form of the M-curve may be concave, convex, concave/convex or convex/concave. In order to demonstrate this fact observe that the first derivative of M is non-negative and that the second derivative of M is given by

(3.4)
$$M''(u) = -\frac{1}{\mu u^3} \int_0^u \frac{t^2 f'\left(F^{-1}(t)\right)}{f'''\left(F^{-1}(t)\right)} dt,$$

provided that $\left[u^2 / f\left(F^{-1}(u)\right)\right] \to 0$ when $u \to 0+$. The expression (3.4) for the second derivative of M demonstrates that there is a close relationship between the shape of the distribution function F and the shape of the M-curve. For example, when F is convex, i.e. strongly skew to the left, then M is concave. By contrast, when F is concave, i.e. strongly skew to the right, then M is convex. Moreover, a symmetric and convex/concave distribution function F implies a concave/convex shape of the corresponding M-curve, whereas a symmetric and concave/convex F implies a convex/concave M-curve. Note that a concave/convex distribution function occurs when there is a tendency of stratification in the population. Now, let us explore how the MC-moments, the moments of the M-curve, reflect this flexibility with regard to functional form.

The k^{th} order MC-moment of M for income distribution F, $C_k(F)$, is defined by

(3.5)
$$C_k(F) = \int_0^1 u^k dM(u).$$

[1] The estimates are based on data of 621 804 persons from Statistics Norway's Tax Assessment Files.

By recalling the properties of M we immediately realize from (3.5) that the MC-moments $\{C_k : k = 1,2,\ldots\}$ constitute a family of inequality measures with range [0,1]. Thus, without loss of generalization we can restrict the examination of the inequality in F to the MC-moments. The following alternative expression of C_k,

$$C_k(F) = k\int_0^1 u^{k-1}(1-M(u))du, \quad k=1,2,\ldots \tag{3.6}$$

demonstrates that C_k for k>1 is adding up weigthed differences between the M-curve and its egalitarian line. The mean of M is defined by

$$A = C_1(F) = \int_0^1 (1-M(u))du. \tag{3.7}$$

Thus, A is equal to the area between the M-curve and its egalitarian line, the horizontal line joining the points (0,1) and (1,1) of Figure 2.

Now, inserting (3.3) into (3.6) for k=2 we find that the second order MC-moment is equal to the Gini coefficient (G), while the third order MC-moment, denoted B, is given by

$$B = C_3(F) = 3\int_0^1 u^2(1-M(u))du. \tag{3.8}$$

Note that $C_{k+1} = D_k$ for k=1,2,..., which means that $B = D_2$.

Inserting (3.2) and (3.1) for M in (3.7) and changing the order of integration yield the following alternative expression for A,

$$A = \frac{1}{\mu}\int_0^\infty F(x)\log(F(x))^{-1}\, dx. \tag{3.9}$$

Like the integrands of G and B (see Section 2) the integrand in (3.9) is a non-negative and convex function of F which is equal to 0 when F is equal to 1 and approaches 0 when F approaches 0. As opposed to G and B, however, the integrand in the expression for A takes its maximum value at the lower part of F. This means that A in general is more concerned with the lower part of the income distribution than G and B. Note that the indicated differences between A, G and B turns out to be a reflection of the measures sensitivity to transfers at different parts of the underlying distribution. The transfer sensitivity properties of these measures will be more closely examined in the following section.

Since A, G and B represent the first, the second and the third order moments of the M-curve, they jointly may represent a fairly good summarization of M. Thus, employed together these three measures should reflect the major aspects of inequality exhibited by the M-curve. Moreover, they prove to supplement each other with regard to focus on the lower, the central and the upper part of the income distribution. Note that these properties are preserved when μA, μG and μB are considered as measures of dispersion of the distribution function F. However, although μA and μB are particularly concerned with the tails of F they turn out to be more robust against outliers in the data than the variance. For symmetric distributions it follows by straightforward calculation that 4B=3G. This relationship between the MC-moments of order two and three suggests

$$\eta = 4\frac{B}{G} - 3 \tag{3.10}$$

as a skewness measure of F. Note that Sillitto (1951) gives an alternative justification of η as a measure of skewness.

