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Consumer Demand with Unobservable Product Attributes
Part I: Theory

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Abstract:
This paper develops a new framework for empirical modelling of consumer demand with particular reference to products that are differentiated with respect to quality and location attributes. The point of departure is a flexible representation of the distribution of product attributes and consumer tastes. From this representation and additional behavioral assumptions we derive a structural model for the distribution of the chosen product attributes and the associated quantities. Furthermore, an explicit relationship between the distribution of prices and unit values is obtained.

Keywords: Price distribution, differentiated products, quality attributes, hedonic price indexes.

JEL classification: C25, C43, D11

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1. Introduction

The textbook theory of consumer demand assumes that the products consumed are homogeneous. This is obviously a rather stylized setting since a typical feature is that products are differentiated with respect to quality attributes. Also prices may vary with respect to these attributes as well as with respect to geographical location of the stores. This paper develops a new framework for analyzing consumer demand for differentiated products in the presence of quality- and location attributes when some of these attributes are unobservable to the analyst.

Following Lancaster (1979), p. 16; "the problem of analyzing economic systems in which the goods (or many of them) can be infinitely varied in design and specification has always been that of finding a workable framework of analysis." The traditional way of dealing with quality aspects is either simply to increase the number of goods or to apply Hicks aggregation. Unfortunately, in practice it turns out to be difficult to treat each variant as a separate observable commodity category. This is related to the fact that it is problematic to quantify quality precisely. In other words, quality is typically a latent variable which only to a limited extent can be accounted for by classifying variants of the product under consideration into a large number of categories. Although many variants can in principle be classified in observable categories, there will, in practice, be a limit to how many variants one can treat as separate goods in a demand system. To aggregate goods into composite ones is also problematic. If consumers have heterogeneous preferences the corresponding price indexes will be individual specific and can therefore not readily be implemented in empirical demand analyses.[1]

When unobserved quality attributes are present the econometrician faces a simultaneity problem. The reason for this is that the random terms in the demand system depend on latent quality attributes, which in turn are correlated with prices. It is known (see for example Trajtenberg (1989)) that ignoring this simultaneity problem can lead to upward sloping demand curves. Moreover, the aggregate demand function in the presence of differentiated products may depend on the whole distribution of prices—not only on the mean price across all variants of each product.

[1] It seems that some authors, such as Chamberlin (1933), p. 79, have thought it impossible to carry through a full formal analysis of variable product design: ...."product variations are in this essence quantitative; they cannot, therefore, be measured along an axis and displayed in a single diagram".

Early studies that discuss this problem are the contributions by Houthakker and Prais (1952 and 1955). While most early work was concerned with practical problems of econometrics within a simple theoretical framework, contributions such as Fisher and Shell (1971), Muellbauer (1974) and Lancaster (1979) have dealt with the theoretical foundations. Rosen (1974), Bartik (1987), Epple (1987), Brown and Rosen (1982) and Berry et al. (1995), analyse the econometric problems related to estimating demand and supply of differentiated products. Deaton (1987, 1988) analyses demand in the presence of price variation due to heterogeneity with respect to latent quality aspects and spatial location of stores, with particular reference to developing countries. The reason why spatial variation in prices may occur is because transportation is expensive and it may also be the case that different stores offer different types of services.

The approach taken here differs from the contributions above. First, the choice setting is viewed as a discrete/continuous one where the discrete dimension corresponds to the product variants. Thus, in contrast to the framework which is commonly used (see for example Rosen (1974)) we perceive the consumer as making his choice from a set of discrete "packages" of attribute combinations. Second, the random variables associated with unobserved product attributes and taste-shifters are, similarly to Dagsvik (1994), treated as integral parts of the choice model. The basic idea of the approach is as follows: Consumers face a variety of product variants and locations, each of which is characterized by its price and a variable that may be interpreted as a "quality" index, cf. Lancaster (1977). This quality index may depend on observable product characteristics (attributes). We thus represent a consumer's set of feasible variants/locations by a collection of pairs of prices and quality indexes. Specifically, this set of prices and quality indexes may be summarized in a distribution function which represents the fraction of feasible variants/locations with prices and quality indexes less than or equal to a given level of price and quality. As mentioned above, the consumers are assumed to have preferences over variants/locations which governs their choice of products and the corresponding quantities. To this end a particular discrete/continuous random utility choice model is developed, in which the probability distribution of the prices and quality indexes of the chosen variants/locations is expressed as a function of parameters of the consumers utility functions and the distribution of prices and quality indexes associated with the feasible location/variants. In other words, the fraction of (observational identical) consumers who choose variants/locations with a given price and quality is in this model expressed as a function of the distribution of preferences and offered prices and quality attributes. In the presence of latent quality/location attributes and unobserved heterogeneity in preferences, the distribution of unit values, i.e., expenditure to quantity for each commodity, may differ from the distribution of prices. That is, the number of (latent) variants in the market within a commodity group (observable) with prices below a given level may differ from the corresponding number of variants purchased by the consumers. This is due to a selection effect that arises from consumers having preferences over the variants. By means of the model developed below it is possible to derive a convenient expression for the distribution of unit values as a function of the distribution of prices and parameters related to preferences. Furthermore, one can construct price indexes (which we denote virtual prices) for each (observational) commodity group which account for the possibility that consumers have preferences over variants/locations that are unobservable to the analyst. As a result, it follows that the chosen quantities, within the commodity groups, can be expressed as in a conventional demand system with prices replaced by virtual prices. Although the virtual prices are unobservable random variables it is possible to identify and estimate their probability distribution function. Accordingly, one can identify and estimate parameters of the demand system. In Part II of this paper we discuss estimation issues related to the framework developed in Part I.

Among the contributions mentioned above, the paper by Berry et al. (1995) is the one which is the closer to the present paper. However, in contrast to this paper, Berry et al. only consider the discrete choice case and they also assume that the classification of product variants relevant to the consumer is observable to the analyst.

The paper is organized as follows: In Sections 2 and 3 we present the theoretical model and we derive the distribution function for the demand and for the unit values. In Section 4 we discuss identification in a modified AIDS demand system, cf. Deaton and Muellbauer (1980). The final section is devoted to the case where consumers only buy one unit of a product at a time. In this section we also introduce observable (nonpecuniary) product attributes.

2. A model for individual purchase that accounts for horizontal and vertical product differentiation: The basic assumptions

A major problem the analyst faces when dealing with the demand for differentiated products is that taste-shifters and quality attributes that affect preferences are unobservable. Below we shall introduce a particular framework that enables us to analyze consumer demand under aggregation of quality/location attributes.