The above results demonstrate that the mean μ and the inequality measures A, G and B jointly provide a good summarization of the main features of the distribution function F (and its corresponding M-curve).

4. Normative characterizations

In order to make explicitly use of social welfare functions in deriving measures of inequality Kolm (1969) and Atkinson (1970) noted that one can exploit the parallel with the expected utility theory of choice under uncertainty. Since then, this approach has been considered as preferable to the conventional descriptive/statistical approach. Conventional measures, as the Gini coefficient, have therefore frequently been critizised because they are not explicitly expressed in terms of a social welfare function. The critisism against the Gini coefficient refers to Newberry (1970) who proved that there does not exist a von Neumann-Morgenstern utility function whose expected value gives the same ranking of distributions (with the same mean) as the Gini coefficient. However, Sen (1974) provided an axiomatization of the Gini coefficient and demonstrated that the Gini coefficient could be interpreted in terms of a social welfare function. It seems, however, that Sen's result has not been fully understood until Yaari (1987) proposed a rank-dependent utility theory for choice under uncertainty and noted that this theory implies a rationale for the Gini coefficient. Thus, the Gini coefficient both possesses attractive statistical/geometrical and normative interpretations, while the Atkinson measures suffer from not having a clear statistical/geometrical interpretation.

The welfare-based approach of Kolm (1969) and Atkinson (1970) rely on the additive welfare function

$$W_U(F) = \int U(x)\, dF(x) \tag{4.1}$$

where F is the income distribution function and U is the social decision-maker's utility function. An alternative rationale for (4.1) is to interpret W as the sum of identical individual welfare functions.

When the welfare or inequality comparisons are restricted to distributions with equal means, Atkinson (1970) proved that W_U for strictly concave U always provides rankings in accordance with the rankings given by the criterion of non-intersecting dominating Lorenz curves. In order to extend the inequality comparisons to distributions with different means, Atkinson (1970) proposed to rank distributions according to

$$I = 1 - \frac{x_e}{\mu}, \tag{4.2}$$

where μ is the mean income and x_e is the equally distributed equivalent income defined by

$$U(x_e) = W_u(F). \tag{4.3}$$

Moreover, Atkinson (1970) noted that I is invariant to scale transformations of the distribution F and thus is independent of the level of income if and only if U has a functional form of the Box-Cox type. The advantage of applying I is that inequality can be interpreted as the relative welfare loss or the fraction of total income which could be sacreficed with no loss of social welfare if the rest were equally distributed. Note that Sen (1973) has proposed an analogous definition to (4.2) for more general welfare functions.

The major arguments in favour of the Atkinson approach is that the derived measures of inequality have an explicit normative foundation in contrast to measures which solely have a statistical rationale. Therefore, the Gini coefficient has traditionally been considered as an atheoretic measure of inequality with an obscure normative content, see e.g. Atkinson (1983). However, by considering alternative welfare functions to the one given by (4.1), Yaari (1987, 1988) demonstrated that the Gini coefficient has an explicit and convincing normative foundation. This interpretation emerged during the development of Yaari's so-called dual theory of choice under risk as an alternative to expected utility theory. Based on the dual theory the following social welfare function is derived

$$\tilde{W}_P(F) = \int x \, dP(F(x)), \tag{4.4}$$

where P is a distribution function, representing the preferences of the social decision-maker. The preference function P assigns weights to the incomes of the individuals in accordance with their positions in the income distribution. Therefore, the functional form of P reveals the social decision-maker's attitude towards inequality. Drawing on results from the theory of choice under uncertainty Atkinson (1970) defined inequality aversion as equivalent to risk aversion. This was motivated by the fact that the principle of transfers is identical to the principle of mean preserving spread introduced by Rotschield and Stiglitz (1970). Note, however, that the application of these two principles requires that the distributions in question have equal means. In this respect Yaari (1988) proved that $\tilde{W}_P$ exhibits inequality aversion if and only if P(t) is a strictly concave function in t. This means that a social decision-maker with a strictly concave preference function supports the principle of transfers. Note that $\tilde{W}_P$ is the level of income, if equally distributed, which would give the same social welfare as the distribution in question. Thus, $\tilde{W}_P$ is the equally distributed equivalent income. Accordingly, $\mu - \tilde{W}_P$ is the income which could be sacreficed in order to achieve complete equality.