Consider a consumer (household) which faces a set of products characterized by quality attributes and price. There are m categories of goods indexed by j, $j=1,2,...,m$. Within each category, let $z=1,2,...$, index an infinite set of stores (location of the stores) and product variants that are offered for sale in the market. For example, the categories may be beer and cereals in which case the corresponding variants are the brands of beer and cereals. Let $Q_j(z)$ be the quantity of observable type j and unobservable location and variant z and let $T_j(z)>0$, be an unobservable quality/location attribute associated with good (j,z), $j\leq m$. The attributes $\{T_j(z)\}$ are objectively measured in the sense that all consumers' perceptions about aggregation and ranking of characteristics embodied in each good are identical. Consistent with Lancaster (1979), p. 27, the T-attributes correspond to the notion of vertical product differentiation. If some characteristics associated with the variants are observable, $\{T_j(z)\}$ can be specified as a function of z-specific observable attributes. In this case T_j will be a (hedonic) quality index. Let $P_j(z)$ be the price of variant/location z of type j. As regards the spatial dimension, a natural "unit" is the store because prices of a given variant do not vary within stores. In general, $P_j(z)$ and $T_j(z)$ may be correlated, cf. Stiglitz (1987). How the distribution of prices and T-attributes is determined in the market will briefly be discussed below. The consumer is assumed to be perfectly informed about the distributions of product locations, variants and prices.

Let

$$(\mathbf{Q}^*,\mathbf{T}^*) \equiv \left(Q_1(1),T_1(1),Q_1(2),T_1(2),...,Q_2(1),T_2(1),Q_2(2),T_2(2),...,Q_m(1),T_m(1),...\right)$$

represent the bundle of quantities and quality attributes of different types and variants. Without loss of generality we may rearrange the components of $(\mathbf{Q}^*,\mathbf{T}^*)$ as

$$(\mathbf{Q},\mathbf{T}) \equiv \underset{z}{\times}\big(Q_1(z),T_1(z),Q_2(z),T_2(z),...,Q_m(z),T_m(z)\big)$$

which is notationally more convenient than $(\mathbf{Q}^*,\mathbf{T}^*)$. The setup above is analogous to the characteristics approach of Lancaster (1966), where $\{T(z)\}$ represents the characteristics dimension. The enumeration in different commodity categories are of course totally indpendent.

Let $U(\mathbf{Q},\mathbf{T})$ be the associated utility of a (particular) consumer. We make the following assumption:

<u>Assumption A1</u>

The utility function has the structure $U(Q,T) = u(S_1,S_2,...,S_m)$, where

$$S_j = \sum_z Q_j(z)T_j(z)\xi_j(z), \tag{2.1}$$

$u(\cdot)$ is a mapping, $u:R_+\rightarrow R_+$, that is increasing and quasi-concave, and $\{\xi_k(z), z=1,2,...\}$ are random positive taste-shifters that account for unobservable variables that reflect heterogeneity in consumer taste.

Note that the interpretation of the taste-shifters, $\{\xi_k(z)\}$, is also consistent with psychological choice theories in which the decision maker is seen as having difficulties with assessing the precise value (to him) of the consumption bundle. Thus, this notion accounts for unobserved variables that affect preferences and are known to the household as well as factors that affect preferences and are random to the household.

The utility structure (2.1) implies that within subgroup j the different qualities are perfect substitutes, cf. Haneman (1984), p. 548. A consequence of (2.1) is thus that the consumer will only buy one quality variant at a time, i.e., for a given set of taste-shifters,

$\{\xi_j(z), z=1,2,..., j=1,...\}$, only one variant will be chosen.[2] Thus this setup is a version of the "Ideal Variety Approach" proposed by Lancaster (1979). Krugman (1989) argues that the Ideal Variety Approach is more realistic than the "Love of Variety Approach" proposed by Spence (1976), and Dixit and Stiglitz (1977). The structure in (2.1) is also analogous to the models investigated by Fisher and Shell (1971), and Gorman (1976). According to Lancaster, op cit. the taste-shifters, $\{\xi_j(z)\}$, correspond to the notion of horizontal product differentiation.

In the special case where the only latent choice variable is associated with location, i.e., z indexes the stores, then the assumption that the consumer only chooses one z at a time seems natural since it may be fair to view the consumer as choosing a single store each time he goes shopping.

Yet another interpretation is possible: We may think of z as an indexation of (latent) "categories", or "baskets", of which basket z, say, consists of "similar" commodities. Examples of baskets are; sports gear and breakfast menus.

Now let us return to the general analysis, where we shall proceed to derive the structure of the demand functions. The budget constraint is given by

$$\sum_{j=1}^{m} \sum_{z} Q_j(z) P_j(z) \le y. \tag{2.2}$$

In order to link the present setup to conventional demand theory we shall now introduce some additional notation that will facilitate the formal analysis. Let

$$R_j(z) = P_j(z) / \big(\xi_j(z) T_j(z)\big). \tag{2.3}$$

If (2.3) is inserted into (2.2) we can express the budget constraint as

[2]The formulation postulated in (2.1) is less restrictive than it appears: We may namely interpret (2.1) as representing the consumer's preferences at a specific moment of purchase — where we allow the set of taste-shifters change each time the consumer goes shopping — or fixed within a short period of time. Of course, this interpretation entails questions about aggregation over time and about the consumers planning horizon. The problem of assessing a realistic planning horizon is also important for the interpretation of data from conventional expenditure surveys since these surveys typically contain data on household expenditure based on short periods of observation (one to three weeks), while a more reasonable notion of "period" in a (static) model is perhaps one year. In Appendix B we outline a possible simple approach for modifying the model so as to allow for savings/borrowing within a year.

$$\sum_{j=1}^{m} \sum_{z} S_j(z) R_j(z) \leq y \tag{2.4}$$

where

$$S_j(z) = Q_j(z) T_j(z) \xi_j(z).$$

Note that maximizing $u(S_1,S_2,...,S_m)$ subject to (2.2) is equivalent to maximizing

$$u\left(\sum_z S_1(z), \sum_z S_2(z), ..., \sum S_m(z)\right)$$

subject to (2.4). Moreover, this optimizing problem is formally equivalent to a conventional consumer demand problem where $S_j(z)$, $z=1,2,...$, are perfect substitutes with corresponding "prices", $\{R_j(z)\}$. Thus, we realize that this implies that the consumer will choose only one (unobservable) quality variant within each observable category. Let $\hat{z}_j$ be the index of the chosen store and variant within category j. Clearly, $\hat{z}_j$ is determined by

$$R_j(\hat{z}_j) = \min_z R_j(z), \tag{2.5}$$

which mean that $\hat{z}_j$ is the variant with the lowest "price". For notational simplicity, let $\hat{R}_j \equiv R_j(\hat{z}_j)$, $\hat{Q}_j = Q_j(\hat{z}_j)$, $\hat{S}_j = S_j(\hat{z}_j)$ and $\hat{P}_j = P_j(\hat{z}_j)$. Let $\overline{x}_j(\mathbf{r},y)$ be the expenditure of type j that follows from maximizing

$$u\left(s_1, s_2, ..., s_m\right) \tag{2.6a}$$

subject to

$$\sum_{j=1}^{m} r_j s_j \leq y, \tag{2.6b}$$

where $\mathbf{r}=(r_1,r_2,...,r_m)$. Evidently, we have that

$$\hat{Q}_j \equiv Q_j(\hat{z}_j) = \frac{\hat{S}_j \hat{R}_j}{\hat{P}_j} = \frac{\overline{x}_j(\hat{\mathbf{R}}, y)}{\hat{P}_j} \tag{2.7}$$

where

$$\hat{\mathbf{R}} = \left(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_m\right).$$

Thus, from (2.7) we realize that formally, we can account for the heterogeneity in quality and prices by replacing prices in an ordinary demand (expenditure) system by the corresponding components of $\hat{\mathbf{R}}$. We shall call $\{\hat{R}_j\}$ <u>virtual prices</u>. The virtual price vector, $\hat{\mathbf{R}}$, is endogenous since by (2.5) it is the result from the consumer's choice of location and quality ($\hat{\mathbf{z}}$). The virtual prices are <u>taste-and-quality-adjusted-prices</u> in the sense that if these virtual prices were known, consumer behavior could be represented by an ordinary demand system that does not depend on quality attributes nor taste-shifters. Note that the virtual prices are <u>unobservable</u>. Note also that $\{\hat{P}_j\}$ represent unit values, i.e., the respective ratios of expenditure to quantity for each commodity, and they therefore depend on the consumer's choice.