However, if the social decision-maker is solely concerned about the distributional aspects and not about "the size of the cake" when judging between distribution functions, the ranking criterions given by (4.1) and (4.4) are only useful when "the size of the cake" is fixed. In order to deal with situations with different means the standard approach is to rank distributions according to the corresponding Lorenz curve ranking. As will be demonstrated below Lorenz curve ranking is equivalent to maximizing relative welfare when the welfare function is given by (4.4). Analogously to (4.1) and (4.2), (4.4) forms the basis for the following class of inequality measures,

$$\tilde{J}_P(F) = 1 - \frac{\tilde{W}_P(F)}{\mu}, \tag{4.5}$$

which, by using integration by parts, we can write as

$$\tilde{J}_P(F) = 1 - P'(1) + \int_0^1 P''(t) L(t) \, dt. \tag{4.6}$$

where P' and P'' are the first and second derivatives of P.

Since the Lorenz curve is invariant with respect to scale transformations of the related distribution function it follows readily from (4.6) that $\tilde{J}_P$ satisfies the scale invariance condition irrespective the form of the related preference function. Moreover, $\tilde{J}_P$ supports the criterion of dominating non-intersecting Lorenz curves if and only if the preference function is strictly concave. Thus, ranking distributions with different means according to $\tilde{J}_P$ for strictly concave preference functions means that inequality depends solely on relative incomes.

As criterion for ranking $\tilde{J}_P$ is only unique up to a positive affine transformation, which means that the social preference function P is unique up to the transformation aP(t)+bt. For the social preference function still to be a distribution function, b must be set equal to 1-a. The conclusion, therefore, is that the social preference function is unique up to a mixture with the inequality neutral preference function. For the purpose of comparison, however, it is convenient to agree on a fixed scale for the measurement of inequality. In this respect we follow the standard practice by imposing the unit interval as range for measures of inequality. For $\tilde{J}_P$ to fulfill this requirement it is according to (4.6) necessary to impose the restriction P'(1)=0 on the preference functions. Thus, we will restrict attention to the following class of preference functions,

$$\wp_1 = \{P : P' \text{ and } P'' \text{ are continuous on } [0,1],\ P'(t) > 0 \text{ and } P''(t) < 0 \text{ for } t \in \langle 0,1 \rangle \text{ and } P'(1) = 0\}.$$

Now, let J_P be the class of inequality measures defined by (4.6) where $P \in \wp_1$. Restricting to J_P and hence to the class $\wp_1$ of social preference functions it follows easily from (4.6) that the social decision-makers support the principle of transfers and moreover agree on total income as the cost of inequality for distributions displaying complete inequality.

Consider the preference function

$$P_k(t) = \frac{1}{k-1}\left(kt - t^k\right), \quad k \geq 0, k \neq 1 \tag{4.7}$$

and

$$P_1(t) = \begin{cases} t(1 - \log t) & 0 < t \leq 1 \\ 0, & t = 0 \end{cases} \tag{4.8}$$

which are members of $\wp_1$. We find that

$$\int x\, dP_k(F(x)) = \frac{k}{k-1}\int F^{-1}(t)\left(1-t^{k-1}\right)dt = \mu\left(1-\int u^k\, dM(u)\right) = \mu\left(1-C_k(F)\right),\quad k=2,3,\ldots$$

and

$$\int x\, dP_1(F(x)) = -\int F^{-1}(t)\log t\, dt = \mu\int M(u)\, du = \mu\left(1-C_1(F)\right).$$

Hence

$$C_k(F) = 1 - \frac{\int x\, dP_k(F)}{\mu},\quad k=1,2,\ldots \tag{4.9}$$

Consequently, even though the family $\{C_k\}$ defined by (3.5) has a descriptive/statistical origin the result above shows that it has an explicit normative foundation similarly to the Atkinson measures of inequality. By applying the normatively founded expression (4.9) for the family $C_k(F)$, $k=1,2,\ldots$, the selection of specific measures of inequality can be transferred to the selection of social preference functions. Since $P_k(t) > P_{k+1}(t)$, $0<t<1$ for $k\geq 0$, the preference functions given by (4.8) is lying entirely above each other. Then it follows easily from (4.4) and (4.5) that P_k exhibits more inequality aversion than P_{k+1}. Hence, the inequality measures A, G and B which correspond to P_1, P_2 and P_3, respectively, can be judged according to their inequality aversion properties. As k rises, aversion to inequality declines. The limiting case when $k\to\infty$, corresponds to a social decision-making that tends towards being inequality neutral. At the other extreme as $k=0$, the decision-making criterion (4.4) corresponds to the Rawlsian leximin criterion.