To obtain analytic expressions we need to make further assumptions about the distribution of the unobservables. Recall that according to the setup above each variant/location z of type j is characterized by the attribute vector $(P_j(z), T_j(z), \xi_j(z))$ where $(P_j(z), T_j(z))$ are objectively perceived attributes in the sense that they have the same value relative to any consumer, while $\xi_j(z)$, (for fixed (j,z)) may vary across consumers — or over time for a given consumer. Let

$$\wp_j = \left\{\left(P_j(z), T_j(z), \xi_j(z)\right), z = 1, 2, \ldots\right\}$$

denote the (consumer-specific) collection of prices, quality attributes and taste-shifters associated with the feasible variants/stores. To make the setup above operational we need to introduce assumptions about the distribution of the elements in $\wp_j$. The next assumption is a convenient representation of the distribution of taste-shifters and feasible attributes (prices and quality attributes).

<u>Assumption A2</u>

The vectors in $\wp_j$, j=1,2,...,m, are points of independent inhomogeneous Poisson processes on R_+^3 with intensity measure associated with $\wp_j$ equal to

$$G_j(dp,dt) \cdot \mu_j(\varepsilon)d\varepsilon. \tag{2.8}$$

Moreover, the Poisson points associated with different consumers are realizations from independent copies of the Poisson process.

Recall that the Poisson process framework means that the (vector) points in $\wp_j$ are independently distributed and the probability that there is a point in $\wp_j$ for which $P_j(z) \in (p,p+dp)$, $T_j(z) \in (t,t+dt)$ and $\xi_j(z) \in (\varepsilon,\varepsilon+d\varepsilon)$ is equal to $G_j(dp,dt)\mu_j(\varepsilon)d\varepsilon$.

The reason why the Poisson points differ across consumers is because the taste-shifters $\{\xi_j(z)\}$ are individual specific: Different consumers evaluate the variants differently and the tastes of a given consumer may also fluctuate randomly over time. The fact that the value of a store to a given consumer may depend on the distance between him and the store is also captured in this formulation. Note that this formulation allows prices to vary across products with given quality. As mentioned in the introduction, a rationale for this is that prices may vary across stores because different stores/producers have different cost functions due to transportation and the quality of the services offered by the stores.

In Dagsvik (1994) it is demonstrated that $G_j(p,t)$ can be interpreted as the distribution of feasible prices and quality attributes. This means that $G_j(p,t)$ is the fraction of all variants of type j in the market that have prices and quality attributes less than or equal to (p,t). The multiplicative structure of (2.8) means that the taste-shifters are independent of the prices, location and quality attributes. A justification for assuming the prices and quality attributes be independent of the taste-shifters stems from the view that the market forces operate on an aggregate level in the sense that the respective distributions of supply and demand are dependent. This means that the intensity measure (2.8) has a functional form that depends both on the systematic parts of the consumers' utility functions and the producers' profit functions. However, the distribution of supply and demand may not necessarily coincide because the firms may have limited information about the distribution of demands when (and if) they set prices. To perform policy analyses (in a rigorous sense) requires a structural specification of the price distribution as a function of parameters that characterize both consumers utility functions and the producers' profit functions.

An interesting point of departure for developing a structural version of the distribution $G_j(p,t)$ is the approach discussed in Anderson et al. (1992), ch. 6 and 7. The typical argument goes as follows: Each firm produces a single variant of a differentiated product. Firms are uncertain about consumer demand in the market. It is assumed that the firms know the probability distribution of the aggregate demand for each variant within each commodity group. From this distribution they can calculate the expected profit conditional on the T-attributes and the prices. Each firm maximizes the expected profit function with respect to own price and T-attributes, taking the prices and quality attributes of the variants produced by other firms as given. Anderson et al. demonstrate that a price equilibrium exists under general assumptions. It is, however, beyond the scope of the present paper to discuss the existence and the structure of G_j.

The assumptions about the Poisson process above are rather weak. Specifically, the independence between Poisson points means that there is a random device that influence the distribution of points — say in each period — but that on average the distribution of points is precisely determined by the intensity measure (2.8).

Assumption A2 is, however, too general to produce useful apriori restrictions. The next assumption regards the functional form of $\mu_j(\varepsilon)$ and the problem of its theoretical justification.

Assumption A3

The structure of μ_j is given by

$$\mu_j(\varepsilon) = \varepsilon^{-\alpha_j-1} c_j \tag{2.9}$$

where $c_j>0$ and $\alpha_j>0$ are constants. Moreover, the Poisson processes ρ_j, $j=1,2,\ldots,m$, are independent.

The structure of (2.9) can be justified as follows: Consider the choice of variant/location within category j. This choice $\hat{z}_j$ is determined by

$$\hat{z}_j = \operatorname{argmin} R_j(z) = \operatorname{argmax}\left\{\xi_j(z)\cdot v\left(T_j(z),P_j(z)\right)\right\} \tag{2.10}$$

where $v(x,y) = x/y$. But (2.10) shows that this maximization problem corresponds to a version of the pure choice-of-attribute model in Dagsvik (1994), p. 1196. In particular, Dagsvik demonstrates that (2.9) follows from a particular version of the "Independence from Irrelevant

Alternatives" assumption, (IIA), (cf. Dagsvik (1994), Theorem 2 and Remark 1, p. 1185).

As discussed in Dagsvik (1994), (2.9) implies that there are (with probability one) a finite number of points of the Poisson process for which the taste-shifters are bounded from below. We realize from (2.9) that the intensity measure tends towards infinity when $\varepsilon \rightarrow 0$. We may interpret this as follows: Although any combination of price and quality attributes that are generated from the $G_j(\cdot)$ are feasible, most of the corresponding variants/stores (in fact infinitely many) are not perceived as "interesting" to the consumer due to the low values of the associated taste-shifters. Only a finite number of the variants/stores is therefore taken into account in the consumer's decision process.[3]

It can easily be demonstrated that (2.9) implies that the dispersion of $\{\xi_j(z)\}$ increases when α_j decreases. On the other hand, when $\alpha_j \rightarrow \infty$, then $\{\xi_j(z)\}$ converges towards one in probability.

3. Aggregate relations

In this section we shall discuss the implications from the general setting introduced above with particular reference to the distributions of the quality attributes, prices and the virtual prices associated with the chosen variants. Recall that the prices associated with the chosen variants are, in the present setting, equivalent to unit values.

Let $\hat{G}_j(p,t)$ be the c.d.f. of $(\hat{P}_j,\hat{T}_j)$, given that a variant of type j is demanded, and let $\hat{g}_j(p,t)$ be the corresponding density. In other words, $\hat{G}_j(p,t)$ is the probability that an agent shall make a choice such that $(\hat{P}_j \leq p,\ \hat{T}_j \leq t)$ given that he purchases a positive quantity of commodity, type j.

Theorem 1

Under assumptions A1 to A3, the virtual price, $\hat{R}_j$, for any j, is stochastically independent of the set $\{(\hat{P}_k,\hat{T}_k,\ k=1,2,...,m\}$. Furthermore, $\hat{R}_1,\hat{R}_2,...\hat{R}_m$, are independent with c.d.f.

[3] A more rigorous argument can be given. By Lemma 1 in Appendix A it follows that for any open set A there is, with probability one, a Poisson point z in $\wp_j$ such that $(P_j(z),T_j(z)) \in A$. Esentially, this means that any price and quality attribute drawn from $G_j(\cdot)$ is feasible.

$$P\left(\hat{R}_j \leq r\right) = 1 - exp\left(-r^{\alpha_j} K_j\right) \tag{3.1}$$

for $r \geq 0$, where

$$K_j \equiv c_j \iint_{R_+^2} \left(\frac{y}{x}\right)^{\alpha_j} G_j(dx, dy) \equiv c_j E\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j}. \tag{3.2}$$

The proof of Theorem 1 is given in Appendix A.

The constant c_j can be viewed as a "normalizing" constant. Since $\hat{R}_j$ is not observable one can therefore fix c_j at any value that is convenient — as we shall do below.

In Theorem 1 it is stated that the virtual prices of the chosen variants are stochastically independent of the corresponding unit values of the price and quality attributes. At first glanse this seems counterintuitive. However, the important fact here is that the distribution of the virtual prices depends on the distribution of the offered prices and quality attributes, since K_j depends on $G_j(\cdot)$. That is, due to the "noisy" structure of the preferences and the choice environment, which follows from the Poisson process setting, the virtual prices and offered prices are mutually independent across purchases, while the corresponding aggregates are functionally dependent. Thus, this property corresponds to the notion of bounded rationality where rational behavior is only assumed to hold on an aggregate level while the choices from one moment to the next may be erratic and inconsistent.

A c.d.f. with the structure given in (3.1) is called a Weibull distribution, cf. Johnson and Kotz (1972).

Corollary 1

Under the assumptions of Theorem 1 it follows that

$$E\hat{R}_j = K_j^{-1/\alpha_j} \Gamma\left(1 + \frac{1}{\alpha_j}\right), \tag{3.3}$$

$$E\left(\hat{R}_j^2\right) = K_j^{-2/\alpha_j}\Gamma\left(1+\frac{2}{\alpha_j}\right), \tag{3.4}$$

$$E\log\hat{R}_j = -\frac{\gamma}{\alpha_j} - \frac{1}{\alpha_j}\log K_j \tag{3.5}$$

and

$$E\left(\log\hat{R}_j\right)^2 = \left(E\log\hat{R}_j\right)^2 + \frac{\pi^2}{6\alpha_j^2}, \tag{3.6}$$

where γ is Euler's constant; $\gamma \approx 0.5772...$.

Proof:

Eq. (3.3) and (3.4) follow readily from the definition of the Gamma function. Eq. (3.5) and (3.6) follow from the fact that (3.1) implies that, $-\log\hat{R}_j$ is type III extreme value distributed.[4]

Q.E.D.

From (3.3) and (3.4) it follows that α_j satisfies

$$\frac{\Gamma\left(1+\frac{2}{\alpha_j}\right)}{\Gamma\left(1+\frac{1}{\alpha_j}\right)^2} - 1 = \frac{\mathrm{Var}\hat{R}_j}{\left(E\hat{R}_j\right)^2}. \tag{3.7}$$

where we recocognize the right hand side of (3.7) as the coefficient of variation. It can be demonstrated that the left hand side of (3.7) is (strictly) decreasing in α_j and accordingly, large α_j corresponds to a small coefficient of variation in the distribution of $\hat{R}_j$.

[4] There is some confusion in the literature: In the terminology of Johnson and Kotz (1972), (3.1) is a type III extreme value distribution, while it is denoted type I by other authors.

Theorem 2

Suppose A1 to A3 hold. Then

$$\hat{G}_j(dp,dt) = \frac{\left(\frac{t}{p}\right)^{\alpha_j} G_j(dp,dt)}{\iint_{R_+^2} \left(\frac{y}{x}\right)^{\alpha_j} G_j(dx,dy)}. \tag{3.8}$$

The proof of Theorem 2 follows directly from Dagsvik (1994), Theorem 7.

The result of Theorem 2 shows that, in general, the distribution of the unit values may differ from the price distribution of the variants that are offered for sale in the stores. This is so due to the fact that the random taste-shifters, $\{\xi_j(z)\}$, induce a selection effect. When α_j decreases this selection effect eventually becomes negligible. The intuition is that when the dispersion of $\{\xi_j(z)\}$ is large the choice of variant and location will be completely random because in a distributional sense,

$$Q_j(z)\,T_j(z)\,\xi_j(z)^{\alpha_j} \sim \xi_j(z)^{\alpha_j}$$

so that $\hat{z}_j$ is determined from the maximization of $\xi_j(z)^{\alpha_j}$. In other words, the consumer has in this case no systematic preference for any attribute and consequently no systematic selection effect will take place since the choice of variant is completely random.

Corollary 2

If and only if prices are determined such that

$$T_j(z) = b_j P_j(z), \tag{3.9}$$

(with probability one) where $b_j>0$, $j=1,2,...,m$, are constants, then $\hat{G}_j(p,t) = G_j(p,t)$, which means that supply equals demand. In this case (3.2) reduces to

$$K_j = c_j b_j^{-\alpha_j}. \tag{3.10}$$

Proof:

Assumption (3.9) means that $G_j(dp|t)=1$, when $t/b_j \in (p,p+dp)$ and zero otherwise, where $G_j(p|t)$ is the conditional distribution of prices in category j given that $T_j(z)=t$. From Theorem 2 and Theorem 1 the result of Corollary 2 follows.

Q.E.D.

If both prices, T_j-attributes as well as unit values and chosen attributes, $\{(\hat{P}_j,\hat{T}_j)\}$, were observed then (3.8) could be utilized to estimate α_j without additional assumptions about $g_j(\cdot)$. Moreover, one could test the utility specification indirectly by testing the particular functional form of (3.8) on the basis of non-parametric estimates of $G_j(\cdot)$ and $\hat{G}_j(\cdot)$. Unfortunately, the quality attributes are rarely observable and accordingly (3.8) is not readily applicable for empirical analyses. To this end the next result is useful.

Corollary 3

Let

$$\lambda_j(p) \equiv E\left(T_j(z)^{\alpha_j} | P_j(z) = p\right), \tag{3.11}$$

and assume that the density, $g_j(p)$, of the marginal price distribution exists. Eq. (3.8) implies that the marginal density of unit values can be expressed as

$$\hat{g}_j(p) = \frac{p^{-\alpha_j}\lambda_j(p) g_j(p)}{\int_{R_+} x^{-\alpha_j}\lambda_j(x) g_j(x) dx}. \tag{3.12}$$

Moreover,

$$K_j = c_j \int_{R_+} x^{-\alpha_j} \lambda_j(x) g_j(x) dx. \tag{3.13}$$

The result of Corollary 3 demonstrates that when $\{T_j(z)\}$ is unobservable then at most $p^{-\alpha_j}\lambda_j(p)$ can be identified from the relationship between the respective distribution of prices and unit values.