The stated inequality aversion properties of the P_k-functions imply that the moments of the M-curve can be arranged strictly according to the inherent degree of inequality aversion. This means that the A-coefficient (the mean of M) exhibits more inequality aversion than the Gini coefficient (the second order moment of M) and the Gini coefficient exhibits more inequality aversion than the B-coefficient (the third order moment of M).

Now what is the implications of these differences in inequality aversion on the preference functions sensivity to transfers? As is well-known an inequality averse social decision-maker approves transfers from richer to poorer persons, i.e. he supports the principle of transfers. In order to place more emphasis on a transfer between persons with a given income difference if these incomes are lower than if they are higher, Kolm (1976) introduced the more demanding principle of diminishing transfers. Kolm (1976) proved that an inequality averse social decision-maker using the expected utility framework, defined by (4.1) and (4.2), favors the principle of diminishing transfers if and only if $U'''(x)$ is greater than zero for all $x>0$. Accordingly, within this framework an inequality averse social decision-maker with preference function having positive third derivative assigns more weight to transfers taking place lower down in the distribution, irrespective of the form of the distribution in question. However, as demonstrated by Atkinson (1970), this is not the case for the Gini coefficient. The following result, which concerns the transfer sensitivity of the measures of inequality defined by (4.4) and (4.5), can be regarded as a reinterpretation as well as a generalization of the result of Atkinson (1970).

THEOREM 2. Let F be a cumulative distribution function with mean μ and let $P \in \wp$ be a social preference function. Then $\tilde{J}_P$ satisfies the principle of diminishing transfers for F if and only if

$$-\frac{P'''(F(x))}{P''(F(x))} > \frac{F''(x)}{(F'(x))^2}.$$

PROOF. Since $\int x dP(F(x)) = \int x P'(F(x)) dF(x)$, P places more emphasis on an infinitesimal redistribution from a person with income $x+h_1$ to a person with income x than from a person with income $x+h_1+h_2$ to a person with income $x+h_2$ if and only if

$$P'(F(x)) - P'(F(x+h_1)) > P'(F(x+h_2)) - P'(F(x+h_1+h_2)),$$

which for small h_1 is equivalent to

$$-P''(F(x))F'(x) > -P''(F(x+h_2))F'(x+h_2)$$

or after rearranging

$$P''(F(x+h_2)) - P''(F(x)) > -\frac{P''(F(x))}{F'(x+h_2)}\left(F'(x+h_2) - F'(x)\right).$$

For small h_2 this is equivalent to

$$P'''(F(x)) \cdot F'(x) > -\frac{P''(F(x))}{F'(x)} F''(x)$$

and the proof is complete.

Q.E.D.

Theorem 2 shows that the transfer sensitivity properties of the inequality measures defined by (4.6) depend on the form of the social preference function as well as on the form of the income distribution. For a given preference function P the related measure $\tilde{J}_P$ satisfies the principle of diminishing transfers solely for a subclass of income distributions. As will be demonstrated by Corollaries 1 and 2 below the size of this subclass depends on the degree of inequality aversion exhibited by the social preference function. For distribution functions which are not members of the subclass in question, the weighting-profile on transfers will depend on the relative occurence of small, medium-sized and large incomes.

The following result follows immediately from Theorem 2.

COROLLARY 1. An inequality averse social decision-maker who acts according to (4.5) and has preference function with non-negative third derivative always supports the principle of diminishing transfers for all concave distributions of income.