<u>Proof:</u>

By (3.8) we can express $\hat{g}_j(p)$ as

$$\hat{g}_j(p) = \frac{p^{-\alpha_j} g_j(p) E\left(T_j(z)^{\alpha_j} | P_j(z) = p\right)}{\int_{R_+} x^{-\alpha_j} g_j(x) E\left(T_j(z)^{\alpha_j} | P_j(z) = x\right) dx}.$$

Eq. (3.12) now follows immediately and (3.13) follows from (3.2).

Q.E.D.

The function $\lambda_j(\cdot)$ is in fact a (preference-adjusted) <u>conditional aggregate quality index</u>. Specifically, $\lambda_j(p)$ expresses the conditional mean value across variants of the quality/location attributes given price level p. It is adjusted for heterogeneity in tastes through the parameter α_j. In general it is desirable to link $\lambda_j(p)$ to observable attributes of the product variants. For simplicity, we shall defer the introduction of observable nonpecuniary attributes till Section 6. From the discussion there it will become evident that the approach in Section 6 also applies to the present case with divisible products.

From Corollary 3 the next result is immediate.

Corollary 4

If

$$E\left(T_j(z)^{\alpha_j} | P_j(z) = p\right) = w_j p^{\alpha_j}, \tag{3.14}$$

where $w_j>0$ is a constant, then by (3.12) the distribution of unit values equals the distribution of prices.

Note, however, that $\hat{g}_j(p) = g_j(p)$ does not necessarily mean that the equilibirum condition, $\hat{g}_j(p,t) = g_j(p,t)$ holds, cf. Corollary 2.

Corollary 5

The parameter K_j of the virtual price distribution can be expressed as

$$K_j \equiv c_j E\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j} = c_j E\left(P_j(z)^{-\alpha_j} \lambda_j(P_j(z))\right) = \frac{c_j E\left(T_j(z)^{\alpha_j}\right)}{E\left(\hat{P}_j^{\alpha_j}\right)} = c_j / E\left(\frac{\hat{P}_j^{\alpha_j}}{\lambda_j(\hat{P}_j)}\right). \tag{3.15}$$

Moreover,

$$g_j(p) = \frac{p^{\alpha_j} \dfrac{\hat{g}_j(p)}{\lambda_j(p)}}{\displaystyle\int_{R_+} \frac{x^{\alpha_j} \hat{g}_j(x) dx}{\lambda_j(x)}}. \tag{3.16}$$

Proof:

The first and second equality of (3.15) follow from (3.2) and (3.13). From (3.12) we get

$$\frac{K_j \hat{g}_j(p) p^{\alpha_j}}{\lambda_j(p)} = g_j(p) c_j \tag{3.17}$$

which implies that

$$K_j \int_{R_+} x^{\alpha_j} \hat{g}_j(x)\,dx \equiv K_j E\left(\hat{P}_j^{\alpha_j}\right) = c_j \int_{R_+} \lambda_j(x)\, g_j(x)\,dx \equiv c_j E\left(\lambda_j(P_j(z))\right) = c_j E\left(T_j(z)^{\alpha_j}\right) \tag{3.18}$$

and

$$K_j \int_{R_+} \frac{x^{\alpha_j} \hat{g}_j(x)\,dx}{\lambda_j(x)} \equiv K_j E\left(\frac{\hat{P}_j^{\alpha_j}}{\lambda_j\left(\hat{P}_j\right)}\right) = c_j. \tag{3.19}$$

and thus the last equality in (3.15) has been proved. Also (3.16) follows from (3.17) and (3.18).

Q.E.D.

Corollary 5 demonstrates that once α_j and $\lambda_j(\cdot)$ have been estimated it is sufficient to have observations on unit values to obtain an estimate of K_j.

Although it is not of primary focus in this paper the results concerning the c.d.f. of virtual prices can be applied to obtain price indexes. From Theorem 1, Corollary 2 and Corollary 5 it follows that K_j^{-1/α_j} can be interpreted as a price index. By (3.15) we can express this price index as

$$K_j^{-1/\alpha_j} = \left(c_j E\left(P_j(z)^{-\alpha_j} \lambda_j(P_j(z))\right)\right)^{-1/\alpha_j} \equiv \left(c_j \int_{R_+} x^{-\alpha_j} \lambda_j(x)\, g_j(x)\,dx\right)^{-1/\alpha_j} \tag{3.20}$$

where c_j can be chosen so as to adjust the level of the index to be equal to the corresponding price level in a reference year. In contrast to the conventional price indexes, (3.20) is in fact a price index functional, since is depends on the whole probability distribution of prices.

A fundamental question is to which extent we can identify the distribution of the virtual prices and the corresponding demand system.

Corollary 6

Assume that $\hat{g}_j(p)$ and $g_j(p)$ are known. Under the assumptions of Theorem 1 it follows that $p^{-\alpha_j}\lambda_j(p)$ is non-parametrically identified apart from a multiplicative constant.

Proof:

From (3.12) we get

$$p^{-\alpha_j}\lambda_j(p) = \frac{\lambda_j(1)\,\hat{g}_j(p)\,g_j(1)}{g_j(p)\,\hat{g}_j(1)}$$

which yields the above results.

Q.E.D.

We found above that $\lambda_j(p)p^{-\alpha_j}$ is only identified up to a multiplicative constant. Unless we impose additional structure on $\lambda_j(p)$ or on the corresponding utility function introduced in A1 we cannot pursue this matter further. We shall next introduce an additional assumption and examine the implications thereof.

Assumption A4

For each commodity group j,

$$\lambda_j(p) \equiv E\left(T_j(z)^{\alpha_j} \middle| P_j(z) = p\right) = \frac{p^{\alpha_j\kappa_j} E\left(T_j(z)^{\alpha_j}\right)}{E\left(P_j(z)^{\alpha_j\kappa_j}\right)} \tag{3.21}$$

where κ_j is a constant (possible time dependent).

Before we discuss the interpretation of A4 we state an immediate result.

Corollary 7

Under A4, (3.12) and (3.13) reduce to

$$\hat{g}_j(p) = \frac{p^{\alpha_j\kappa_j-\alpha_j} g_j(p)}{\int_{R_+} x^{\alpha_j\kappa_j-\alpha_j} g_j(x)\,dx}, \tag{3.22}$$

and

$$K_j = c_j \frac{E\left(T_j(z)^{\alpha_j}\right) E\left(P_j(z)^{\alpha_j\kappa_j-\alpha_j}\right)}{E\left(P_j(z)^{\alpha_j\kappa_j}\right)}. \tag{3.23}$$

Under Assumption A4 it follows that

$$\frac{\partial^2 E\left(T_j(z)^{\alpha_j} \mid P_j(z)^{\alpha_j}=y\right)}{\partial y^2} = \frac{\kappa_j(\kappa_j-1)\, y^{\kappa_j-2} E\left(T_j(z)^{\alpha_j}\right)}{E\left(P_j(z)^{\alpha_j\kappa_j}\right)}.$$

Consequently, under A4 the function $\lambda_j\left(y^{1/\alpha_j}\right)$ is convex when $\kappa_j>1$ and concave when $\kappa_j<1$. The interpretation is that when $\kappa_j<1$, then the dependence between $T_j(z)^{\alpha_j}$ and $P_j(z)^{\alpha_j}$ is weakened when the price level increases, while when $\kappa_j>1$ this dependence is strenghtened when the price level increases. The latter case means that price is perceived as an increasingly more "reliable" proxy for quality as the price level increases. Without real loss of generality, suppose now that the probability mass of prices less than one is negligible. From (3.22) we realize that $\kappa_j<1$ implies that $\hat{g}_j(p)$ is more skew to the left than $g_j(p)$, while the opposite is true when $\kappa_j>1$. The reason is of course that when $\kappa_j>1$, increasing prices do not reduce the attractiveness of the product variants as much as when $\kappa_j\leq 1$, because high prices are perceived as a strong indication of high quality, and vice versa.

As regards identification, Assumption A4 is not sufficient to fully resolve the issue unless we are willing to make assumptions about how $E\left(T_j(z)^{\alpha_j}\right)$, j=1,2,...,m, changes over time (cf. (3.23)).

4. A special case: AIDS demand system

Let $w_{j\tau}$ denote the budget share of type j for a particular consumer in period τ and let $\{\hat{R}_{j\tau}\}$, $\{P_{j\tau}(z)\}$ and $\{\hat{P}_{j\tau}\}$ be the corresponding virtual prices, prices and unit values. Now assume that the appropriate demand system is an AIDS model given by

$$w_{j\tau} = h_j + \sum_k \delta_{jk} \log \hat{R}_{k\tau} + \beta_j \log(y_\tau/q_\tau) \tag{4.1}$$

and

$$\log q_\tau = h_0 + \sum_k h_k \log \hat{R}_{k\tau} + \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} \delta_{jk} \log \hat{R}_{j\tau} \log \hat{R}_{k\tau} \tag{4.2}$$

where y_τ is total expenditure for the consumer in period τ, $\{\beta_j\}$, $\{h_k\}$ and $\{\delta_{ik}\}$ are unknown parameters which satisfy

$$\sum_j h_j = 1, \quad \delta_{jk} = \delta_{kj},$$

and

$$\sum_k \delta_{jk} = \sum_j \delta_{jk} = \sum_j \beta_j = 0,$$

(cf. Deaton and Muellbauer, 1980). For simplicity, we rule out the possibility of corner solutions. By Corollary 1 we have

$$E w_{j\tau} = h_j - \sum_{k=1}^{m} \delta_{jk} \left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k} \log K_{k\tau} \right) + \beta_j E \log y_{i\tau} - \beta_j E \log q_{i\tau} \tag{4.3}$$

where

$$E \log q_\tau = h_0 - \sum_{k=1}^{m} h_k \left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k} \log K_{k\tau} \right) + \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} \delta_{jk} \left(\frac{\gamma}{\alpha_j} + \frac{1}{\alpha_j} \log K_{j\tau} \right)\left(\frac{\gamma}{\alpha_k} + \frac{1}{\alpha_k} \log K_{k\tau} \right) \tag{4.4}$$

and γ=0.5772..., is Euler's constant. Provided one is willing to assume that $E\left(T_{j\tau}(z)^{\alpha_j}\right)$ is constant over time, there is no loss of generality in assuming that

$$c_j E\left(T_{j\tau}(z)^{\alpha_j}\right) = 1 \tag{4.5}$$

which by (3.15) imply that

$$\log K_{j\tau} = -\log E\left(\hat{P}_{j\tau}^{\alpha_j}\right). \tag{4.6}$$

The identification and estimation of this — and the linear expenditure system will be discussed in Part II of this paper.

The analysis in this section can, as mentioned above, easily be extended to the case in which nonpecuniary attributes are present. We refer to the next section for details about this extension.

5. Discrete choice

In the context of qualitative choice — such as the demand for durables, choice among jobs and schooling decisions — the set of feasible alternatives is typically discrete. Berry et al. (1995) have developed a discrete choice technique to analyze demand with latent quality attributes. Their approach differs from the one described below in that Berry et al. assume that the classification of product variants relevant to the consumers is observable. In addition, their assumptions about the stochastic elements of the model differ from the assumptions invoked here.

In this section we shall modify the analysis above so as to apply in the discrete choice setting in which the consumer only purchases one unit of a product variant at a time. Thus the vector of quantities, **Q**, has components that are either zero or one.

Specifically, we now assume the following:

Assumption A5

The utility function has the structure

$$U(\boldsymbol{Q},\boldsymbol{T}) = u\left(\sum_{j=1}^{m}\sum_{z} Q_j(z)T_j(z)\xi_j(z)\right) \tag{5.1}$$

where $Q_j(z) \in \{0,1\}$, *for all j and z.*

In contrast to Assumption A1, A5 implies that apart from the quality attributes and taste-shifters, the goods enter symmetrically in the model.

Assumption A5 means that there is no difference between product types and product variants within types, as regards the structure of the preferences. Below we shall, however, distinguish between "types" and "variants" by letting product heterogeneity be larger between types than within types.

The implication from (5.1) is that the consumer will choose a single variant within a single commodity group. Similarly to the analysis of Section 2 it follows that

$$Q_j(\hat{z}_j) = 1 \Leftrightarrow \hat{R}_j = \min_{k,r} R_k(r) = \min_k \hat{R}_k. \tag{5.2}$$

Let H_j denote the choice probability defined by

$$H_j \equiv P\left(\hat{R}_j = \min_k \hat{R}_k\right). \tag{5.3}$$

Theorem 3

Under assumptions A2, A3 and A5 the probability that a consumer shall purchase a product variant of type j is given by

$$H_j = \frac{K_j}{\sum_{r=1}^{m} K_r}. \tag{5.4}$$

Proof:

Let $V_j \equiv -\alpha_j \log\hat{R}_j$. From (3.1) it follows that V_j has c.d.f.

$$P\left(V_j \le v\right) = P\left(\hat{R}_j > e^{-v/\alpha_j}\right) = \exp\left(-e^{-v} K_j\right). \tag{5.5}$$

But this means that $V_j = \log K_j + \eta_j$, where $\eta_1, \eta_2, ..., \eta_m$, are independent with extreme value c.d.f., $\exp(-e^{-y})$. But then (5.4) follows immediately from a familiar result in discrete choice theory (cf. Ben-Akiva and Lerman, 1985), because minimization of $\hat{R}_j$ is equivalent to maximization of V_j.

Q.E.D.

In many fields of discrete choice, such as choice of housing, residential location, tourist destinations, etc., the consumer faces many product variants of each type, and it is often the case that in addition to prices, nonpecuniary attributes associated with the chosen variants are observable to the analyst. We shall now extend the framework developed above to take into account observable attributes that characterize the variants. Let $X_j(z)$ denote the observable nonpecuniary attribute (possible vector-valued) associated with variant z of type j.

To focus on the potential for empirical applications we shall express the choice probabilities under additional assumptions about the distribution of $\{T_j(z)\}$.