To be more specific let us examine the implications of Theorem 2 for the preference function P_k defined by (4.7) and (4.8). Deriving the expression $P'''(t) / P''(t)$ for P_k gives

(4.10)
$$-\frac{P_k'''(t)}{P_k''(t)} = \frac{2-k}{t}, \quad k>0$$

Note that P_k has positive third derivative for $0 < k < 2$.

Next, by noting that

$$\frac{2-k}{F(x)} > \frac{F''(x)}{(F'(x))^2}, \quad k>0, \quad k \neq 1$$

is equivalent to $F^{k-1}(x)$ being strictly convex for $0<k<1$ and strictly concave for $k>1$, and for $k=1$ that

$$\frac{1}{F(x)} > \frac{F''(x)}{(F'(x))^2}$$

is equivalent to $\log F(x)$ being strictly concave we obtain the following result.

COROLLARY 2. Let F be a cumulative distribution function and let P_k, $k > 0$, be a social preference function defined by (4.7) and (4.8). Then C_k defined by (3.5) and (4.9) satisfies the principle of diminishing transfers for all distribution functions F for which $F^{k-1}(x)$ is strictly convex when $0<k<1$

and strictly concave when k>1. Moreover, C_1 satisfies the principle of diminishing transfers for all distribution functions F for which logF(x) is strictly concave.

As k rises, i.e., decreasing inequality aversion occurs, it follows from Corollary 2 that the class of distribution functions for which P_k is in favor of the principle of diminishing transfers decreases.

Now, by recalling that the preference functions P_1, P_2 and P_3 characterize the inequality measures A, G and B defined in Section 3, we can examine the transfer-sensitivity properties for these three measures by applying Corollary 2. As stated by Corollary 2, P_1 and hence A satisfies the principle of diminishing transfers for all distribution functions F for which logF is strictly concave. This class includes uniform, exponential, Gamma, Laplace, Weibull and Wishart distributions. For log-concave distribution functions there are, as were also noted by Heckman and Honoré (1990) and Caplin and Nalebuff (1991), a rising gap between the income of the richest and the average income of those units with income lower than the richest as we move up the income distribution, i.e. $x - E(Y \mid Y \leq x)$ is an increasing function of x. Observe that if X and Y are distributed according to F (with mean μ) we have

(4.11) $$E\{X - E(Y \mid Y \leq X)\} = \mu - \int x \, dP_1(F(x)) = \mu A.$$

which means that A is equal to the ratio between the mean of these income gaps and the mean income. Consequently, the A-coefficient assigns more weight to transfers taking place lower down in the distribution for all distributions which are strongly skew to the right and even for some distributions which are strongly skew to the left. Distributions which are strongly skew to the left exhibit a minority of "poor" individuals/households and a majority of "rich" individuals/households.

The preference function P_2, which characterizes the Gini coefficient, has the property that $P_2'''(t)$ is equal to zero. It follows from Corollary 2 that P_2, and hence the Gini coefficient, satisfies the principle of diminishing transfers for all strictly concave distribution functions, i.e. distribution functions which are strongly skew to the right. Hence, if the income is uniformly distributed over [0,a] for an abitrary positive a, the Gini coefficient attaches an equal weight to a given transfer irrespective of whether it takes place in the upper, the middle or the lower part of the distribution. By contrast, when the income distribution is strongly skew to the left the Gini coefficient assigns more weight to a given transfer between persons with a given income difference if these incomes are higher than if they are lower. For unimodal distributions which are neither strongly skew to the right nor to the left, the

Gini coefficient assigns more weight to transfers at the central part (around the mode) of the distributions than at the tails. The latter property was also indicated by Atkinson (1970).

Finally, let us examine the transfer sensitivity of the inequality measure B defined by (3.8). As observed above the social preference function of B is P_3. Hence, Corollary 2 implies that B satisfies the principle of diminishing transfers for all distribution functions F for which F^2 is strictly concave. Consequently, B attaches an equal weight to a given transfer irrespective where it occurs in the distribution if and only if the distribution in question is the power distribution $F(x) = (x/a)^{1/2}$, $0 \leq x \leq a$. Note that this specific distribution function is strongly skew to the right and thus is predominated by units with low incomes. When the incomes are uniformly distributed B assigns more weight to transfers at the upper than at the central and lower part of the distribution.

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