Assumption A6

The vectors in $\tilde{\wp}_j \equiv \{(P_j(z), T_j(z), X_j(z), \xi_j(z)),\ z=1,2,...\},\ j=1,2,...,m,$ *are points of independent inhomogeneous Poisson processes on* $R_+^3 \times R \times R_+$ *with intensity measure associated with* $\wp_j$ *equal to*

$$G_j(dp, dt, dx)\, \varepsilon^{-\alpha_j - 1} c_j d\varepsilon, \tag{5.6}$$

where $\alpha_j > 0,\ c_j > 0$.

Clearly, A6 is an immediate extension of A2 and A3.

Assumption A7

For each commodity group j,

$$E\left(T_j(z)^{\alpha_j} | X_j(z) = x\right) = e^{x\theta_j \alpha_j} \tag{5.7}$$

where θ_j is a parameter (vector).

Assumption A7 implies that we can predict the mean value of $T_j(z)^{\alpha_j}$ if we know the distribution of $\{X_j(z)\}$.

Theorem 4

Under assumptions A5 to A7,

$$K_j = c_j / E\left(\hat{P}_j^{\alpha_j} exp\left(-\hat{X}_j \theta_j \alpha_j\right)\right) \tag{5.8}$$

where $\hat{X}_j$ denotes the chosen X-attribute.

Proof:

Similarly to Theorem 2 it follows that

$$\hat{G}_j(dp, dt, dx) = \frac{\left(\frac{t}{p}\right)^{\alpha_j} G_j(dp, dt, dx)}{E\left(\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j}\right)} = \frac{c_j \left(\frac{t}{p}\right)^{\alpha_j} G_j(dp, dt, dx)}{K_j}, \tag{5.9}$$

where the last equality follows from Corollary 5. From (5.9) we obtain that

$$p^{\alpha_j}\hat{G}_j(dp,dt,dx) = \frac{c_j t^{\alpha_j} G_j(dp,dt,dx)}{K_j} \tag{5.10}$$

which together with A7 implies that

$$E\left(\hat{P}_j^{\alpha_j}\exp\left(-\hat{X}_j\theta_j\alpha_j\right)\right) = EE\left(\hat{P}_j^{\alpha_j}\exp\left(-\hat{X}_j\theta_j\alpha_j\right)|\hat{X}_j\right)$$

$$= E\left(\exp\left(-\hat{X}_j\theta_j\alpha_j\right)E\left(\hat{P}_j^{\alpha_j}|\hat{X}_j\right)\right) = \frac{c_j E\left(\exp\left(-X_j(z)\theta_j\alpha_j\right)E\left(T_j(z)^{\alpha_j}|X_j(z)\right)\right)}{K_j} = \frac{c_j}{K_j}.$$

Q.E.D.

Theorems 3 and 4 suggest that it may be possible to estimate $\{\alpha_j\}$ and $\{\theta_j\}$ solely from observations on unit values and chosen X-attributes without imposing restrictions on the conditional moment, $E\left(T_j(z)^{\alpha_j}|P_j(z),X_j(z)\right)$. However, for the purpose of analyzing the effect from a change in the distribution of prices and nonpecuniary attributes, it is necessary also to make assumptions about the conditional moment. A direct extension from Assumption A4 would be to make the following assumption:

<u>Assumption A8</u>

For each commodity group j,

$$E\left(T_j(z)^{\alpha_j}|P_j(z)=p,X_j(z)=x\right) = \frac{p^{\alpha_j\kappa_j}e^{x\theta_j\alpha_j}}{E\left(P_j(z)^{\alpha_j\kappa_j}\right)} \tag{5.11}$$

where θ_j and κ_j are parameters.

From Assumption A8 it follows that we can express K_j as

$$K_j = \frac{E\left(P_j(z)^{\alpha_j\kappa_j - \alpha_j}\exp\left(X_j(z)\theta_j\alpha_j\right)\right)c_j}{E\left(P_j(z)^{\alpha_j\kappa_j}\right)}. \tag{5.12}$$

Let us finally consider the relevance of the present setup for the theory and estimation of hedonic price indexes. Recall from the discussion following Theorem 1 that within the Poisson process framework proposed in this paper, dependence between virtual prices and attributes and prices must be understood in a "distributional sense". Consequently, we shall interpret the hedonic price index as the mean virtual price when the set of feasible attributes and prices are generated by the conditional distributions, $G_j(p,t|x)$, given x, j=1,2,...,m. The corresponding virtual price distributions (cf. Theorem 1) thus take the form

$$P\left(\hat{R}_j(x)\le r\right) = 1 - \exp\left(-r^{\alpha_j}K_j(x)\right) \tag{5.13}$$

where

$$K_j(x) \equiv c_j E\left(\left(\frac{T_j(z)}{P_j(z)}\right)^{\alpha_j} \middle| X_j(z) = x\right) \tag{5.14}$$

Similarly to (3.3) we get

$$E\hat{R}_j(x) = K_j(x)^{-1/\alpha_j}\Gamma\left(1 + \frac{1}{\alpha_j}\right). \tag{5.15}$$

We shall interpret (5.15) as our hedonic price index. Recall that $E\hat{R}_j$ may also be interpreted as a price index. To distinguish the two price index definitions we may call $E\hat{R}_j(x)$ and $E\hat{R}_j$ the conditional and unconditional hedonic price indexes, respectively. While $E\hat{R}_j(x)$ depends on the conditional distribution of offered prices given level x of the nonpecuniary attributes, $E\hat{R}_j$ depends on the unconditional distribution of offered prices and attributes.

When Assumption A8 holds we get immediately from (5.14) and (5.10) that

$$E\hat{R}_j(x) = e^{-x\theta_j}\left[\frac{E\left(P_j(z)^{\alpha_j\kappa_j}\right)}{c_j E\left(P_j(z)^{\alpha_j\kappa_j-\alpha_j}\right)}\right]^{1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right) = e^{-x\theta_j}\left(E\left(\hat{P}_j^{\alpha_j}\right)\right)^{1/\alpha_j}\Gamma\left(1+\frac{1}{\alpha_j}\right)c_j^{-1/\alpha_j}. \quad (5.16)$$

6. Conclusions

When consumer goods differ by quality and location, traditional demand analysis is no longer appropriate. In this paper we have demonstrated that by means of a particular probabilistic framework for discrete and continuous choice, it is possible to modify standard consumer theory to accommodate a rather general environment with a rich variety of product variants with product characteristics that may partly be unobservable to the analyst.

Finally, we present the corresponding analysis for the case of demand for indivisible goods of which some of the attributes may be observable and we also outline how hedonic price indexes can be established.

Appendix A

Lemma 1

Suppose that A2 holds and that

$$\lim_{\varepsilon_1 \downarrow 0} \int_{\varepsilon_1}^{\varepsilon_2} \mu_j(\varepsilon)\, d\varepsilon = 0.$$

Then with probability one there exists a point z in $\wp_j$ for which $(P_j(z), T_j(z) \in A)$, where A is an arbitrary open Borel set in R_+^2.

Proof:

Recall first that if A is a Borel set in R_+^2 the probability density of the number of points within $\tilde{A} \equiv A \times [\varepsilon_1, \varepsilon_2]$, $N_j(\tilde{A})$, for $\varepsilon_1 < \varepsilon_2$, is given by

$$P\left(N_j(\tilde{A}) = n\right) = \frac{M_j(\tilde{A})^n}{n!} \exp\left(-M_j(\tilde{A})\right)$$

where

$$M_j(\tilde{A}) = \int_{\tilde{A}} G_j(dp,dt)\, \mu_j(\varepsilon)\, d\varepsilon .$$

Hence, it follows that for A = [p,p+dp] × [t,t+dt] we get

$$P\left(N_j(\tilde{A}) \geq 1\right) = 1 - \exp\left(-G_j(dp,dt) \int_{\varepsilon_1}^{\varepsilon_2} \mu_j(\varepsilon)\, d\varepsilon\right).$$

Now suppose that μ_j has the property that

$$\lim_{\varepsilon_1 \downarrow 0} \int_{\varepsilon_1}^{\varepsilon_2} \mu_j(\varepsilon)\, d\varepsilon \rightarrow \infty.$$

Then it follows that $P(N_j(\tilde{A})\geq 1)$ converges towards one as $\varepsilon_1 \rightarrow 0$.

Q.E.D.

Proof of Theorem 1:

Recall that $(\hat{P}_j,\hat{T}_j)$ is defined by $\hat{P}_j=P_j(\hat{z}_j)$, $\hat{T}_j=T_j(\hat{z}_j)$ where

$$\hat{z}_j = \arg\max_z \left(\alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right)+\xi_j^*(z)\right) \tag{A.1}$$

where $\xi_j^*(z)=\alpha_j \log \xi_j(z)$. Let A be a Borel set in R_+^2 and define

$$L_j(A) = \max_{z \atop (P_j(z),T_j(z))\in A} \left(\alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right)+\xi_j^*(z)\right) \tag{A.2}$$

and

$$L_j(\bar{A}) = \max_{z \atop (P_j(z),T_j(z))\notin A} \left(\alpha_j \log\left(\frac{T_j(z)}{P_j(z)}\right)+\xi_j^*(z)\right). \tag{A.3}$$

It follows that $L_j(A)$ and $L_j(\bar{A})$ are stochastically independent because maximum is taken over Poisson points in disjoint sets, A and $\bar{A}$. Moreover, it can be demonstrated that $L_j(A)$ and $L_j(\bar{A})$ are extreme value distributed (type III) i.e.

$$P\left(L_j(C)\leq x\right) = \exp\left(-D_j(C)e^{-x}\right) \tag{A.4}$$

for C equal to A and $\bar{A}$, respectively, where

$$D_j(C) = c_j \iint_{(x,y)\in C} \left(\frac{y}{x}\right)^{\alpha_j} g(j,x,y)\,dx\,dy. \tag{A.5}$$

(See Dagsvik (1994), for a proof of this.) We have

$$P\left((\hat{P}_j, \hat{T}_j) \in A, \hat{R}_j > r\right)$$

$$= P\left(L_j(A) > L_j(\bar{A}),\ \max\left(L_j(A), L_j(\bar{A})\right) < r^{-\alpha_j}\right) \tag{A.6}$$

$$= P\left(L_j(A) > L_j(\bar{A}),\ L_j(A) < r^{-\alpha_j}\right).$$

Now by straight forward calculus (A.4) and (A.6) yield

$$P\left((\hat{P}_j, \hat{T}_j) \in A, \hat{R}_j > r\right)$$

$$= \frac{D_j(A)}{D_j(A) + D_j(\bar{A})} \exp\left(-r^{-\alpha_j}\left(D_j(A) + D_j(\bar{A})\right)\right)$$

$$= P\left(L_j(A) > L_j(\bar{A})\right) P\left(\max\left(L_j(A), L_j(\bar{A})\right) < r^{-\alpha_j}\right) \tag{A.7}$$

$$= P\left((\hat{P}_j, \hat{T}_j) \in A\right) P\left(\hat{R}_j > r\right)$$

which proves that $\hat{R}_j$ and $(\hat{P}_j, \hat{T}_j)$ are independent. Moreover, (A.7) also implies that

$$P\left(\hat{R}_j > r\right) = \exp\left(-r^{-\alpha_j}\left(D_j(A) + D_j(\bar{A})\right)\right). \tag{A.8}$$

Since

$$D_j(A) + D_j(\bar{A}) = D_j\left(A \cup \bar{A}\right) = D_j\left(R_+^2\right) = K_j,$$

(3.2) follows.

Q.E.D.

Appendix B

Allowing for savings/borrowing within a year

In the model framework developed above we treated the total expenditure, y_τ, in period τ as exogenous. We shall now allow y_τ to be determined according to the maximization of

$$\sum_{\tau=0}^{T-1} a^\tau U(\mathbf{Q}_\tau, \mathbf{T}_\tau) \tag{B.1}$$

subject to

$$\sum_{\tau=0}^{T-1} \sum_{j=1}^{m} \sum_{z} Q_j(z) P_{j\tau}(z) d_\tau \leq y \tag{B.2}$$

where

$$d_\tau = (1+r)^{-\tau},$$

r is the interest rate and $a=(1+f)^{-1}$, where f is the rate of time preference, and y is now the total income over T periods which is assumed to be exogenous.

Due to the structure of (B.1) and (B.2) the optimizing problem can be formulated as a two-stage budgeting problem where the incomes to be spent in each period are determined in the first stage and consumptions within periods are determined in the second stage. If y_τ is the (chosen) income to be spent in period τ the corresponding indirect utility as of period τ, $v(y_\tau)$, follows from (2.7) i.e.

$$v_\tau(y_\tau) = u\left(\bar{x}_1\left(\hat{\mathbf{R}}_\tau, y_\tau\right)/\hat{P}_{1\tau}, \dots, \bar{x}_m\left(\hat{\mathbf{R}}_\tau, y_\tau\right)/\hat{P}_{m\tau}\right). \tag{B.3}$$

The first stage optimization problem can thus be expressed as maximizing

$$\sum_{\tau=0}^{T-1} a^\tau v_\tau(y_\tau) \tag{B.4}$$

subject to

$$\sum_{\tau=0}^{T-1} y_\tau d_\tau \le y. \tag{B.5}$$

In the case where the demand system has the structure (5.1) it follows that the period-specific indirect utility is given by

$$v_\tau(y_\tau) = \log\left(y_\tau - \sum_{j=1}^{m} \gamma_j \hat{R}_{j\tau}\right) - \sum_{j=1}^{m} \beta_j \log \hat{R}_{j\tau}. \tag{B.6}$$

From (B.4), (B.5) and (B.6) it follows readily that the corresponding first stage solution is given by

$$y_\tau = \sum_{j=1}^{m} \gamma_j \hat{R}_{j\tau} + \frac{a^\tau \left(y - \sum_{k=0}^{T-1} d_k \sum_{j=1}^{m} \gamma_j \hat{R}_{jk}\right)(1-a)}{(1-a^T) d_\tau}. \tag{B.7}$$

In the particular case when r=0 and a=1, (B.7) reduces to

$$y_\tau = \sum_{j=1}^{m} \gamma_j \hat{R}_{j\tau} + \left(y - \sum_{k=0}^{T-1} \sum_{j=1}^{m} \gamma_j \hat{R}_{jk}\right) / T. \tag{B.8}$$

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